

CS365 Mathematics of Computer Science
Fall 2005 – Dr. George Bebis
Midterm Exam

Name: _____

1. (25 points) True/False Questions – To get credit, you must give brief reasons for each answer!

T F The logical expression $\exists x (P(x) \wedge (P(y) \wedge y \neq x))$ represents a valid proposition.

T F The value of the implication $"(1=0) \rightarrow \text{pigs can fly}"$ is always TRUE.

T F The composition $f \circ g$ of any two functions f and g is always defined.

T F Suppose set A contains 3 elements and set B contains 2 elements. Then, cardinality of $A \times B$ is 5.

T F Suppose $p = "I \text{ am elected}"$ and $q = "I \text{ will lower taxes}"$. The proposition "*If I am not elected then, I will not lower taxes*" represents a fallacy of affirming the hypothesis.

2. (15 points) Using symbolic derivations (i.e., not truth tables), show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent.

3. (15 points) Construct an argument using rules of inference to show that the hypotheses "Randy works hard", "If Randy works hard, then he is a dull boy", and "if Randy is a dull boy, then he will not get the job", imply the conclusion "Randy will not get the job". Indicate clearly which inference rule you use at each step of your derivation.

4. **(15 points)** Prove or disprove that if A, B, and C are sets then
 $A - (B \cap C) = (A - B) \cap (A - C)$

5. **(15 points)** Prove the following statement using proof by contradiction:
"The sum of an even integer and an odd integer is always odd"

6. **(15 points)** Consider the function given below where x is a real number. What are the domain, co-domain, and range of f ? Is f invertible?. If yes, find its inverse. If not, what would be the necessary conditions for f to become invertible?

$$f(x) = (x-1)/(x-2)$$