Sample Final

**CS474/674 Image Processing and Interpretation**
**Sample Final Exam**

**Name:** Solutions

1. **[20 points – 4pts each]** True/False Questions – To get credit, **you must give brief reasons for each answer!**

**T F** The lower the frequency of a sinusoidal, the more samples must be taken to gain an accurate representation of the wave.

F From Nyquist Theorem: $\Delta x \leq \frac{1}{2w}$ w: max freq.
if $w_1 < w_2 \Rightarrow \frac{1}{2w_1} > \frac{1}{2w_2}$ or $\Delta x_1 > \Delta x_2$
bigger sampling step implies less samples

**T F** Arithmetic coding requires knowledge of pixel intensity frequencies.

T The interval [0,1) is subdivided based on the ~~probabity~~ of the pixel intensities
probability

**T F** The Fourier transform of the product of two functions is the product of the Fourier transforms of the functions (i.e., F[f(x)g(x)] = F[f(x)] F[g(x)] )

F $\mathcal{F}\{f(x)g(x)\} = \mathcal{F}\{f(x)\} * \mathcal{F}\{g(x)\}$

convolution

**T F** Inverse filtering will yield as good results as Wiener filtering when processing an image which has been degraded by motion blurring **only** .

T If there is no noise, both filters will yield equally good results.

**T F** Median filtering can be implemented efficiently using convolution.

F Median-filtering is non-linear

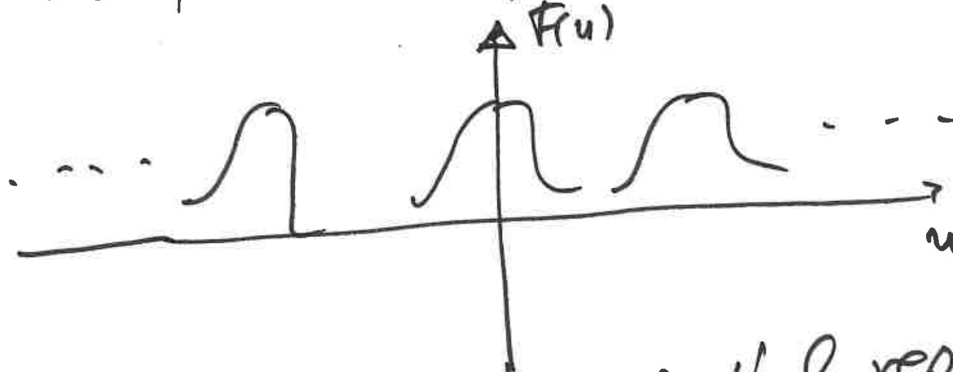
2. **[15 points]** Prove the following property of the Fourier Transform:

$$f(x)e^{j2\pi u_0 x} \leftrightarrow F(u-u_0)$$

< see class notes >

3. **[15 pts]** When sampling a band-limited function, aliasing can be avoided by obtaining a sufficient number of samples, as stated by the Nyquist criterion. In practice, however, aliasing cannot be avoided in general, even if the Nyquist criterion is satisfied. Explain why this is the case and justify your answer.

- The DFT of a sampled $f(x)$ is periodic:
- Choosing the sampling step using $\Delta x \leq \frac{1}{2W}$ ensures that the periods do <u>not</u> overlap




- However, since the spatial representation of a band-limited function has infinite duration, we need to truncate the samples by multiplying $f(x)$ with a $rect(x)$ function

$$f(x)\, rect(x) \longleftrightarrow F(u) * \underbrace{Sinc(u)}_{\text{infinite duration}}$$

which implies convolution with a $Sinc()$

i.e., cannot separate a "single" period anymore!

4. **[15 pts]** The following image is given:

21 21 21 95 169 243 243 243
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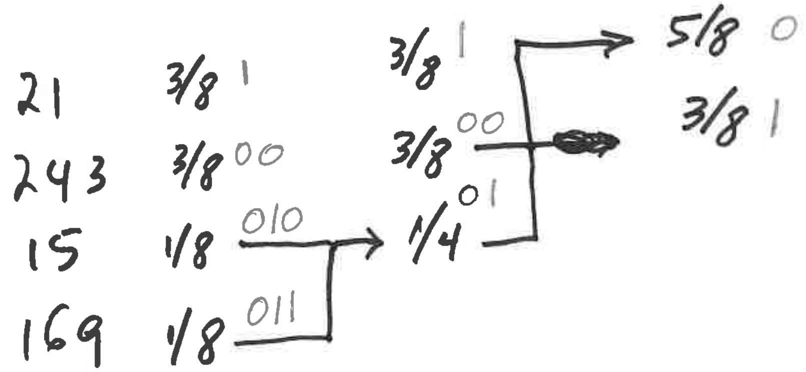
$$H = -\sum_{k=0}^{3} P(r_k)\log(P(r_k)) =$$

1.81 bits/pixel

(a) Compute its first order entropy.

| Gray level | Count | Prob |
|---|---|---|
| 21 | 12 | 3/8 |
| 15 | 4 | 1/8 |
| 169 | 4 | 1/8 |
| 243 | 12 | 3/8 |

(b) Construct the Huffman code for this image.




(c) What is the average number of bits/pixel in this case? Is this a good code?

$$L_{avg} = \sum_{i=0}^{3} l(r_k)\,P(r_k) = 1\times 3/8 + 2\times 3/8 + 3\times 1/8 + 3\times 1/8$$

$$= 1.875 \text{ bits/pixel}$$

$L_{avg}$ is very close to $H$, so Yes

data

information

**5. [15 points]** Explain how homomorphic filtering works. What is the reason that we first apply the log() function on the image?

We assume $f(x,y) = i(x,y)\, r(x,y)$

since $\mathcal{F}\{f(x,y)\} = \underbrace{\mathcal{F}\{i(x,y)\}}_{\text{low freq. content}} * \underbrace{\mathcal{F}\{r(x,y)\}}_{\text{high freq content}}$

We use $\log(f(x,y))$ to separate the low freq from $i(x,y)$ from the high freq from $r(x,y)$

i.e., $\mathcal{F}\{\log(f(x,y))\} = \mathcal{F}\{\log(i(x,y))\} + \mathcal{F}\{\log(r(x,y))\}$

Steps of Homomorphic filtering:

1. Take log() of $f(x,y)$
2. Compute DFT
3. Apply filter $F(u,v)\, H(u,v) = G(u,v)$
4 Compute inverse DFT
5 Apply exp() on $g(x,y)$

**6. [20 points]** How does progressive JPEG work? Discuss at least two different progressive JPEG methods.

Progressive JPEG: encode image in multiple scans (instead of sequentially)

(1) Progressive spectral selection

(2) Progressive successive approximation

(See class notes)

**7. (Graduate Students Only)** This problem is on image restoration.

**(a) [10 points]** What do we mean in image restoration when we say that the degradation function H is linear and shift-invariant?

Linearity: $H[f_1+f_2]=H[f_1]+H[f_2]$

$H[af]=aH[f]$

Shift invariance:

if $H[f(x,y)]=g(x,y)$ then

$H[f(x-a,y-b)]=g(x-a,y-b)$

**(b) [10 points]** How is the degradation process modeled assuming linearity and shift invariance? **Prove it.**

$H[f(x,y)] = f(x,y) * h(x,y)$

where $h(x,y)=H[\delta(x,y)]$

(impulse response)

Proof: see class notes