

Fourier Transform Table

$x(t)$	$X(f)$	$X(\omega)$
$\delta(t)$	1	1
1	$\delta(f)$	$2\pi\delta(\omega)$
$\delta(t - t_0)$	$e^{-j2\pi ft_0}$	$e^{-j\omega t_0}$
$e^{j2\pi f_0 t}$	$\delta(f - f_0)$	$2\pi\delta(\omega - \omega_0)$
$\cos(2\pi f_0 t)$	$\frac{1}{2}[\delta(f - f_0) + \delta(f + f_0)]$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
$\sin(2\pi f_0 t)$	$\frac{1}{2j}[\delta(f - f_0) - \delta(f + f_0)]$	$-j\pi[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
$rect(t)$	$sinc(f)$	$sinc\left(\frac{\omega}{2\pi}\right)$
$sinc(t)$	$rect(f)$	$rect\left(\frac{\omega}{2\pi}\right)$
$\Lambda(t)$	$sinc^2(f)$	$sinc^2\left(\frac{\omega}{2\pi}\right)$
$sinc^2(t)$	$\Lambda(f)$	$\Lambda\left(\frac{\omega}{2\pi}\right)$
$e^{-\alpha t}u(t), \alpha > 0$	$\frac{1}{\alpha + j2\pi f}$	$\frac{1}{\alpha + j\omega}$
$te^{-\alpha t}u(t), \alpha > 0$	$\frac{1}{(\alpha + j2\pi f)^2}$	$\frac{1}{(\alpha + j\omega)^2}$
$e^{-\alpha t }, \alpha > 0$	$\frac{2\alpha}{(\alpha^2 + (2\pi f)^2)}$	$\frac{2\alpha}{(\alpha^2 + \omega^2)}$
$e^{-\pi t^2}$	$e^{-\alpha f^2}$	$e^{-\alpha f^2}$
$sgn(t)$	$\frac{1}{j\pi f}$	$\frac{2}{j\omega}$
$u(t)$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$	$\pi\delta(\omega) + \frac{1}{j\omega}$
$\frac{d}{dt}\delta(t)$	$j2\pi f$	$j\omega$
$\sum_{n=-\infty}^{\infty} \delta(t - nT_0)$	$\frac{1}{T_0} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_0}\right)$	$\frac{1}{T_0} \sum_{n=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi n}{T_0}\right)$

$u(t)e^{-\alpha t} \sin(\omega_0 t)$	$\frac{\omega_0}{\omega_0^2 + (\alpha + j\omega)^2}$
$e^{-\alpha t }$	$\frac{2\alpha}{\alpha^2 + \omega^2}$
$e^{-t^2/(2\sigma^2)}$	$\sigma\sqrt{2\pi} e^{-\sigma^2\omega^2/2}$
$u(t)e^{-\alpha t}$	$\frac{1}{\alpha + j\omega}$
$u(t)te^{-\alpha t}$	$\frac{1}{(\alpha + j\omega)^2}$

➤ **Trigonometric Fourier Series**

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(\omega_0 nt) + b_n \sin(\omega_0 nt))$$

where

$$a_0 = \frac{1}{T} \int_0^T f(t) dt, \quad a_n = \frac{2}{T} \int_0^T f(t) \cos(\omega_0 nt) dt, \quad \text{and}$$

$$b_n = \frac{2}{T} \int_0^T f(t) \sin(\omega_0 nt) dt$$

➤ **Complex Exponential Fourier Series**

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{j\omega_0 nt}, \quad \text{where} \quad F_n = \frac{1}{T} \int_0^T f(t) e^{-j\omega_0 nt} dt$$

Some Useful Mathematical Relationships

$\cos(x) = \frac{e^{jx} + e^{-jx}}{2}$
$\sin(x) = \frac{e^{jx} - e^{-jx}}{2j}$
$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$
$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$
$\cos(2x) = \cos^2(x) - \sin^2(x)$
$\sin(2x) = 2\sin(x)\cos(x)$
$2\cos^2(x) = 1 + \cos(2x)$
$2\sin^2(x) = 1 - \cos(2x)$
$\cos^2(x) + \sin^2(x) = 1$
$2\cos(x)\cos(y) = \cos(x - y) + \cos(x + y)$
$2\sin(x)\sin(y) = \cos(x - y) - \cos(x + y)$
$2\sin(x)\cos(y) = \sin(x - y) + \sin(x + y)$

Useful Integrals

$\int \cos(x) dx$	$\sin(x)$
$\int \sin(x) dx$	$-\cos(x)$
$\int x \cos(x) dx$	$\cos(x) + x \sin(x)$
$\int x \sin(x) dx$	$\sin(x) - x \cos(x)$
$\int x^2 \cos(x) dx$	$2x \cos(x) + (x^2 - 2) \sin(x)$
$\int x^2 \sin(x) dx$	$2x \sin(x) - (x^2 - 2) \cos(x)$
$\int e^{\alpha x} dx$	$\frac{e^{\alpha x}}{\alpha}$
$\int x e^{\alpha x} dx$	$e^{\alpha x} \left[\frac{x}{\alpha} - \frac{1}{\alpha^2} \right]$
$\int x^2 e^{\alpha x} dx$	$e^{\alpha x} \left[\frac{x^2}{\alpha} - \frac{2x}{\alpha^2} + \frac{2}{\alpha^3} \right]$
$\int \frac{dx}{\alpha + \beta x}$	$\frac{1}{\beta} \ln \alpha + \beta x $
$\int \frac{dx}{\alpha^2 + \beta^2 x^2}$	$\frac{1}{\alpha\beta} \tan^{-1}\left(\frac{\beta x}{\alpha}\right)$