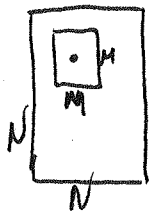
CS474/674 Image Processing and Interpretation

Fall 2009 – Dr. George Bebis

Homework 4 - Solutions

(a) What is the complexity of convolving an N x N image with an M x M mask? Use big-O notation.

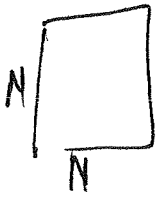


at each point of the image, we need to make M^2 multiplications and M^2 additions. There are N^2 image points thus $O(N^2M^2)$

($N \times N$ image)

(b) What is the complexity of 2D FFT? (*hint:* use the severability property of the FT)

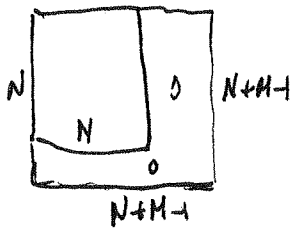
To compute 1D FFT, it takes $O(N \log N)$

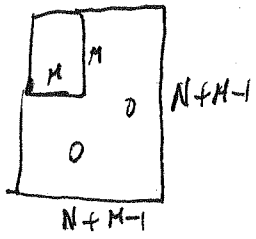


N FFTs of the rows: $O(N^2 \log N)$
N FFTs of the columns: $O(N^2 \log N)$ } $O(N^2 \log N)$

(c) How could we compute the convolution using FFTs? (from (a))

Padd with zeros both image and mask





1. Take FFT of padded image and mask
2. Perform multiplication in frequency domain
3. Take inverse FFT

(d) What is the complexity of computing the convolution in the frequency domain? Use big-O notation.

1) FFT image: $O((N+M-1)^2 \log(N+M-1))$
FFT mask: $O((N+M-1)^2 \log(N+M-1))$
2) Multiplication $O((N+M-1)^2)$
3) Inverse FFT: $O((N+M-1)^2 \log(N+M-1))$

Total

$O((N+M-1)^2 \log(N+M-1))$

$= O((N^2+M^2) \log(N+M-1))$

$= O(N^2 \log(N+M-1) + M^2 \log(N+M-1))$