CS365 – Midterm Exam Review

Propositional Logic

- Proposition definition
- Logical Operators
  - NOT,AND, OR, X-OR
  - Implication, Contrapositive, Biconditional (study very well)
- Precedence of logical operators
- Tautologies, Contradictions
- Propositional Equivalence, Equivalnce laws
- Proving equivalences (truth table, symbolic derivations using equivalence laws)

Predicate Logic

- Predicate definition - predicates are not propositions!
- The Universal Quantifier $\forall$
- The Existential Quantifier $\exists$
- Quantifier Equivalence Laws
- Scope of quantifiers, free and bound variables, how to bind a variable
- Order of quantifiers is VERY important!

Proofs

- What is a proof?
- Rules of inference – prove theorems using rules of inference
- Fallacies (e.g., *affirming the conclusion*, *denying the hypothesis, circular reasoning*)
- Methods of proof for implications
  - Direct
  - Indirect
  - Vacuous, Trivial
  - Proof by Contradiction
  - Proof by Cases
  - Proof of Equivalence
  - Proof by counterexample
  - Proving existentials (constructive, non-constructive proofs)

Set Theory

- Definitions, Vein Diagrams, Membership Notation, Empty Set
- Subsets, Supersets, Set equality
- Cardinality, Power Set, Cartesian Product
- Set Operations
    - Union
    - Intersection
    - Difference
    - Complement
- Proving Set Identities (mutual subsets, membership tables)

Functions

- Definitions, Terminology (domain, co-domain, range, image, pre-image), Graphs
- Function composition
- One-to-One functions
- Onto functions, Sufficient Conditions
- Bijections
- Inverse of a function, Identity function
- Floor and Ceiling functions

Propositional Logic - Equivalence Laws

- *Identity*: $p \wedge \mathbf{T} \Leftrightarrow p$ $\quad p \vee \mathbf{F} \Leftrightarrow p$
- *Domination*: $p \vee \mathbf{T} \Leftrightarrow \mathbf{T}$ $\quad p \wedge \mathbf{F} \Leftrightarrow \mathbf{F}$
- *Idempotent*: $p \vee p \Leftrightarrow p$ $\quad p \wedge p \Leftrightarrow p$
- *Double negation:* $\neg\neg p \Leftrightarrow p$
- *Commutative:* $p \vee q \Leftrightarrow q \vee p$ $\quad p \wedge q \Leftrightarrow q \wedge p$
- *Associative:* $(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$
  $(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$
- *Distributive*: $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$
  $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$
- *De Morgan's*:
  $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$
  $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$
- *Absorption*:
  $p \vee (p \wedge q) \Leftrightarrow p$
  $p \wedge (p \vee q) \Leftrightarrow p$
- *Trivial tautology/contradiction*:
  $p \vee \neg p \Leftrightarrow \mathbf{T}$ $\quad p \wedge \neg p \Leftrightarrow \mathbf{F}$
- $p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p$
- $p \rightarrow q \Leftrightarrow \neg p \vee q$

Predicate Logic - Equivalence Laws

- $\forall x\, P(x) \Leftrightarrow P(\text{a}) \wedge P(\text{b}) \wedge P(\text{c}) \wedge \ldots$
  $\exists x\, P(x) \Leftrightarrow P(\text{a}) \vee P(\text{b}) \vee P(\text{c}) \vee \ldots$

- $\forall x\, P(x) \Leftrightarrow \exists\neg x\, \neg P(x)$
  $\exists x\, P(x) \Leftrightarrow \neg\forall x\, \neg P(x)$

- $\exists\neg x\, P(x) \Leftrightarrow \forall x\, \neg\, P(x)$
  $\neg\forall x\, P(x) \Leftrightarrow \exists x\, \neg\, P(x)$

- $\forall x\, \forall y\, P(x,y) \Leftrightarrow \forall y\, \forall x\, P(x,y)$
  $\exists x\, \exists y\, P(x,y) \Leftrightarrow \exists y\, \exists x\, P(x,y)$

- $\forall x\, \mathbf{(}P(x) \wedge Q(x)\mathbf{)} \Leftrightarrow \mathbf{(}\forall x\, P(x)\mathbf{)} \wedge \mathbf{(}\forall x\, Q(x)\mathbf{)}$
  $\exists x\, \mathbf{(}P(x) \vee Q(x)\mathbf{)} \Leftrightarrow \mathbf{(}\exists x\, P(x)\mathbf{)} \vee \mathbf{(}\exists x\, Q(x)\mathbf{)}$

Inference Rules

- p Rule of Addition
  $\therefore p \vee q$

- $p \wedge q$ Rule of Simplification
  $\therefore p$

- p
  q
  $\therefore p \wedge q$ Rule of Conjunction

- p Rule of *modus ponens*
  $\underline{p \to q}$
  $\therefore q$

- $\neg q$
  $\underline{p \to q}$ Rule of *modus tollens*
  $\therefore \neg p$

- $p \to q$ Rule of hypothetical
  $q \to r$ syllogism
  $\therefore p \to r$

- $p \vee q$ Rule of disjunctive
  $\neg p$ syllogism
  $\therefore q$

- $\forall x\, P(x)$
  $\therefore P(o)$ (substitute *any* object o)

- $P(g)$ (for g a *general* element of u.d.)
  $\therefore \forall x\, P(x)$

- $\exists x\, P(x)$
  $\therefore P(c)$ (substitute a *new constant* c)

- $P(o)$ (substitute any extant object o)
  $\exists \therefore x\ P(x)$

Set Identities

- Identity: $A\emptyset\cup=A$ $A\cap U=A$
- Domination: $A\cup U=U$ $A\emptyset\cap=\emptyset$
- Idempotent: $A\cup A = A = A\cap A$
- Double complement: $\overline{(\overline{A})} = A$
- Commutative: $A\cup B=B\cup A$ $A\cap B=B\cap A$
- Associative: $A\cup(B\cup C)=(A\cup B)\cup C$
  $A\cap(B\cap C)=(A\cap B)\cap C$
- DeMorgan's Laws

$$\overline{A\cup B} = \overline{A}\cap\overline{B}$$
$$\overline{A\cap B} = \overline{A}\cup\overline{B}$$