CS474/674 Image Processing and Interpretation

Homework 1 - Solutions

1.

Image $f(x,y)$ with $f_{min} = 4$, $f_{max} = 18$

Image $g(x,y)$ with $g_{min} = 10$, $g_{max} = 50$

$T(m) = ma + b = n$ where $T(m)$: a linear transformation function

m : a gray level in image $f(x,y)$

n : the corresponding gray level in image $g(x,y)$

Solving the equation for $m = f_{min} = 4$ and $n = g_{min} = 10$

$4a + b = 10 \qquad b = 10 - 4a$

Solving the equation for $m = f_{max} = 18$ and $n = g_{max} = 50$

$18a + b = 50 \qquad b = 50 - 18a$

Then,

$50 - 18a = 10 - 4a = b \qquad 14a = 40 \qquad a = \frac{20}{7} \qquad b = 10 - \frac{80}{7} = -\frac{10}{7}$

Transformation function is:

$$\boxed{T(m) = \frac{20}{7} m - \frac{10}{7}}$$

2.

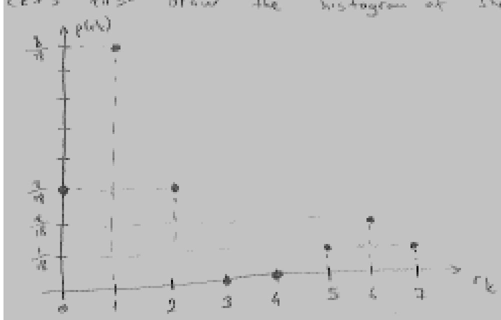
$p(0) = \frac{3}{18} \qquad p(4) = 0$

$p(1) = \frac{8}{18} \qquad p(5) = \frac{1}{18}$

$p(2) = \frac{3}{18} \qquad p(6) = \frac{2}{18}$

$p(3) = 0 \qquad p(7) = \frac{1}{18}$

Let's first draw the histogram of the original image



The transformation to be used is:

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p(r_j) \qquad \text{where} \quad L = 8 \text{ (eight possible grey level)}$$

Applying the transformation, we get:

$s_0 = 7 \cdot p(0) = \frac{21}{18} \approx 1$

$s_1 = 7 \cdot (p(0)+p(1)) = \frac{77}{18} \approx 4$

$s_2 = 7 \cdot \sum_{j=0}^{2} p(r_j) = \frac{98}{18} \approx 5$

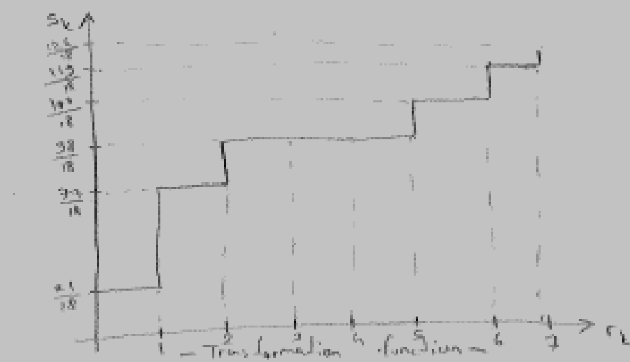
$s_3 = 7 \cdot \sum_{j=0}^{3} p(r_j) = \frac{98}{18} \approx 5$

$s_4 = 7 \cdot \sum_{j=0}^{4} p(r_j) = \frac{98}{18} \approx 5$

$s_5 = 7 \cdot \sum_{j=0}^{5} p(r_j) = \frac{105}{18} \approx 6$

$s_6 = 7 \cdot \sum_{j=0}^{6} p(r_j) = \frac{119}{18} \approx 7$

$s_7 = 7 \cdot \sum_{j=0}^{7} p(r_j) = \frac{126}{18} = 7$



Finally, calculating the distribution values:

$p(0) = 0$

$p(1) = r_0 = \frac{3}{18}$

$p(2) = 0$

$p(3) = 0$

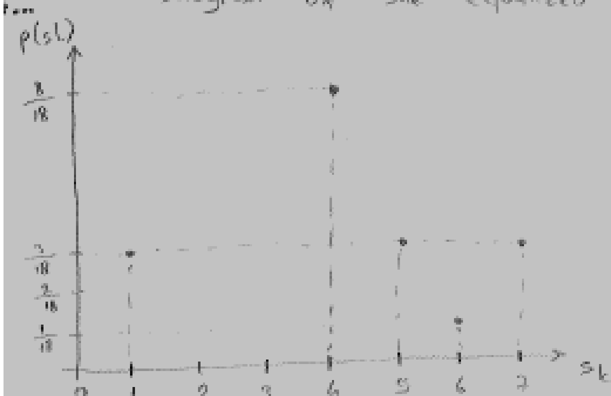
$p(4) = r_1 = \frac{8}{18}$

$p(5) = r_2 + r_3 + r_4 = \frac{3}{18}$

$p(6) = r_5 = \frac{1}{18}$

$p(7) = r_6 + r_7 = \frac{3}{18}$

c) The histogram of the equalized image:



The equalized image is given below

4	5	4	4	5	1
1	4	6	4	1	4
4	7	7	7	4	5

3. Problem 3.11 (page 194)

First, we obtain the histogram equalization transformation:

$$s = T(r) = \int_0^r p_r(w)\,dw = \int_0^r (-2w+2)\,dw = -r^2 + 2r.$$

Next we find

$$v = G(z) = \int_0^z p_z(w)\,dw = \int_0^z 2w\,dw = z^2.$$

Finally,

$$z = G^{-1}(v) = \pm\sqrt{v}.$$

But only positive intensity levels are allowed, so $z = \sqrt{v}$. Then, we replace v with s, which in turn is $-r^2 + 2r$, and we have

$$z = \sqrt{-r^2 + 2r}.$$

5. Problem 3.7 (page 194)

Let $n = MN$ be the total number of pixels and let n_{r_j} be the number of pixels in the input image with intensity value r_j. Then, the histogram equalization transformation is

$$s_k = T(r_k) = \sum_{j=0}^{k} n_{r_j}/n = \frac{1}{n}\sum_{j=0}^{k} n_{r_j}.$$

Because every pixel (and no others) with value r_k is mapped to value s_k, it follows that $n_{s_k} = n_{r_k}$. A second pass of histogram equalization would produce values v_k according to the transformation

$$v_k = T(s_k) = \frac{1}{n}\sum_{j=0}^{k} n_{s_j}.$$

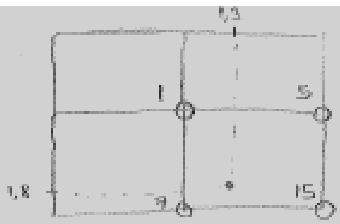
But, $n_{s_j} = n_{r_j}$, so

$$v_k = T(s_k) = \frac{1}{n}\sum_{j=0}^{k} n_{r_j} = s_k$$

which shows that a second pass of histogram equalization would yield the same result as the first pass. We have assumed negligible round-off errors.

5.

(i)



(i) Zero-order interpolation is the nearest neighbour interpolation

$I_0(1.3, 1.8) = 7$

(ii) First-order interpolation using average

$$I_1(1.3, 1.8) = \frac{1+5+7+15}{4} = 7$$

(iii) First-order interpolation using a bilinear function.

$$I_1'(x, y) = ax + by + cxy + d$$

To determine the constants, we put the nearest neighbours into the equation.

(a) $I_1'(1, 1) = a + b + c + d = 1$

(b) $I_1'(2, 1) = 2a + b + 2c + d = 5$

(c) $I_1'(1, 2) = a + 2b + 2c + d = 7$

(d) $I_1'(2, 2) = 2a + 2b + 4c + d = 15$

(e) $a + c = 4$ (from a & b)

(f) $a + 2c = 8$ (from c & d)

(g) $c = 4, a = 0$ (from e & f)

(h) $b + d = -3$ (from a & g)

(i) $2b + d = -1$ (from c & g)

(j) $b = 2, d = -5$ (from h & i)

$$I_1'(x, y) = 2y + 4xy - 5$$

$$I_1'(1.3, 1.8) = 3,6 + 9.36 - 5 = 7.96 \approx 8$$