CS485/685 Computer Vision

Spring 2009 – Dr. George Bebis

Homework 2 - Due Date: 2/26/2009

1. As we have discussed in class, the solution of an over-determined system $Ax = b$ (A is mxn with $m > n$) can be computed as follows: $x = (A^T A)^{-1} A^T b$ where A^+ is the pseudo-inverse of A. Show that this is equivalent to computing $x = VD_0^{-1}U^T b$ where $A = UDV^T$ is the SVD decomposition of A and

$$D_0^{-1} = \begin{cases} 1/\sigma_i & \text{if } \sigma_i > t \\ 0 & otherwise \end{cases}$$

2. Consider the 3D point $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$. What would be coordinates of the point after applying the following composite transformation: (i) rotation of 90 degrees about the x-axis, (ii) translation by $d_x = -2$, $d_y = 1$, $d_z = 1$ and, (iii) scaling by $s_x = 1$, $s_y = 2$ and $s_z = 0.5$. Show your calculations clearly.

3. Prove that the following matrix represents a rigid transformation ($a = \frac{\sqrt{2}}{2}$).

$$\begin{bmatrix} a & -a & 0 \\ a & a & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4 Consider the following camera model. Assume first that the camera and world coordinate systems are aligned. Next, the camera is translated by amount (0,2,2), and it is rotated by an angle 90 degrees around the Z axis, followed by rotation of 45 degrees around the X axis. Both rotations are in clockwise directions. After that the image plane is displaced by amount (0.02, 0.01, 0.03) with respect to the camera center. Finally, perspective projection is applied to form the image. Assume that the focal length of the camera is 0.030. Find the image coordinates of the world point (1,1,0.2).

Graduate Students Only

5 Suppose A is a real mxn matrix. Prove that the squares of the singular values of A are the eigenvalues of $A^T A$. (*hint*: if A is a symetric matrix, it can be written as $A = P \Lambda P^T$ where the columns of P are the eigenvectors of A and Λ is a diagonal matrix with diagonal elements equal to the eigenvalues of A).