We summarize the preceding approach as follows. Let $f(x, y)$ represent the value of an image at any image coordinates (x, y), and let $g(x, y)$ represent the corresponding enhanced value at those coordinates. Then,

$$g(x, y) = \begin{cases} E \cdot f(x, y) & \text{if } m_{S_{xy}} \leq k_0 m_G \text{ AND } k_1 \sigma_G \leq \sigma_{S_{xy}} \leq k_2 \sigma_G \\ f(x, y) & \text{otherwise} \end{cases} \tag{3.3-24}$$

for $x = 0, 1, 2, \ldots, M - 1$ and $y = 0, 1, 2, \ldots, N - 1$, where, as indicated above, E, k_0, k_1, and k_2 are specified parameters, m_G is the global mean of the input image, and σ_G is its standard deviation. Parameters $m_{S_{xy}}$ and $\sigma_{S_{xy}}$ are the local mean and standard deviation, respectively. As usual, M and N are the row and column image dimensions.

Choosing the parameters in Eq. (3.3-24) generally requires a bit of experimentation to gain familiarity with a given image or class of images. In this case, the following values were selected: $E = 4.0$, $k_0 = 0.4$, $k_1 = 0.02$, and $k_2 = 0.4$. The relatively low value of 4.0 for E was chosen so that, when it was multiplied by the levels in the areas being enhanced (which are dark), the result would still tend toward the dark end of the scale, and thus preserve the general visual balance of the image. The value of k_0 was chosen as less than half the global mean because we can see by looking at the image that the areas that require enhancement definitely are dark enough to be below half the global mean. A similar analysis led to the choice of values for k_1 and k_2. Choosing these constants is not difficult in general, but their choice definitely must be guided by a logical analysis of the enhancement problem at hand. Finally, the size of the local area S_{xy} should be as small as possible in order to preserve detail and keep the computational burden as low as possible. We chose a region of size 3×3.

As a basis for comparison, we enhanced the image using global histogram equalization. Figure 3.27(b) shows the result. The dark area was improved but details still are difficult to discern, and the light areas were changed, something we did not want to do. Figure 3.27(c) shows the result of using the local statistics method explained above. In comparing this image with the original in Fig. 3.27(a) or the histogram equalized result in Fig. 3.27(b), we note the obvious detail that has been brought out on the right side of Fig. 3.27(c). Observe, for example, the clarity of the ridges in the dark filaments. It is noteworthy that the light-intensity areas on the left were left nearly intact, which was one of our initial objectives. ■

3.4 Fundamentals of Spatial Filtering

In this section, we introduce several basic concepts underlying the use of spatial filters for image processing. Spatial filtering is one of the principal tools used in this field for a broad spectrum of applications, so it is highly advisable that you develop a solid understanding of these concepts. As mentioned at the beginning of this chapter, the examples in this section deal mostly with the use

The name *filter* is borrowed from frequency domain processing, which is the topic of the next chapter, where "filtering" refers to accepting (passing) or rejecting certain frequency components. For example, a filter that passes low frequencies is called a *lowpass* filter. The net effect produced by a lowpass filter is to blur (smooth) an image. We can accomplish a similar smoothing directly on the image itself by using spatial filters (also called spatial *masks*, *kernels*, *templates*, and *windows*). In fact, as we show in Chapter 4, there is a one-to-one correspondence between linear spatial filters and filters in the frequency domain. However, spatial filters offer considerably more versatility because, as you will see later, they can be used also for nonlinear filtering, something we cannot do in the frequency domain.

See Section 2.6.2 regarding linearity.

3.4.1 The Mechanics of Spatial Filtering

In Fig. 3.1, we explained briefly that a spatial filter consists of (1) a *neighborhood*, (typically a small rectangle), and (2) a *predefined operation* that is performed on the image pixels encompassed by the neighborhood. Filtering creates a new pixel with coordinates equal to the coordinates of the center of the neighborhood, and whose value is the result of the filtering operation.† A processed (filtered) image is generated as the center of the filter visits each pixel in the input image. If the operation performed on the image pixels is linear, then the filter is called a *linear spatial filter*. Otherwise, the filter is *nonlinear*. We focus attention first on linear filters and then illustrate some simple nonlinear filters. Section 5.3 contains a more comprehensive list of nonlinear filters and their application.

Figure 3.28 illustrates the mechanics of linear spatial filtering using a 3×3 neighborhood. At any point (x, y) in the image, the response, $g(x, y)$, of the filter is the sum of products of the filter coefficients and the image pixels encompassed by the filter:

$$\begin{aligned} g(x, y) = {} & w(-1, -1) f(x - 1, y - 1) + w(-1, 0) f(x - 1, y) + \ldots \\ & + w(0, 0) f(x, y) + \ldots + w(1, 1) f(x + 1, y + 1) \end{aligned}$$

Observe that the center coefficient of the filter, $w(0, 0)$, aligns with the pixel at location (x, y). For a mask of size $m \times n$, we assume that $m = 2a + 1$ and $n = 2b + 1$, where a and b are positive integers. This means that our focus in the following discussion is on filters of odd size, with the smallest being of size 3×3. In general, linear spatial filtering of an image of size $M \times N$ with a filter of size $m \times n$ is given by the expression:

It certainly is possible to work with filters of even size or mixed even and odd sizes. However, working with odd sizes simplifies indexing and also is more intuitive because the filters have centers falling on integer values.

$$g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x + s, y + t)$$

where x and y are varied so that each pixel in w visits every pixel in f.

†The filtered pixel value typically is assigned to a corresponding location in a new image created to hold the results of filtering. It is seldom the case that filtered pixels replace the values of the corresponding

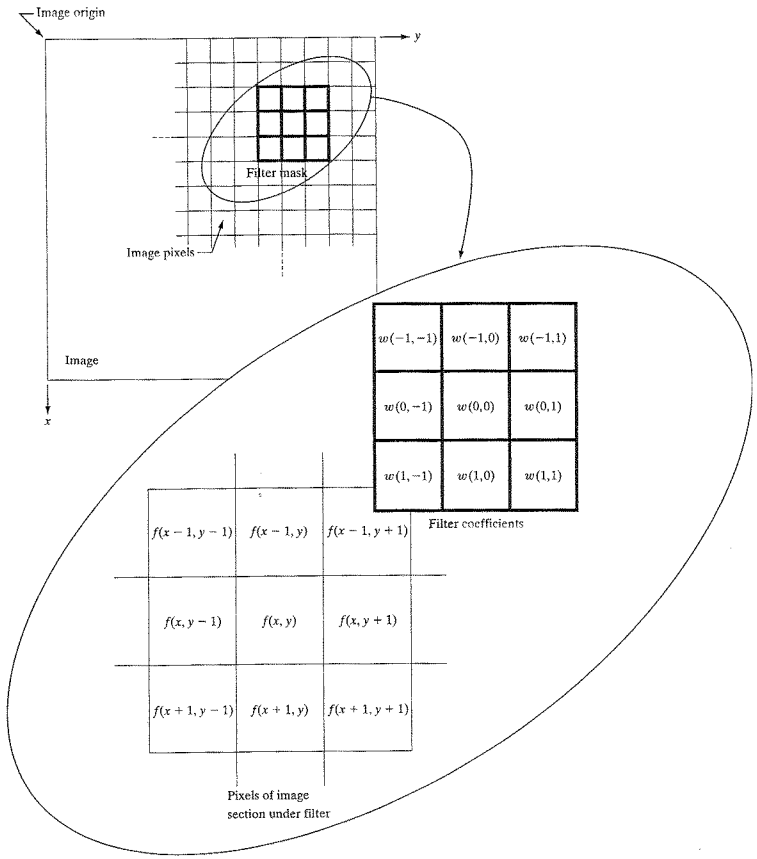


FIGURE 3.28 The mechanics of linear spatial filtering using a 3×3 filter mask. The form chosen to denote the coordinates of the filter mask coefficients simplifies writing expressions for linear filtering.

3.4.2 Spatial Correlation and Convolution

There are two closely related concepts that must be understood clearly when performing linear spatial filtering. One is *correlation* and the other is *convolution*. Correlation is the process of moving a filter mask over the image and computing the sum of products at each location, exactly as explained in the previous section. The mechanics of convolution are the same, except that the filter is first rotated by 180°. The best way to explain the differences between the two concepts is by example. We begin with a 1-D illustration.

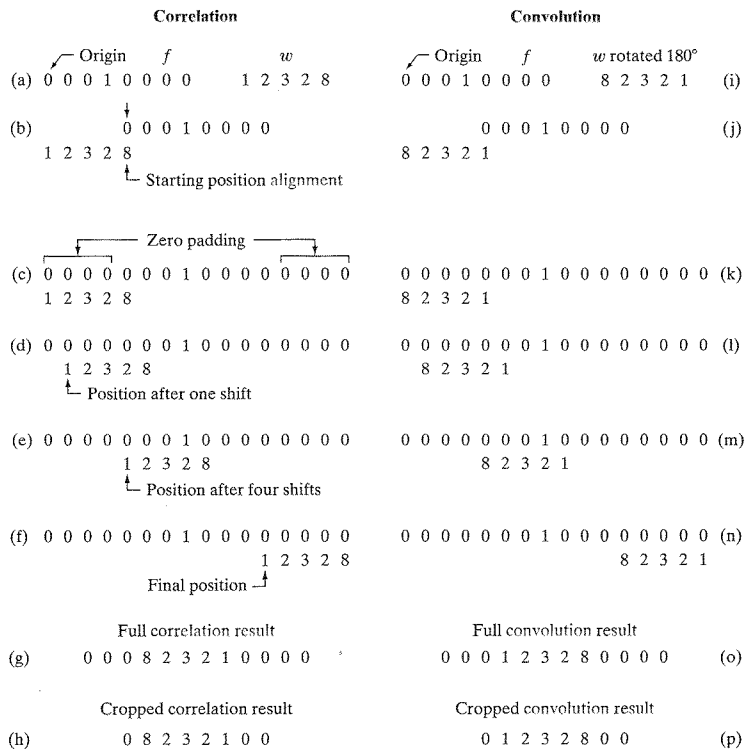


FIGURE 3.29 Illustration of 1-D correlation and convolution of a filter with a discrete unit impulse. Note that correlation and convolution are functions of *displacement*.

are parts of the functions that do not overlap. The solution to this problem is to pad f with enough 0s on either side to allow each pixel in w to visit every pixel in f. If the filter is of size m, we need $m - 1$ 0s on either side of f. Figure 3.29(c) shows a properly padded function. The first value of correlation is the sum of products of f and w for the initial position shown in Fig. 3.29(c) (the sum of products is 0). This corresponds to a displacement $x = 0$. To obtain the second value of correlation, we shift w one pixel location to the right (a displacement of $x = 1$) and compute the sum of products. The result again is 0. In fact, the first nonzero result is when $x = 3$, in which case the 8 in w overlaps the 1 in f and the result of correlation is 8. Proceeding in this manner, we obtain the full correlation result in Fig. 3.29(g). Note that it took 12 values of x (i.e., $x = 0, 1, 2, \ldots, 11$) to fully slide w past f so that each pixel in w visited every pixel in f. Often, we like to work with correlation arrays that are the same size as f, in which case we crop

Zero padding is not the only option. For example, we could duplicate the value of the first and last element $m - 1$ times on each side of f, or mirror the first and last $m - 1$ elements and use the mirrored values for padding.

There are two important points to note from the discussion in the preceding paragraph. First, correlation is a function of *displacement* of the filter. In other words, the first value of correlation corresponds to zero displacement of the filter, the second corresponds to one unit displacement, and so on. The second thing to notice is that correlating a filter w with a function that contains all 0s and a single 1 yields a result that is a *copy* of w, but *rotated* by 180°. We call a function that contains a single 1 with the rest being 0s a *discrete unit impulse*. So we conclude that correlation of a function with a discrete unit impulse yields a rotated version of the function at the location of the impulse.

The concept of convolution is a cornerstone of linear system theory. As you will learn in Chapter 4, a fundamental property of convolution is that convolving a function with a unit impulse yields a copy of the function at the location of the impulse. We saw in the previous paragraph that correlation yields a copy of the function also, but rotated by 180°. Therefore, if we *pre-rotate* the filter and perform the same sliding sum of products operation, we should be able to obtain the desired result. As the right column in Fig. 3.29 shows, this indeed is the case. Thus, we see that to perform convolution all we do is rotate one function by 180° and perform the same operations as in correlation. As it turns out, it makes no difference which of the two functions we rotate.

Note that rotation by 180° is equivalent to flipping the function horizontally.

The preceding concepts extend easily to images, as Fig. 3.30 shows. For a filter of size $m \times n$, we pad the image with a minimum of $m - 1$ rows of 0s at the top and bottom and $n - 1$ columns of 0s on the left and right. In this case, m and n are equal to 3, so we pad f with two rows of 0s above and below and two columns of 0s to the left and right, as Fig. 3.30(b) shows. Figure 3.30(c) shows the initial position of the filter mask for performing correlation, and Fig. 3.30(d) shows the full correlation result. Figure 3.30(e) shows the corresponding cropped result. Note again that the result is rotated by 180°. For convolution, we pre-rotate the mask as before and repeat the sliding sum of products just explained. Figures 3.30(f) through (h) show the result. You see again that convolution of a function with an impulse copies the function at the location of the impulse. It should be clear that, if the filter mask is symmetric, correlation and convolution yield the same result.

In 2-D, rotation by 180° is equivalent to flipping the mask along one axis and then the other.

If, instead of containing a single 1, image f in Fig. 3.30 had contained a region identically equal to w, the value of the correlation function (after normalization) would have been maximum when w was centered on that region of f. Thus, as you will see in Chapter 12, correlation can be used also to find *matches* between images.

Summarizing the preceding discussion in equation form, we have that the correlation of a filter $w(x, y)$ of size $m \times n$ with an image $f(x, y)$, denoted as $w(x, y) \star f(x, y)$, is given by the equation listed at the end of the last section, which we repeat here for convenience:

$$w(x, y) \star f(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x + s, y + t) \qquad (3.4\text{-}1)$$

This equation is evaluated for all values of the displacement variables x and y so that all elements of w visit every pixel in f, where we assume that f has been

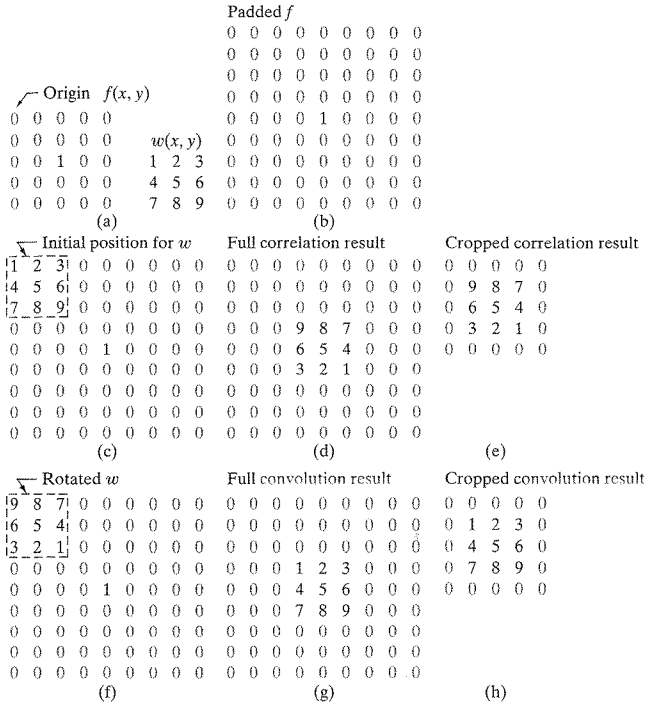


FIGURE 3.30 Correlation (middle row) and convolution (last row) of a 2-D filter with a 2-D discrete, unit impulse. The 0s are shown in gray to simplify visual analysis.

In a similar manner, the convolution of $w(x, y)$ and $f(x, y)$, denoted by $w(x, y) \star f(x, y)$,[†] is given by the expression

$$w(x, y) \star f(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x - s, y - t) \qquad (3.4\text{-}2)$$

where the minus signs on the right flip f (i.e., rotate it by 180°). Flipping and shifting f instead of w is done for notational simplicity and also to follow convention. The result is the same. As with correlation, this equation is evaluated for all values of the displacement variables x and y so that every element of w visits every pixel in f, which we assume has been padded appropriately. You should expand Eq. (3.4-2) for a 3×3 mask and convince yourself that the result using this equation is identical to the example in Fig. 3.30. In practice, we frequently work with an algorithm that implements

Often, when the meaning is clear, we denote the result of correlation or convolution by a function $g(x, y)$, instead of writing $w(x, y) \star f(x, y)$ or $w(x, y) \star f(x, y)$. For example, see the equation at the end of the previous section, and Eq. (3.5-1).

Eq. (3.4-1). If we want to perform correlation, we input w into the algorithm; for convolution, we input w rotated by 180°. The reverse is true if an algorithm that implements Eq. (3.4-2) is available instead.

As mentioned earlier, convolution is a cornerstone of linear system theory. As you will learn in Chapter 4, the property that the convolution of a function with a unit impulse copies the function at the location of the impulse plays a central role in a number of important derivations. We will revisit convolution in Chapter 4 in the context of the Fourier transform and the convolution theorem. Unlike Eq. (3.4-2), however, we will be dealing with convolution of functions that are of the same size. The form of the equation is the same, but the limits of summation are different.

Using correlation or convolution to perform spatial filtering is a matter of preference. In fact, because either Eq. (3.4-1) or (3.4-2) can be made to perform the function of the other by a simple rotation of the filter, what is important is that the filter mask used in a given filtering task be specified in a way that corresponds to the intended operation. All the linear spatial filtering results in this chapter are based on Eq. (3.4-1).

Finally, we point out that you are likely to encounter the terms, *convolution filter*, *convolution mask* or *convolution kernel* in the image processing literature. As a rule, these terms are used to denote a spatial filter, and not necessarily that the filter will be used for true convolution. Similarly, "convolving a mask with an image" often is used to denote the sliding, sum-of-products process we just explained, and does not necessarily differentiate between correlation and convolution. Rather, it is used generically to denote either of the two operations. This imprecise terminology is a frequent source of confusion.

3.4.3 Vector Representation of Linear Filtering

When interest lies in the characteristic response, R, of a mask either for correlation or convolution, it is convenient sometimes to write the sum of products as

$$\begin{aligned} R &= w_1 z_1 + w_2 z_2 + \ldots + w_{mn} z_{mn} \\ &= \sum_{k=1}^{mn} w_k z_k \\ &= \mathbf{w}^T \mathbf{z} \end{aligned} \tag{3.4-3}$$

Consult the Tutorials section of the book Web site for a brief review of vectors and matrices.

where the ws are the coefficients of an $m \times n$ filter and the zs are the corresponding image intensities encompassed by the filter. If we are interested in using Eq. (3.4-3) for correlation, we use the mask as given. To use the same equation for convolution, we simply rotate the mask by 180°, as explained in the last section. It is implied that Eq. (3.4-3) holds for a particular pair of coordinates (x, y). You will see in the next section why this notation is convenient for explaining the characteristics of a given linear filter.

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

FIGURE 3.31 Another representation of a general 3×3 filter mask.

As an example, Fig. 3.31 shows a general 3×3 mask with coefficients labeled as above. In this case, Eq. (3.4-3) becomes

$$\begin{aligned} R &= w_1 z_1 + w_2 z_2 + \ldots + w_9 z_9 \\ &= \sum_{k=1}^{9} w_k z_k \\ &= \mathbf{w}^T \mathbf{z} \end{aligned} \tag{3.4-4}$$

where $\mathbf{w}$ and $\mathbf{z}$ are 9-dimensional vectors formed from the coefficients of the mask and the image intensities encompassed by the mask, respectively.

3.4.4 Generating Spatial Filter Masks

Generating an $m \times n$ *linear* spatial filter requires that we specify mn mask coefficients. In turn, these coefficients are selected based on what the filter is supposed to do, keeping in mind that all we can do with linear filtering is to implement a sum of products. For example, suppose that we want to replace the pixels in an image by the average intensity of a 3×3 neighborhood centered on those pixels. The average value at any location (x, y) in the image is the sum of the nine intensity values in the 3×3 neighborhood centered on (x, y) divided by 9. Letting z_i, $i = 1, 2, \ldots, 9$, denote these intensities, the average is

$$R = \frac{1}{9} \sum_{i=1}^{9} z_i$$

But this is the same as Eq. (3.4-4) with coefficient values $w_i = 1/9$. In other words, a linear filtering operation with a 3×3 mask whose coefficients are $1/9$ implements the desired averaging. As we discuss in the next section, this operation results in image smoothing. We discuss in the following sections a number of other filter masks based on this basic approach.

In some applications, we have a continuous function of two variables, and the objective is to obtain a spatial filter mask based on that function. For example, a Gaussian function of two variables has the basic form

$$h(x, y) = e^{-\frac{x^2 + y^2}{2\sigma^2}}$$

where σ is the standard deviation and, as usual, we assume that coordinates x and y are integers. To generate, say, a 3×3 filter mask from this function, we

sample it about its center. Thus, $w_1 = h(-1, -1), w_2 = h(-1, 0), \ldots, w_9 = h(1, 1)$. An $m \times n$ filter mask is generated in a similar manner. Recall that a 2-D Gaussian function has a bell shape, and that the standard deviation controls the "tightness" of the bell.

Generating a *nonlinear* filter requires that we specify the size of a neighborhood and the operation(s) to be performed on the image pixels contained in the neighborhood. For example, recalling that the max operation is nonlinear (see Section 2.6.2), a 5×5 max filter centered at an arbitrary point (x, y) of an image obtains the maximum intensity value of the 25 pixels and assigns that value to location (x, y) in the processed image. Nonlinear filters are quite powerful, and in some applications can perform functions that are beyond the capabilities of linear filters, as we show later in this chapter and in Chapter 5.

3.5 Smoothing Spatial Filters

Smoothing filters are used for blurring and for noise reduction. Blurring is used in preprocessing tasks, such as removal of small details from an image prior to (large) object extraction, and bridging of small gaps in lines or curves. Noise reduction can be accomplished by blurring with a linear filter and also by nonlinear filtering.

3.5.1 Smoothing Linear Filters

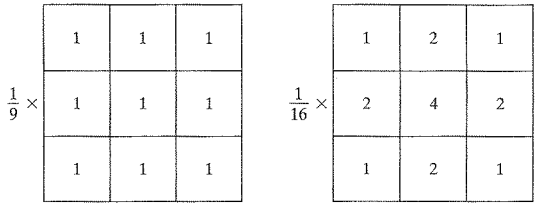
The output (response) of a smoothing, linear spatial filter is simply the average of the pixels contained in the neighborhood of the filter mask. These filters sometimes are called *averaging filters*. As mentioned in the previous section, they also are referred to a *lowpass filters*.

The idea behind smoothing filters is straightforward. By replacing the value of every pixel in an image by the average of the intensity levels in the neighborhood defined by the filter mask, this process results in an image with reduced "sharp" transitions in intensities. Because random noise typically consists of sharp transitions in intensity levels, the most obvious application of smoothing is noise reduction. However, edges (which almost always are desirable features of an image) also are characterized by sharp intensity transitions, so averaging filters have the undesirable side effect that they blur edges. Another application of this type of process includes the smoothing of false contours that result from using an insufficient number of intensity levels, as discussed in Section 2.4.3. A major use of averaging filters is in the reduction of "irrelevant" detail in an image. By "irrelevant" we mean pixel regions that are small with respect to the size of the filter mask. This latter application is illustrated later in this section.

Figure 3.32 shows two 3×3 smoothing filters. Use of the first filter yields the standard average of the pixels under the mask. This can best be seen by substituting the coefficients of the mask into Eq. (3.4-4):

$$R = \frac{1}{9}\sum_{i=1}^{9} z_i$$

which is the average of the intensity levels of the pixels in the 3×3 neighbor



a b

FIGURE 3.32 Two 3×3 smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to 1 divided by the sum of the values of its coefficients, as is required to compute an average.

the coefficients of the filter are all 1s. The idea here is that it is computationally more efficient to have coefficients valued 1. At the end of the filtering process the entire image is divided by 9. An $m \times n$ mask would have a normalizing constant equal to $1/mn$. A spatial averaging filter in which all coefficients are equal sometimes is called a *box filter*.

The second mask in Fig. 3.32 is a little more interesting. This mask yields a so-called *weighted average*, terminology used to indicate that pixels are multiplied by different coefficients, thus giving more importance (weight) to some pixels at the expense of others. In the mask shown in Fig. 3.32(b) the pixel at the center of the mask is multiplied by a higher value than any other, thus giving this pixel more importance in the calculation of the average. The other pixels are inversely weighted as a function of their distance from the center of the mask. The diagonal terms are further away from the center than the orthogonal neighbors (by a factor of $\sqrt{2}$) and, thus, are weighed less than the immediate neighbors of the center pixel. The basic strategy behind weighing the center point the highest and then reducing the value of the coefficients as a function of increasing distance from the origin is simply an attempt to reduce blurring in the smoothing process. We could have chosen other weights to accomplish the same general objective. However, the sum of all the coefficients in the mask of Fig. 3.32(b) is equal to 16, an attractive feature for computer implementation because it is an integer power of 2. In practice, it is difficult in general to see differences between images smoothed by using either of the masks in Fig. 3.32, or similar arrangements, because the area spanned by these masks at any one location in an image is so small.

With reference to Eq. (3.4-1), the general implementation for filtering an $M \times N$ image with a weighted averaging filter of size $m \times n$ (m and n odd) is given by the expression

$$g(x, y) = \frac{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s, t) f(x + s, y + t)}{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s, t)} \tag{3.5-1}$$

The parameters in this equation are as defined in Eq. (3.4-1). As before, it is understood that the complete filtered image is obtained by applying Eq. (3.5-1)

Eq. (3.5-1) is simply the sum of the mask coefficients and, therefore, it is a constant that needs to be computed only once.

EXAMPLE 3.13: Image smoothing with masks of various sizes.

■ The effects of smoothing as a function of filter size are illustrated in Fig. 3.33, which shows an original image and the corresponding smoothed results obtained using square averaging filters of sizes $m = 3, 5, 9, 15$, and 35 pixels, respectively. The principal features of these results are as follows: For $m = 3$, we note a general slight blurring throughout the entire image but, as expected, details that are of approximately the same size as the filter mask are affected considerably more. For example, the 3×3 and 5×5 black squares in the image, the small letter "a," and the fine grain noise show significant blurring when compared to the rest of the image. Note that the noise is less pronounced, and the jagged borders of the characters were pleasingly smoothed.

The result for $m = 5$ is somewhat similar, with a slight further increase in blurring. For $m = 9$ we see considerably more blurring, and the 20% black circle is not nearly as distinct from the background as in the previous three images, illustrating the blending effect that blurring has on objects whose intensities are close to that of its neighboring pixels. Note the significant further smoothing of the noisy rectangles. The results for $m = 15$ and 35 are extreme with respect to the sizes of the objects in the image. This type of aggresive blurring generally is used to eliminate small objects from an image. For instance, the three small squares, two of the circles, and most of the noisy rectangle areas have been blended into the background of the image in Fig. 3.33(f). Note also in this figure the pronounced black border. This is a result of padding the border of the original image with 0s (black) and then trimming off the padded area after filtering. Some of the black was blended into all filtered images, but became truly objectionable for the images smoothed with the larger filters. ■

As mentioned earlier, an important application of spatial averaging is to blur an image for the purpose of getting a gross representation of objects of interest, such that the intensity of smaller objects blends with the background and larger objects become "bloblike" and easy to detect. The size of the mask establishes the relative size of the objects that will be blended with the background. As an illustration, consider Fig. 3.34(a), which is an image from the Hubble telescope in orbit around the Earth. Figure 3.34(b) shows the result of applying a 15×15 averaging mask to this image. We see that a number of objects have either blended with the background or their intensity has diminished considerably. It is typical to follow an operation like this with thresholding to eliminate objects based on their intensity. The result of using the thresholding function of Fig. 3.2(b) with a threshold value equal to 25% of the highest intensity in the blurred image is shown in Fig. 3.34(c). Comparing this result with the original image, we see that it is a reasonable representation of what we would consider to be the largest, brightest ob-

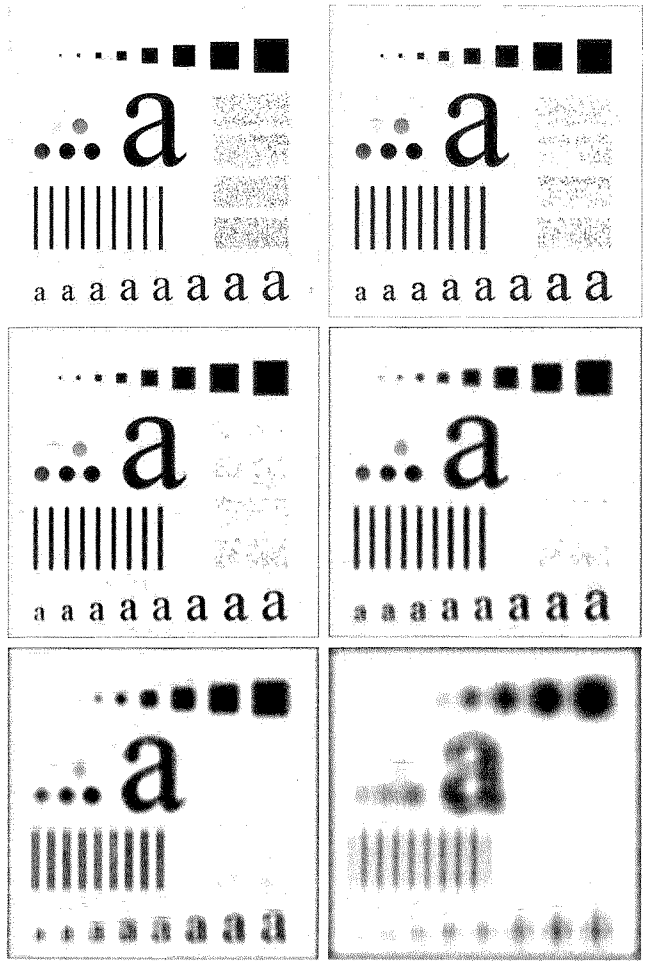
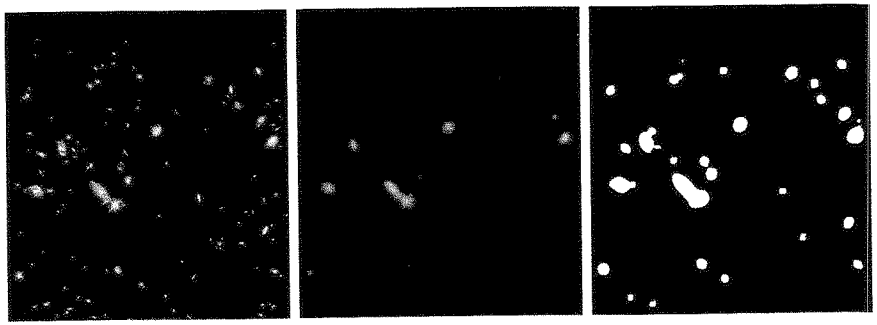


FIGURE 3.33 (a) Original image, of size 500×500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $m = 3, 5, 9, 15$, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their intensity levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50×120 pixels.

a b
c d
e f



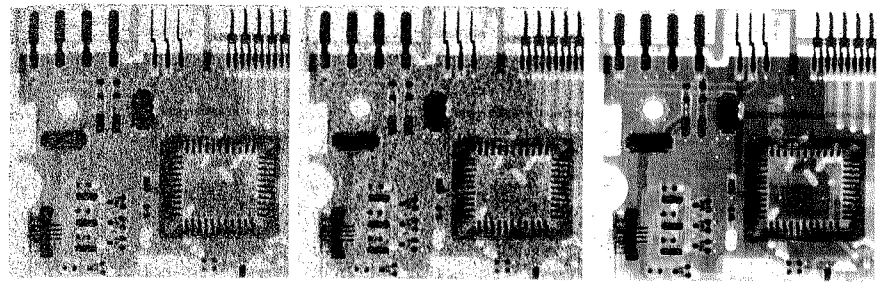
a b c

FIGURE 3.34 (a) Image of size 528 × 485 pixels from the Hubble Space Telescope. (b) Image filtered with a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

3.5.2 Order-Statistic (Nonlinear) Filters

Order-statistic filters are nonlinear spatial filters whose response is based on ordering (ranking) the pixels contained in the image area encompassed by the filter, and then replacing the value of the center pixel with the value determined by the ranking result. The best-known filter in this category is the *median filter*, which, as its name implies, replaces the value of a pixel by the median of the intensity values in the neighborhood of that pixel (the original value of the pixel is included in the computation of the median). Median filters are quite popular because, for certain types of random noise, they provide excellent noise-reduction capabilities, with considerably less blurring than linear smoothing filters of similar size. Median filters are particularly effective in the presence of *impulse noise*, also called *salt-and-pepper noise* because of its appearance as white and black dots superimposed on an image.

The median, ξ, of a set of values is such that half the values in the set are less than or equal to ξ, and half are greater than or equal to ξ. In order to perform median filtering at a point in an image, we first sort the values of the pixel in the neighborhood, determine their median, and assign that value to the corresponding pixel in the filtered image. For example, in a 3 × 3 neighborhood the median is the 5th largest value, in a 5 × 5 neighborhood it is the 13th largest value, and so on. When several values in a neighborhood are the same, all equal values are grouped. For example, suppose that a 3 × 3 neighborhood has values (10, 20, 20, 20, 15, 20, 20, 25, 100). These values are sorted as (10, 15, 20, 20, 20, 20, 20, 25, 100), which results in a median of 20. Thus, the principal function of median filters is to force points with distinct intensity levels to be more like their neighbors. In fact, isolated clusters of pixels that are light or dark with respect to their neighbors, and whose area is less than $m^2/2$ (one-half the filter area), are eliminated by an $m \times m$ median filter. In this case "eliminated" means forced to the median intensity of the neighbors. Larger



a b c

FIGURE 3.35 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3 × 3 averaging mask. (c) Noise reduction with a 3 × 3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Although the median filter is by far the most useful order-statistic filter in image processing, it is by no means the only one. The median represents the 50th percentile of a ranked set of numbers, but recall from basic statistics that ranking lends itself to many other possibilities. For example, using the 100th percentile results in the so-called *max filter*, which is useful for finding the brightest points in an image. The response of a 3 × 3 max filter is given by $R = \max\{z_k | k = 1, 2, \ldots, 9\}$. The 0th percentile filter is the *min filter*, used for the opposite purpose. Median, max, min, and several other nonlinear filters are considered in more detail in Section 5.3.

See Section 10.3.5 regarding percentiles.

EXAMPLE 3.14: Use of median filtering for noise reduction.

■ Figure 3.35(a) shows an X-ray image of a circuit board heavily corrupted by salt-and-pepper noise. To illustrate the point about the superiority of median filtering over average filtering in situations such as this, we show in Fig. 3.35(b) the result of processing the noisy image with a 3 × 3 neighborhood averaging mask, and in Fig. 3.35(c) the result of using a 3 × 3 median filter. The averaging filter blurred the image and its noise reduction performance was poor. The superiority in all respects of median over average filtering in this case is quite evident. In general, median filtering is much better suited than averaging for the removal of salt-and-pepper noise. ■

3.6 Sharpening Spatial Filters

The principal objective of sharpening is to highlight transitions in intensity. Uses of image sharpening vary and include applications ranging from electronic printing and medical imaging to industrial inspection and autonomous guidance in military systems. In the last section, we saw that image blurring could be accomplished in the spatial domain by pixel averaging in a neighborhood. Because averaging is analogous to integration, it is logical to conclude that sharp-

and the discussion in this section deals with various ways of defining and implementing operators for sharpening by digital differentiation. Fundamentally, the strength of the response of a derivative operator is proportional to the degree of intensity discontinuity of the image at the point at which the operator is applied. Thus, image differentiation enhances edges and other discontinuities (such as noise) and deemphasizes areas with slowly varying intensities.

3.6.1 Foundation

In the two sections that follow, we consider in some detail sharpening filters that are based on first- and second-order derivatives, respectively. Before proceeding with that discussion, however, we stop to look at some of the fundamental properties of these derivatives in a digital context. To simplify the explanation, we focus attention initially on one-dimensional derivatives. In particular, we are interested in the behavior of these derivatives in areas of constant intensity, at the onset and end of discontinuities (step and ramp discontinuities), and along intensity ramps. As you will see in Chapter 10, these types of discontinuities can be used to model noise points, lines, and edges in an image. The behavior of derivatives during transitions into and out of these image features also is of interest.

The derivatives of a digital function are defined in terms of differences. There are various ways to define these differences. However, we require that any definition we use for a *first derivative* (1) must be zero in areas of constant intensity; (2) must be nonzero at the onset of an intensity step or ramp; and (3) must be nonzero along ramps. Similarly, any definition of a *second derivative* (1) must be zero in constant areas; (2) must be nonzero at the onset *and* end of an intensity step or ramp; and (3) must be zero along ramps of constant slope. Because we are dealing with digital quantities whose values are finite, the maximum possible intensity change also is finite, and the shortest distance over which that change can occur is between adjacent pixels.

A basic definition of the first-order derivative of a one-dimensional function $f(x)$ is the difference

$$\frac{\partial f}{\partial x} = f(x+1) - f(x) \tag{3.6-1}$$

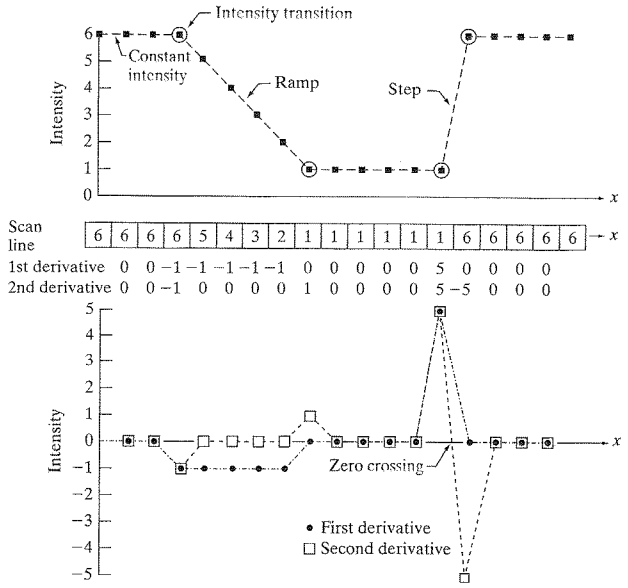
We return to Eq. (3.6-1) in Section 10.2.1 and show how it follows from a Taylor series expansion. For now, we accept it as a definition.

We used a partial derivative here in order to keep the notation the same as when we consider an image function of two variables, $f(x, y)$, at which time we will be dealing with partial derivatives along the two spatial axes. Use of a partial derivative in the present discussion does not affect in any way the nature of what we are trying to accomplish. Clearly, $\partial f/\partial x = df/dx$ when there is only one variable in the function; the same is true for the second derivative.

We define the second-order derivative of $f(x)$ as the difference

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x) \tag{3.6-2}$$

It is easily verified that these two definitions satisfy the conditions stated



a
b
c

FIGURE 3.36 Illustration of the first and second derivatives of a 1-D digital function representing a section of a horizontal intensity profile from an image. In (a) and (c) data points are joined by dashed lines as a visualization aid.

first- and second-order derivatives of a digital function, consider the example in Fig. 3.36.

Figure 3.36(b) (center of the figure) shows a section of a scan line (intensity profile). The values inside the small squares are the intensity values in the scan line, which are plotted as black dots above it in Fig. 3.36(a). The dashed line connecting the dots is included to aid visualization. As the figure shows, the scan line contains an intensity ramp, three sections of constant intensity, and an intensity step. The circles indicate the onset or end of intensity transitions. The first- and second-order derivatives computed using the two preceding definitions are included below the scan line in Fig. 3.36(b), and are plotted in Fig. 3.36(c). When computing the first derivative at a location x, we subtract the value of the function at that location from the next point. So this is a "look-ahead" operation. Similarly, to compute the second derivative at x, we use the previous and the next points in the computation. To avoid a situation in which the previous or next points are outside the range of the scan line, we show derivative computations in Fig. 3.36 from the second through the penultimate points in the sequence.

Let us consider the properties of the first and second derivatives as we traverse the profile from left to right. First, we encounter an area of constant intensity and, as Figs. 3.36(b) and (c) show, both derivatives are zero there, so condition (1) is satisfied for both. Next, we encounter an intensity

the step; similarly, the second derivative is nonzero at the onset *and* end of both the ramp and the step; therefore, property (2) is satisfied for both derivatives. Finally, we see that property (3) is satisfied also for both derivatives because the first derivative is nonzero and the second is zero along the ramp. Note that the sign of the second derivative changes at the onset and end of a step or ramp. In fact, we see in Fig. 3.36(c) that in a step transition a line joining these two values crosses the horizontal axis midway between the two extremes. This *zero crossing* property is quite useful for locating edges, as you will see in Chapter 10.

Edges in digital images often are ramp-like transitions in intensity, in which case the first derivative of the image would result in thick edges because the derivative is nonzero along a ramp. On the other hand, the second derivative would produce a double edge one pixel thick, separated by zeros. From this, we conclude that the second derivative enhances fine detail much better than the first derivative, a property that is ideally suited for sharpening images. Also, as you will learn later in this section, second derivatives are much easier to implement than first derivates, so we focus our attention initially on second derivatives.

3.6.2 Using the Second Derivative for Image Sharpening—The Laplacian

In this section we consider the implementation of 2-D, second-order derivatives and their use for image sharpening. We return to this derivative in Chapter 10, where we use it extensively for image segmentation. The approach basically consists of defining a discrete formulation of the second-order derivative and then constructing a filter mask based on that formulation. We are interested in *isotropic* filters, whose response is independent of the direction of the discontinuities in the image to which the filter is applied. In other words, isotropic filters are *rotation invariant*, in the sense that rotating the image and then applying the filter gives the same result as applying the filter to the image first and then rotating the result.

It can be shown (Rosenfeld and Kak [1982]) that the simplest isotropic derivative operator is the Laplacian, which, for a function (image) $f(x, y)$ of two variables, is defined as

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \tag{3.6-3}$$

Because derivatives of any order are linear operations, the Laplacian is a linear operator. To express this equation in discrete form, we use the definition in Eq. (3.6-2), keeping in mind that we have to carry a second variable. In the x-direction, we have

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y) \tag{3.6-4}$$

and, similarly, in the y-direction we have

$$\frac{\partial^2 f}{\partial y^2} = \cdots \tag{3.6-5}$$

Therefore, it follows from the preceding three equations that the discrete Laplacian of two variables is

$$\nabla^2 f(x, y) = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y) \tag{3.6-6}$$

This equation can be implemented using the filter mask in Fig. 3.37(a), which gives an isotropic result for rotations in increments of 90°. The mechanics of implementation are as in Section 3.5.1 for linear smoothing filters. We simply are using different coefficients here.

The diagonal directions can be incorporated in the definition of the digital Laplacian by adding two more terms to Eq. (3.6-6), one for each of the two diagonal directions. The form of each new term is the same as either Eq. (3.6-4) or (3.6-5), but the coordinates are along the diagonals. Because each diagonal term also contains a $-2f(x, y)$ term, the total subtracted from the difference terms now would be $-8f(x, y)$. Figure 3.37(b) shows the filter mask used to implement this new definition. This mask yields isotropic results in increments of 45°. You are likely to see in practice the Laplacian masks in Figs. 3.37(c) and (d). They are obtained from definitions of the second derivatives that are the negatives of the ones we used in Eqs. (3.6-4) and (3.6-5). As such, they yield equivalent results, but the difference in sign must be kept in mind when combining (by addition or subtraction) a Laplacian-filtered image with another image.

Because the Laplacian is a derivative operator, its use highlights intensity discontinuities in an image and deemphasizes regions with slowly varying intensity levels. This will tend to produce images that have grayish edge lines and other discontinuities, all superimposed on a dark, featureless background. Background features can be "recovered" while still preserving the sharpening

0	1	0	1	1	1
1	−4	1	1	−8	1
0	1	0	1	1	1

0	−1	0	−1	−1	−1
−1	4	−1	−1	8	−1
0	−1	0	−1	−1	−1

a b
c d

FIGURE 3.37
(a) Filter mask used to implement Eq. (3.6-6). (b) Mask used to implement an extension of this equation that includes the diagonal terms. (c) and (d) Two other implementations of the Laplacian found frequently in practice.

effect of the Laplacian simply by adding the Laplacian image to the original. As noted in the previous paragraph, it is important to keep in mind which definition of the Laplacian is used. If the definition used has a negative center coefficient, then we *subtract*, rather than add, the Laplacian image to obtain a sharpened result. Thus, the basic way in which we use the Laplacian for image sharpening is

$$g(x, y) = f(x, y) + c\left[\nabla^2 f(x, y)\right] \tag{3.6-7}$$

where $f(x, y)$ and $g(x, y)$ are the input and sharpened images, respectively. The constant is $c = -1$ if the Laplacian filters in Fig. 3.37(a) or (b) are used, and $c = 1$ if either of the other two filters is used.

EXAMPLE 3.15: Image sharpening using the Laplacian.

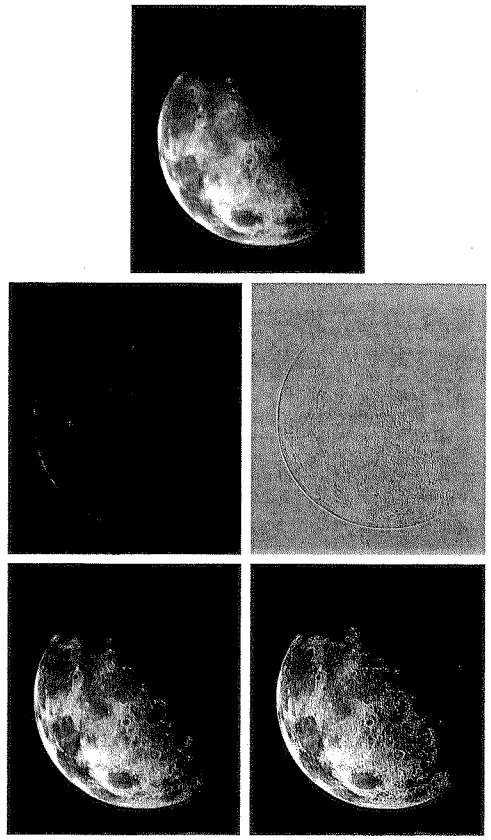
■ Figure 3.38(a) shows a slightly blurred image of the North Pole of the moon. Figure 3.38(b) shows the result of filtering this image with the Laplacian mask in Fig. 3.37(a). Large sections of this image are black because the Laplacian contains both positive and negative values, and all negative values are clipped at 0 by the display.

A typical way to scale a Laplacian image is to add to it its minimum value to bring the new minimum to zero and then scale the result to the full $[0, L - 1]$ intensity range, as explained in Eqs. (2.6-10) and (2.6-11). The image in Fig. 3.38(c) was scaled in this manner. Note that the dominant features of the image are edges and sharp intensity discontinuities. The background, previously black, is now gray due to scaling. This grayish appearance is typical of Laplacian images that have been scaled properly. Figure 3.38(d) shows the result obtained using Eq. (3.6-7) with $c = -1$. The detail in this image is unmistakably clearer and sharper than in the original image. Adding the original image to the Laplacian restored the overall intensity variations in the image, with the Laplacian increasing the contrast at the locations of intensity discontinuities. The net result is an image in which small details were enhanced and the background tonality was reasonably preserved. Finally, Fig. 3.38(e) shows the result of repeating the preceding procedure with the filter in Fig. 3.37(b). Here, we note a significant improvement in sharpness over Fig. 3.38(d). This is not unexpected because using the filter in Fig. 3.37(b) provides additional differentiation (sharpening) in the diagonal directions. Results such as those in Figs. 3.38(d) and (e) have made the Laplacian a tool of choice for sharpening digital images. ■

3.6.3 Unsharp Masking and Highboost Filtering

A process that has been used for many years by the printing and publishing industry to sharpen images consists of subtracting an unsharp (smoothed) version of an image from the original image. This process, called *unsharp masking*, consists of the following steps:

1. Blur the original image.
2. Subtract the blurred image from the original (the resulting difference is called the *mask*.)



a b c d e

FIGURE 3.38 (a) Blurred image of the North Pole of the moon. (b) Laplacian without scaling. (c) Laplacian with scaling. (d) Image sharpened using the mask in Fig. 3.37(a). (e) Result of using the mask in Fig. 3.37(b). (Original image courtesy of NASA.)

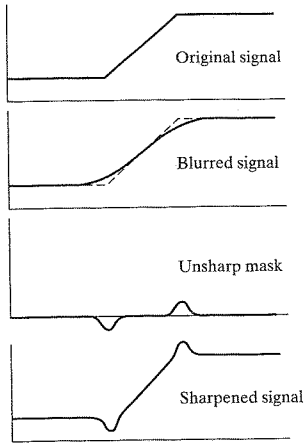
Letting $\bar{f}(x, y)$ denote the blurred image, unsharp masking is expressed in equation form as follows. First we obtain the mask:

$$g_{\text{mask}}(x, y) = f(x, y) - \bar{f}(x, y) \tag{3.6-8}$$

Then we add a weighted portion of the mask back to the original image:

$$g(x, y) = f(x, y) + k * g_{\text{mask}}(x, y) \tag{3.6-9}$$

where we included a weight, k ($k \geq 0$), for generality. When $k = 1$, we have



a
b
c
d

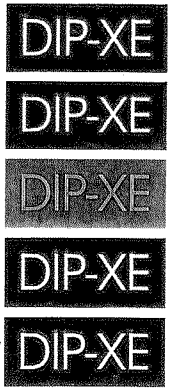
FIGURE 3.39 1-D illustration of the mechanics of unsharp masking. (a) Original signal. (b) Blurred signal with original shown dashed for reference. (c) Unsharp mask. (d) Sharpened signal, obtained by adding (c) to (a).

highboost filtering. Choosing $k < 1$ de-emphasizes the contribution of the unsharp mask.

Figure 3.39 explains how unsharp masking works. The intensity profile in Fig. 3.39(a) can be interpreted as a horizontal scan line through a vertical edge that transitions from a dark to a light region in an image. Figure 3.39(b) shows the result of smoothing, superimposed on the original signal (shown dashed) for reference. Figure 3.39(c) is the unsharp mask, obtained by subtracting the blurred signal from the original. By comparing this result with the section of Fig. 3.36(c) corresponding to the ramp in Fig. 3.36(a), we note that the unsharp mask in Fig. 3.39(c) is very similar to what we would obtain using a second-order derivative. Figure 3.39(d) is the final sharpened result, obtained by adding the mask to the original signal. The points at which a change of slope in the intensity occurs in the signal are now emphasized (sharpened). Observe that negative values were added to the original. Thus, it is possible for the final result to have negative intensities if the original image has any zero values or if the value of k is chosen large enough to emphasize the peaks of the mask to a level larger than the minimum value in the original. Negative values would cause a dark halo around edges, which, if k is large enough, can produce objectionable results.

EXAMPLE 3.16: Image sharpening using unsharp masking

■ Figure 3.40(a) shows a slightly blurred image of white text on a dark gray background. Figure 3.40(b) was obtained using a Gaussian smoothing filter (see Section 3.4.4) of size 5×5 with $\sigma = 3$. Figure 3.40(c) is the unsharp



a
b
c
d
e

FIGURE 3.40 (a) Original image. (b) Result of blurring with a Gaussian filter. (c) Unsharp mask. (d) Result of using unsharp masking. (e) Result of using highboost filtering.

masking [Eq. (3.6-9) with $k = 1$]. This image is a slight improvement over the original, but we can do better. Figure 3.40(e) shows the result of using Eq. (3.6-9) with $k = 4.5$, the largest possible value we could use and still keep positive all the values in the final result. The improvement in this image over the original is significant. ■

3.6.4 Using First-Order Derivatives for (Nonlinear) Image Sharpening—The Gradient

First derivatives in image processing are implemented using the magnitude of the gradient. For a function $f(x, y)$, the gradient of f at coordinates (x, y) is defined as the two-dimensional column *vector*

$$\nabla f \equiv \text{grad}(f) \equiv \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} \tag{3.6-10}$$

We discuss the gradient in detail in Section 10.2.5. Here, we are interested only in using the magnitude of the gradient for image sharpening.

This vector has the important geometrical property that it points in the direction of the greatest rate of change of f at location (x, y).

The *magnitude* (*length*) of vector ∇f, denoted as $M(x, y)$, where

$$M(x, y) = \text{mag}(\nabla f) = \sqrt{g_x^2 + g_y^2} \tag{3.6-11}$$

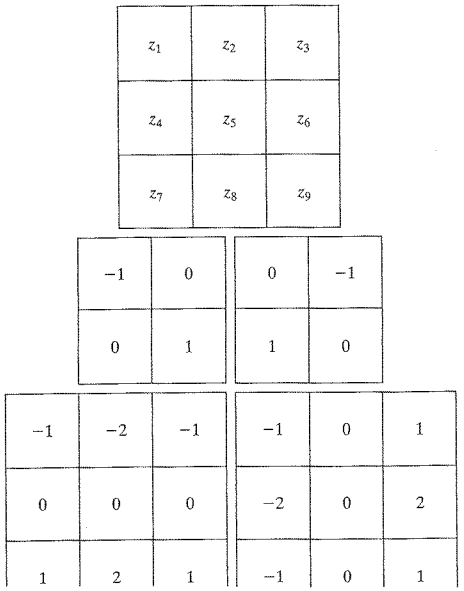
is the *value* at (x, y) of the rate of change in the direction of the gradient vector. Note that $M(x, y)$ is an image of the same size as the original, created when x and y are allowed to vary over all pixel locations in f. It is common practice to refer to this image as the *gradient image* (or simply as the *gradient* when the

Because the components of the gradient vector are derivatives, they are linear operators. However, the magnitude of this vector is not because of the squaring and square root operations. On the other hand, the partial derivatives in Eq. (3.6-10) are not rotation invariant (isotropic), but the magnitude of the gradient vector is. In some implementations, it is more suitable computationally to approximate the squares and square root operations by absolute values:

$$M(x, y) \approx |g_x| + |g_y| \tag{3.6-12}$$

This expression still preserves the relative changes in intensity, but the isotropic property is lost in general. However, as in the case of the Laplacian, the isotropic properties of the discrete gradient defined in the following paragraph are preserved only for a limited number of rotational increments that depend on the filter masks used to approximate the derivatives. As it turns out, the most popular masks used to approximate the gradient are isotropic at multiples of 90°. These results are independent of whether we use Eq. (3.6-11) or (3.6-12), so nothing of significance is lost in using the latter equation if we choose to do so.

As in the case of the Laplacian, we now define discrete approximations to the preceding equations and from there formulate the appropriate filter masks. In order to simplify the discussion that follows, we will use the notation in Fig. 3.41(a) to denote the intensities of image points in a 3 × 3 region. For



a
b c
d e

FIGURE 3.41
A 3 × 3 region of an image (the zs are intensity values).
(b)–(c) Roberts cross gradient operators.
(d)–(e) Sobel operators. All the mask coefficients sum to zero, as expected of a derivative operator.

example, the center point, z_5, denotes $f(x, y)$ at an arbitrary location, (x, y); z_1 denotes $f(x - 1, y - 1)$; and so on, using the notation introduced in Fig. 3.28. As indicated in Section 3.6.1, the simplest approximations to a first-order derivative that satisfy the conditions stated in that section are $g_x = (z_8 - z_5)$ and $g_y = (z_6 - z_5)$. Two other definitions proposed by Roberts [1965] in the early development of digital image processing use cross differences:

$$g_x = (z_9 - z_5) \quad \text{and} \quad g_y = (z_8 - z_6) \tag{3.6-13}$$

If we use Eqs. (3.6-11) and (3.6-13), we compute the gradient image as

$$M(x, y) = \left[(z_9 - z_5)^2 + (z_8 - z_6)^2\right]^{1/2} \tag{3.6-14}$$

If we use Eqs. (3.6-12) and (3.6-13), then

$$M(x, y) \approx |z_9 - z_5| + |z_8 - z_6| \tag{3.6-15}$$

where it is understood that x and y vary over the dimensions of the image in the manner described earlier. The partial derivative terms needed in equation (3.6-13) can be implemented using the two linear filter masks in Figs. 3.41(b) and (c). These masks are referred to as the *Roberts cross-gradient operators*.

Masks of even sizes are awkward to implement because they do not have a center of symmetry. The smallest filter masks in which we are interested are of size 3 × 3. Approximations to g_x and g_y using a 3 × 3 neighborhood centered on z_5 are as follows:

$$g_x = \frac{\partial f}{\partial x} = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3) \tag{3.6-16}$$

and

$$g_y = \frac{\partial f}{\partial y} = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7) \tag{3.6-17}$$

These equations can be implemented using the masks in Figs. 3.41(d) and (e). The difference between the third and first rows of the 3 × 3 image region implemented by the mask in Fig. 3.41(d) approximates the partial derivative in the x-direction, and the difference between the third and first columns in the other mask approximates the derivative in the y-direction. After computing the partial derivatives with these masks, we obtain the magnitude of the gradient as before. For example, substituting g_x and g_y into Eq. (3.6-12) yields

$$M(x, y) \approx |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)| + |(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)| \tag{3.6-18}$$

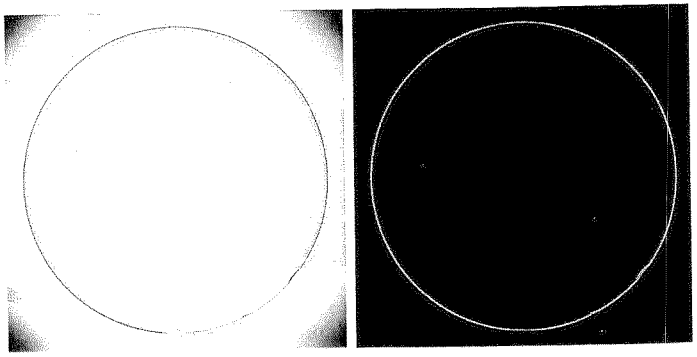
The masks in Figs. 3.41(d) and (e) are called the *Sobel operators*. The idea behind using a weight value of 2 in the center coefficient is to achieve some smoothing by giving more importance to the center point (we discuss this in more detail in Chapter 10). Note that the coefficients in all the masks shown in Fig. 3.41 sum to 0, indicating that they would give a response of 0 in an area of

As mentioned earlier, the computations of g_x and g_y are linear operations because they involve derivatives and, therefore, can be implemented as a sum of products using the spatial masks in Fig. 3.41. The nonlinear aspect of sharpening with the gradient is the computation of $M(x, y)$ involving squaring and square roots, or the use of absolute values, all of which are nonlinear operations. These operations are performed *after* the linear process that yields g_x and g_y.

EXAMPLE 3.17: Use of the gradient for edge enhancement.

■ The gradient is used frequently in industrial inspection, either to aid humans in the detection of defects or, what is more common, as a preprocessing step in automated inspection. We will have more to say about this in Chapters 10 and 11. However, it will be instructive at this point to consider a simple example to show how the gradient can be used to enhance defects and eliminate slowly changing background features. In this example, enhancement is used as a preprocessing step for automated inspection, rather than for human analysis.

Figure 3.42(a) shows an optical image of a contact lens, illuminated by a lighting arrangement designed to highlight imperfections, such as the two edge defects in the lens boundary seen at 4 and 5 o'clock. Figure 3.42(b) shows the gradient obtained using Eq. (3.6-12) with the two Sobel masks in Figs. 3.41(d) and (e). The edge defects also are quite visible in this image, but with the added advantage that constant or slowly varying shades of gray have been eliminated, thus simplifying considerably the computational task required for automated inspection. The gradient can be used also to highlight small specs that may not be readily visible in a gray-scale image (specs like these can be foreign matter, air pockets in a supporting solution, or miniscule imperfections in the lens). The ability to enhance small discontinuities in an otherwise flat gray field is another important feature of the gradient. ■



a b

FIGURE 3.42
(a) Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Pete Sites, Perceptics Corporation.)

3.7 Combining Spatial Enhancement Methods

With a few exceptions, like combining blurring with thresholding (Fig. 3.34), we have focused attention thus far on individual approaches. Frequently, a given task will require application of several complementary techniques in order to achieve an acceptable result. In this section we illustrate by means of an example how to combine several of the approaches developed thus far in this chapter to address a difficult image enhancement task.

The image in Fig. 3.43(a) is a nuclear whole body bone scan, used to detect diseases such as bone infection and tumors. Our objective is to enhance this image by sharpening it and by bringing out more of the skeletal detail. The narrow dynamic range of the intensity levels and high noise content make this image difficult to enhance. The strategy we will follow is to utilize the Laplacian to highlight fine detail, and the gradient to enhance prominent edges. For reasons that will be explained shortly, a smoothed version of the gradient image will be used to mask the Laplacian image (see Fig. 2.30 regarding masking). Finally, we will attempt to increase the dynamic range of the intensity levels by using an intensity transformation.

Figure 3.43(b) shows the Laplacian of the original image, obtained using the filter in Fig. 3.37(d). This image was scaled (for display only) using the same technique as in Fig. 3.38(c). We can obtain a sharpened image at this point simply by adding Figs. 3.43(a) and (b), according to Eq. (3.6-7). Just by looking at the noise level in Fig. 3.43(b), we would expect a rather noisy sharpened image if we added Figs. 3.43(a) and (b), a fact that is confirmed by the result in Fig. 3.43(c). One way that comes immediately to mind to reduce the noise is to use a median filter. However, median filtering is a nonlinear process capable of removing image features. This is unacceptable in medical image processing.

An alternate approach is to use a mask formed from a smoothed version of the gradient of the original image. The motivation behind this is straightforward and is based on the properties of first- and second-order derivatives explained in Section 3.6.1. The Laplacian, being a second-order derivative operator, has the definite advantage that it is superior in enhancing fine detail. However, this causes it to produce noisier results than the gradient. This noise is most objectionable in smooth areas, where it tends to be more visible. The gradient has a stronger average response in areas of significant intensity transitions (ramps and steps) than does the Laplacian. The response of the gradient to noise and fine detail is lower than the Laplacian's and can be lowered further by smoothing the gradient with an averaging filter. The idea, then, is to smooth the gradient and multiply it by the Laplacian image. In this context, we may view the smoothed gradient as a mask image. The product will preserve details in the strong areas while reducing noise in the relatively flat areas. This process can be interpreted roughly as combining the best features of the Laplacian and the gradient. The result is added to the original to obtain a final sharpened image.

Figure 3.43(d) shows the Sobel gradient of the original image, computed using Eq. (3.6-12). Components g_x and g_y were obtained using the masks in Figs. 3.41(d) and (e), respectively. As expected, edges are much more dominant