Math 181 – Review sheet

The first exam will be given on Friday, 22 Feb in class. Exam I will cover §§ 2.1, 2.2, 2.3, 2.5, 2.6, 2.7, 2.8. Please bring your student ID to the exam.

(1) Find the average rate of change of the function $f(x) = x^2 + 2x + 3$ over the interval $[-1, 4]$.

(2) Determine the limit: $\lim_{x \to 1} \dfrac{3x - 3}{x^2 - 4x + 3}$

(3) Find the limit: $\lim_{x \to \infty} \dfrac{4x^2 - 7x + 11}{6 - 2x^2}$

(4) Find the limits: $\lim_{x \to 2^-} \dfrac{x - 1}{x^2 - 2x - 3}$ $\qquad$ $\lim_{x \to -1^+} \dfrac{x + 1}{x^2 - 2x - 3}$

(5) For what value of a is the given function continuous at $x = 1$?

$$f(x) = \begin{cases} x + 2, & x \leq 1 \\ ax, & x > 1 \end{cases}$$

(6) Let $f(x)$ be given as follows:

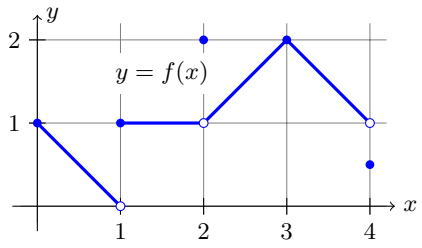
$$f(x) = \begin{cases} 2x, & x \leq -1, \\ x^2, & -1 < x \leq 1, \\ 3 - 2x, & x > 1. \end{cases}$$

Find $\lim_{x \to -1^-} f(x)$, $\lim_{x \to -1^+} f(x)$, $\lim_{x \to 1^-} f(x)$, $\lim_{x \to 1^+} f(x)$. Determine $\lim_{x \to -1} f(x)$, $\lim_{x \to 1} f(x)$, if they exist. Where is f continuous?

(7) Let $f(x) = \dfrac{2x + 3}{x - 1}$. Find the limits of f at ∞ and $-\infty$. Determine all horizontal and vertical asymptotes of the graph $y = f(x)$ and sketch the function.

(8) Let $f(x) = x^2 + 3x$. Find the slope of the secant joining the points $P(-2, f(-2))$ and $Q(-2+h, f(-2+h))$. Find the slope of the curve $y = x^2 + 3x$ at P and then find an equation for the tangent line at P. Find $f'(-2)$. Sketch the curve and the tangent line.

(9) With f as in the given sketch, find both one-sided limits at $a = 1, 2, 3$. In each case determine $\lim_{x \to a} f(x)$, if it exists. Is f is continuous at $x = 1, 2$ or 3?



Does f have any **jump discontinuities**? If so, where?

Does f have any **removable discontinuities**? If so, where?

(10) Let $f(x) = x^3 - 2x + 3$. Use the **definition** of the derivative to find $f'(a)$. Find an equation for the tangent to the graph $y = f(x)$ at $(2, 7)$. Find two points on the curve at which the tangent line has slope 1.

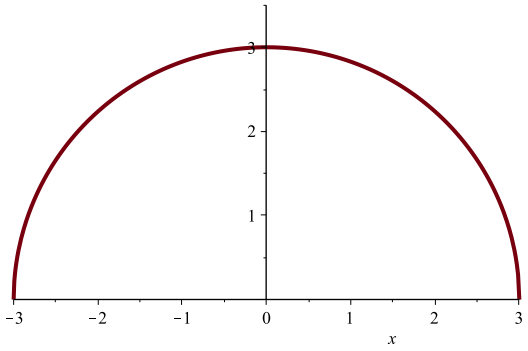
(11) Let $f(x) = \sqrt{x}$. Use the **definition** of the derivative to find $f'(9)$.

(12) Find the limits of both f and g at ∞ where $f(x) = \dfrac{x}{9x^2 + 2x + 3}$ and $g(x) = \dfrac{x}{\sqrt{9x^2 + 2x + 3}}$.

(13) Define $h(2)$ in a way that extends $h(x) = \dfrac{x - 2}{\sqrt{x^2 + 5} - 3}$ to be continuous at $x = 2$.

(14) Determine the limit: $\lim_{x \to 1^-} \dfrac{x^2 - 3}{x^2 - 4x + 3}$.

(15) Sketch the graph of the derivative of $y = f(x)$.



(16) Sketch the graph of the derivative of $y = f(x)$.