

Math 181 – Practice for Exam 2

(1) Find $\frac{dy}{dx}$ where $y = \frac{6}{\sqrt{x}}$

- A) $\frac{3}{x^{3/2}}$ B) $-\frac{3}{x^{3/2}}$ C) $12\sqrt{x}$ D) $6x^{2/3}$ E) none of these

(2) The derivative of the function $f(x) = \frac{2x+1}{x+1}$ is

- A) 1 B) $\frac{1}{x+1}$ C) $\frac{1}{(x+1)^2}$ D) $-\frac{1}{(x+1)^2}$ E) none of these

(3) The derivative of the function $f(x) = x^2e^x$ is

- A) $(x+2)e^x$ B) $2xe^x$ C) $x^2e^x + 2x^2$ D) $x(x+2)e^x$ E) none of these

(4) If $g(t) = \ln(3t^4)$, then its derivative $g'(t)$ is

- A) $\frac{3}{t^4}$ B) $\frac{4}{t}$ C) $\frac{12}{t}$ D) $12\ln(t^3)$ E) none of these

(5) The slope of the tangent line to the graph $y = x + \frac{2}{x}$ at $(2, 3)$ is

- A) -1 B) $-\frac{3}{2}$ C) $\frac{1}{2}$ D) 2 E) none of these

(6) Find the derivative of each of the following. Please simplify your answer.

(a) $y = \frac{x^2}{2} - \frac{2}{x^2} - \sqrt{3}$

(b) $y = (5x^3 + 7)e^x$

(c) $y = \frac{4x}{x^2 + 3}$

(7) In each case find the derivative $\frac{dy}{dx}$. Please simplify your answer.

(a) $y = e^{x^2} \sin x + \ln 3$

(b) $y = \ln(x^3 \cos^2 x)$

(8) Use the *definition* to find the derivative of the function $f(x) = x^3$.

(9) The derivative of $f(x)$ at a is

$$\lim_{\text{something}} \frac{f(x) - f(a)}{x - a}.$$

What is *something*?

(10) The derivative of $f(x)$ at a is

$$\lim_{\text{something}} \frac{f(a + h) - f(a)}{h}.$$

What is *something*?

(11) In the previous two difference quotients, what is the relationship between x and h ?

(12) Suppose the slope of the secant line through $(1, f(1))$ and $(1 + h, f(1 + h))$ is $4 + h$. What is $f'(1)$

(13) Suppose the slope of the secant line through $(1, f(1))$ and $(1 + h, f(1 + h))$ is $4 + h$. (a) What is the slope of the secant line through $(1, f(1))$ and $(x, f(x))$ in terms of x ? (b) How would you find $f'(1)$ using (a)?

(14) Find an equation of the tangent to the graph $y = 2x^{-1/2}$ at the point $(4, 1)$.

$$y = -\frac{x}{8} + \frac{3}{2}$$

(15) Find all points on the graph $y = x^3 - 12x + 9$ with horizontal tangents.

$$(2, -7), (-2, 25)$$

(16) A rock thrown upward on the dwarf planet Eris has height in meters given by the formula $h(t) = 12t - t^2$ where t is given in seconds. Find the velocity and acceleration of the rock. How high is the rock at the time it starts falling. When does it hit the ground? How fast is it going at that time? $12 - 2t$ m/s, -2 m/s², 36 m, 12 s, 12 m/s

(17) The position of an object as a function of time is given by $s(t) = t^3 - 3t + 5$. Find the velocity and acceleration. Find the displacement and average velocity over the interval $-1 \leq t \leq 3$. When if ever does the object change directions? Find its position at these times. $3t^2 - 3$, $6t$, 16, 4, ± 1 , 3, 7

(18) Find an equation tangent to the curve given below at the point (2, 2).

$$y = -x + 4$$

$$x^2y + xy^2 - 16 = 0$$

(19) Find the equations of the normal and tangent lines to the curve at the point (1, 2).

$$y = \frac{-19x+59}{20}, y = \frac{20x+18}{19}$$

$$x^3 + 2x^2y^2 + y^3 = 17.$$

(20) Find the derivative of each of the following. Please simplify your answer.

(a) $y = \sin^{-1}(5x)$

(b) $y = \ln((2x - 3)^{-3} \sin^4 x)$

(c) $y = \ln \sqrt{\frac{2x - 1}{x^2 + 2x + 3}}$

(d) $y = \pi^x + x^\pi$

(21) Use logarithmic differentiations to find the derivative of

$$y = \frac{(3x^2 - 2)^5}{(2x^3 + 5)^7(4x + 1)^{-11}}$$

(22) A circular ripple in a pond caused by a drop of water is increasing in size. Find the rate at which the area of the circle is increasing when its radius is 45 cm and increasing at 7 cm/s. $630\pi \text{ cm}^2/\text{s}$

(23) The length ℓ of a rectangle is decreasing at 1 cm/s and its width w is increasing at a rate of 2 cm/s when $\ell = 20$ cm and $w = 15$ cm. Find the rate of change of the area and the length of the diagonal. $25 \text{ cm}^2/\text{s}, 0.4 \text{ cm/s}$

(24) Determine the limit: $\lim_{x \rightarrow 0} \frac{5x}{\sin 3x}$.