CS365 Mathematics of Computer Science
Fall 2005 – Dr. George Bebis
Final Exam

Name:____

1. **(30 points)** True/False Questions – To get credit, you must give brief reasons for each answer!

   T F If $f(x) \in O(g(x))$, then $f(x) \in \Theta(g(x))$.

   T F The number of selecting five players from a 10-member tennis team is 252.

   T F Recursive algorithms are always more efficient in terms of time and memory than their iterative counterparts.

   T F Any two mutually exclusive events are also independent.

   T F A random variable is a variable in the sample space of a random experiment.

T F Any relation *R* from A to B represents also a function *f* from A to B.

T F In any group of 27 English words, there must be at least two that begin with the same letter.

T F Given any two matrices A and B, then AB=BA.

T F Binary search is always faster than linear search.

T F A problem or algorithm with at most logarithmic time complexity is considered *tractable*.

2. (10 points) Prove that $2^n \in O(3^n)$ but $3^n \notin O(2^n)$

3. (10 points) Prove that every amount of postage of 12 cents or more can be formed using just 4-cent and 5-cent stamps.

4. (10 points) Prove that $\sum_{k=1}^{n} k2^k = (n-1)2^{n+1} + 2$.

5. (10 points) Suppose that a die is biased (or loaded) so 3 appears four times as often as each other number but that the other five outcomes are equally likely. What is the probability that an odd number appears when we roll the dice?

6. (10 points) Each user on a computer system has a password, each being six to eight characters long. Each character is an uppercase letter or a digit. Also, each password must contain at least one digit. How many possible passwords are there?

7. (10 points) Determine whether the relation R: $xy \geq 0$ on the set of all real numbers is reflexive, symmetric, antisymmetric, and/or transitive.

8. (10 points) Sum both sides of the identity $k^2 - (k-1)^2 = 2k - 1$ from k=1 to k=n to find a formula for $\sum_{k=1}^{n} (2k-1)$ (i.e, the sum of the first n odd natural numbers). If needed, you can use the following formula in your proof: $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.