

CS474/674 Image Processing and Interpretation

Fall 2009 – Dr. George Bebis

Homework 1 - Solutions

1.

Image $f(x,y)$ with $f_{\min} = 6$, $f_{\max} = 18$
Image $g(x,y)$ with $g_{\min} = 10$, $g_{\max} = 50$

$T(m) = ma + b = n$ where $T(m)$: a linear transformation function
 m : a gray level in image $f(x,y)$
 n : the corresponding gray level in image $g(x,y)$

Solving the equation for $m = f_{\min} = 6$ and $n = g_{\min} = 10$

$$6a + b = 10 \quad b = 10 - 6a$$

Solving the equation for $m = f_{\max} = 18$ and $n = g_{\max} = 50$

$$18a + b = 50 \quad b = 50 - 18a$$

Then,

$$50 - 18a = 10 - 6a = b \quad 14a = 40 \quad a = \frac{20}{7} \quad b = 10 - \frac{80}{7} = -\frac{10}{7}$$

Transformation function is:

$$T(m) = \frac{20}{7}m - \frac{10}{7}$$

2.

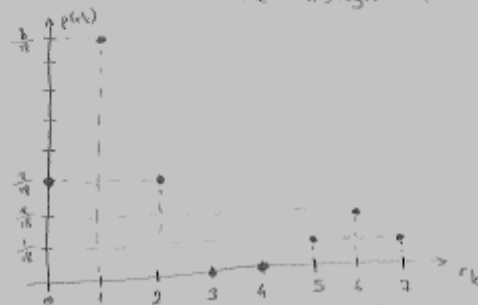
$$p(0) = \frac{3}{18} \quad p(4) = 0$$

$$p(1) = \frac{8}{18} \quad p(5) = \frac{1}{18}$$

$$p(2) = \frac{3}{18} \quad p(6) = \frac{2}{18}$$

$$p(3) = 0 \quad p(7) = \frac{1}{18}$$

Let's first draw the histogram of the original image



The transformation to be used is:

$$s_k = T(r_k) = (L-1) \sum_{j=0}^k p(r_j)$$

where $L = 8$ (eight possible gray level)

Applying the transformation, we get:

$$s_0 = 7 \cdot p(0) = \frac{21}{18} \approx 1$$

$$s_1 = 7 \cdot (p(0) + p(1)) = \frac{33}{18} \approx 4$$

$$s_2 = 7 \cdot \sum_{j=0}^2 p(r_j) = \frac{38}{18} \approx 5$$

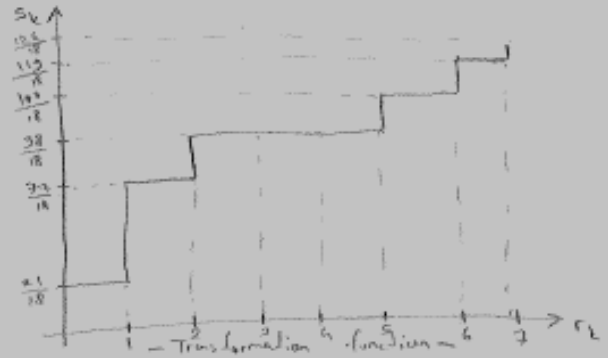
$$s_3 = 7 \cdot \sum_{j=0}^3 p(r_j) = \frac{43}{18} \approx 5$$

$$s_4 = 7 \cdot \sum_{j=0}^4 p(r_j) = \frac{48}{18} \approx 5$$

$$s_5 = 7 \cdot \sum_{j=0}^5 p(r_j) = \frac{105}{18} \approx 6$$

$$s_6 = 7 \cdot \sum_{j=0}^6 p(r_j) = \frac{113}{18} \approx 7$$

$$s_7 = 7 \cdot \sum_{j=0}^7 p(r_j) = \frac{126}{18} = 7$$



Finally, calculating the distribution values:

$$p(0) = 0$$

$$p(1) = r_0 = \frac{3}{18}$$

$$p(2) = 0$$

$$p(3) = 0$$

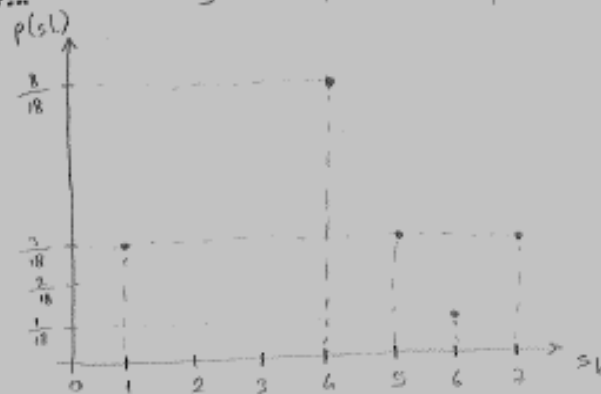
$$p(4) = r_1 = \frac{8}{18}$$

$$p(5) = r_2 + r_3 + r_4 = \frac{3}{18}$$

$$p(6) = r_5 = \frac{1}{18}$$

$$p(7) = r_6 + r_7 = \frac{3}{18}$$

The histogram of the equalized image:



The equalized image is given below

4	5	4	4	5	1
1	4	6	4	1	4
4	7	7	7	4	5

3. Problem 3.11 (page 194)

First, we obtain the histogram equalization transformation:

$$s = T(r) = \int_0^r p_r(w) dw = \int_0^r (-2w + 2) dw = -r^2 + 2r.$$

Next we find

$$v = G(z) = \int_0^z p_z(w) dw = \int_0^z 2w dw = z^2.$$

Finally,

$$z = G^{-1}(v) = \pm\sqrt{v}.$$

But only positive intensity levels are allowed, so $z = \sqrt{v}$. Then, we replace v with s , which in turn is $-r^2 + 2r$, and we have

$$z = \sqrt{-r^2 + 2r}.$$

5. (Graduate Students Only) Problem 3.7 (page 194)

Let $n = MN$ be the total number of pixels and let n_{r_j} be the number of pixels in the input image with intensity value r_j . Then, the histogram equalization transformation is

$$s_k = T(r_k) = \sum_{j=0}^k n_{r_j} / n = \frac{1}{n} \sum_{j=0}^k n_{r_j}.$$

Because every pixel (and no others) with value r_k is mapped to value s_k , it follows that $n_{s_k} = n_{r_k}$. A second pass of histogram equalization would produce values v_k according to the transformation

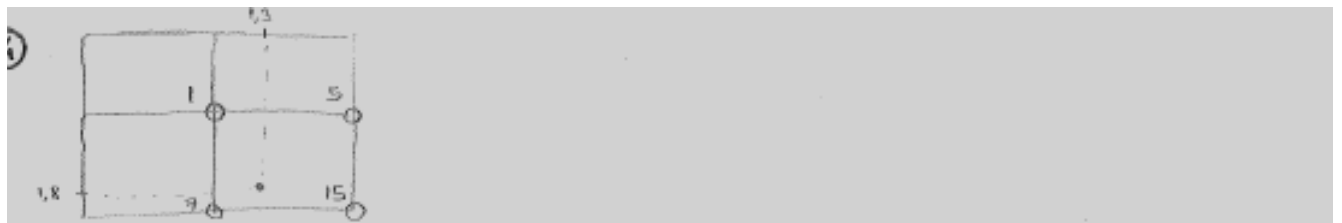
$$v_k = T(s_k) = \frac{1}{n} \sum_{j=0}^k n_{s_j}.$$

But, $n_{s_j} = n_{r_j}$, so

$$v_k = T(s_k) = \frac{1}{n} \sum_{j=0}^k n_{r_j} = s_k$$

which shows that a second pass of histogram equalization would yield the same result as the first pass. We have assumed negligible round-off errors.

5.



(i) Zero-order interpolation is the nearest neighbour interpolation



$$I_0(1.3, 1.8) = 7 //$$

(ii) First-order interpolation using average

$$I_1(1.3, 1.8) = \frac{1 + 5 + 7 + 15}{4} = 7 //$$

(iii) First-order interpolation using a bilinear function.

$$I_1'(x, y) = ax + by + cxy + d$$

To determine the constants, we put the nearest neighbours into the equation.

$$(a) I_1'(1, 1) = a + b + c + d = 1$$

$$(b) I_1'(2, 1) = 2a + b + 2c + d = 5$$

$$(c) I_1'(1, 2) = a + 2b + 2c + d = 7$$

$$(d) I_1'(2, 2) = 2a + 2b + 4c + d = 15$$

$$(e) a + c = 6 \quad (\text{from a \& b})$$

$$(f) a + 2c = 8 \quad (\text{from c \& d})$$

$$(g) c = 4, a = 0 \quad (\text{from e \& f})$$

$$(h) b + d = -3 \quad (\text{from a \& g})$$

$$(i) 2b + d = -1 \quad (\text{from c \& g})$$

$$(j) b = 2, d = -5 \quad (\text{from h \& i})$$

$$I_1'(x, y) = 2y + 4xy - 5$$

$$I_1'(1.3, 1.8) = 3.6 + 9.36 - 5 = 7.96 \approx 8 //$$