CS477/677 Analysis of Algorithms
Fall 2006 – Dr. George Bebis
Final Exam
2:15-4:15 PM

Name:_____

1. **(30 points)** True/False Questions – To get credit, **you must give brief reasons for each answer!**

   T F H is an array representing a heap. The elements of H are in non-increasing order.

   T F Counting Sort will take $O(n^3)$ time to sort n numbers in the range $[1..n^3]$

   T F By Case 3 of Master's method, the solution to the recurrence $T(n)=2T(n/2) + 2n\lg n$ is $\Theta(n\lg n)$

   T F An $O(n^2)$ algorithm is always faster than an $O(n^3)$ algorithm.

T **F** The reason that we use randomization in Quicksort is to eliminate the worst-case scenario.

T **F** A family of hash functions $H=\{h: U \dashrightarrow [0..m-1]\}$ is called **universal** if there are no two functions from H giving the same value on two different keys.

T **F** The maximum number of nodes possible in a heap of height h is $2^{h+1}-1$

T **F** Linear probing can generate at most n^2 different probing sequences.

T **F** MAX-HEAP-INSERT runs in $O(\lg n)$ time.

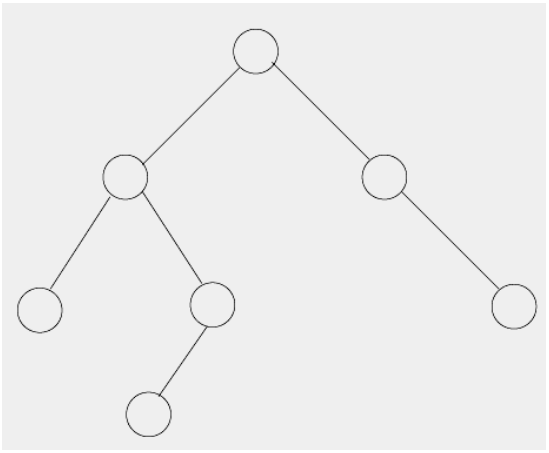
T **F** Both Heapsort and Quicksort are in-place algorithms.

2. **[10 points]** Prove or disprove that $(n+a)^b=\Theta(n^b)$ where a and b are positive constants (non-zero).

3. **[10 points]** Suppose that you are given n numbers and you are asked to find the sum of the largest 10 of them where $10 << n$. Propose an efficient solution to this problem and discuss its running time using asymptotic notation (*note:* your solution must be based only on the algorithms and data structures discussed in class).

4. [15 points] This problem is about red-black trees.

(a) Label the following binary tree with numbers from the set [6,22,9,14,13,1,8] so that it is a legal binary search tree.



(b) Label each node in the figure above with r or b denoting the colors RED and BLACK, respectively, so that the tree is a legal red-black tree.

(c) Make the left child of the root be the root by performing a single rotation. Draw the binary search tree that results and label your tree with the keys from part (a). Is it possible to label the nodes with colors so that the tree is a valid red-black tree? Justify your answer.

5. **[15 points]** Write and solve the recurrence for the running time of the following algorithm:

```
Alg: example(a,n)
  {
   if (n == 1)
       return a
   m = n/2
   return example(a,m) * example(a,n-m)
  }
```

6. **[10 points]** A priority queue can be implemented using heaps or linked lists. Discuss the advantages/disadvantages of each implementation.

6. (Undergraduate Students only) [10 pts] (a) Demonstrate the insertion of keys 5, 28, 19, 15, 20, 33, 12, 17, 10 into a hash table with collisions resolved by chaining. Let the table have 7 slots and the hash function be $h(k)=k \bmod 7$.

(b) Analyze both the worst-case and average-case of searching using hashing with chaining.

7. **(Graduate Students only) [10 pts]** Prove that any comparison sort algorithm requires $\Omega(n \lg n)$ comparisons in the worst case. To get full credit, you must explain your arguments clearly and carefully!