

# Impact of Limiting Number of Links on the Lifetime of Wireless Sensor Networks

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**Abstract**—The lifetime of wireless sensor networks (WSNs) is optimized if the traffic within the network is adjusted in a way that all nodes dissipate their energies in a balanced fashion. To balance the energy dissipation, nodes split their flows and these flows are forwarded to different nodes acting as relays. In consequence some nodes have too many incoming and/or outgoing links. In this letter we investigate the impact of limiting the number of incoming and outgoing links of nodes on the network lifetime of WSNs through a Mixed Binary Linear Programming (MBLP) framework. Our results show that the decrease in WSN lifetime is less than 1.0 % if the limits on the number of incoming and outgoing links are not lower than three.

**Index Terms**—Wireless sensor networks, mixed binary linear programming, energy efficiency, network complexity.

## I. INTRODUCTION

IN WSNs minimizing the energy dissipation of each node individually does not result in maximal network lifetime [1], [2]. Instead nodes should cooperatively optimize their energy dissipation characteristics. Previously, energy balancing for network lifetime maximization was investigated through Linear Programming (LP) [1], [2], [3], [4], [5], [6], [7]. It was shown that to achieve the maximal lifetime, nodes often split their flows into parts and forward each part to a different node. The energy cost of transmission and reception for relaying are evenly shared by the whole network. Although the optimal routing strategy requires data splitting, it results in a complicated network flow pattern. Some nodes have too many incoming and/or outgoing links. In practical WSN applications maintaining a large number of connections and splitting the flows into too many parts are not desirable [8]. Reducing the number of links results in simpler routing patterns and lower network complexity [9].

LP is a powerful tool to analyze and characterize WSNs. In [1] WSN lifetime is investigated by using two multi-hop routing schemes through an LP approach. In [2] strategies for mitigating the WSN hot spot problem is explored by using an LP framework. In [3] an MBLP framework is presented to investigate the impact of one time energy costs on the system lifetime in WSNs. In [4] an LP framework is devised for modeling dynamic data compression in WSNs. In [5] a Mixed Integer Linear Programming (MILP) approach based on multi-commodity flow to ensure uniform energy consumption throughout the network by splitting and re-routing the non-optimal flows over multiple paths is proposed. In [6] energy efficient routing for data aggregation in WSNs to maximize

network lifetime is investigated using LP formulation. In [7] a centralized MILP based column generation approach and a distributed heuristic algorithm are proposed to maximize network lifetime by exploiting sensor spatial redundancy. The novel contribution of our study is to characterize the impact of limiting the number of incoming/outgoing links on the WSN lifetime through an MBLP framework, which has not been investigated previously.

## II. CONCEPT AND MODEL

Throughout this letter, we use the energy model introduced in [2], where the amount of energy to transmit a bit is represented as  $P_{tx,ij} = \rho + \varepsilon d_{ij}^\alpha$ , and the amount of energy to receive a bit is represented as  $P_{rx} = \rho$ , where  $\rho$  models the energy dissipation on electronic circuitry,  $\varepsilon$  denotes the transmitter's efficiency,  $\alpha$  represents the path loss, and  $d_{ij}$  is the distance between node- $i$  and node- $j$ . An important QoS metric for sensor networks (especially WSNs used for surveillance of a sensitive area) is the coverage (e.g., in a surveillance system if one of the critical sensors dies then overall security is compromised). Hence, we adopt the network lifetime definition given in [1], [2], [7] which is the time when the first sensor node exhausts all its battery power. In our framework, each sensor node- $i$  creates  $s_i$  unit of raw data per unit time to be conveyed to the base station.

The network topology is represented by a graph  $G = (V, A)$ , which is a complete directed graph.  $V$  is the set of nodes, including the base station as node-1. We also define set  $W$ , which includes all nodes except node-1.  $A = \{(i, j) : i \in W, j \in V - \{i\}\}$  is the set of arcs. The amount of data to be sent on the directed arc  $(i, j)$  is denoted as  $f_{ij}$ . We also use the binary variables  $a_{ij}$  to be able to control the number of incoming and outgoing links, parametrically. The optimization problem in its general form is formulated as maximizing  $t$  (the minimum lifetime of sensor nodes) subject to the following constraints:

$$f_{ij} \geq 0, \forall (i, j) \in A \quad (1)$$

$$\sum_{j \in V} f_{ij} - \sum_{j \in W} f_{ji} - s_i t = 0, \forall i \in W \quad (2)$$

$$\sum_{j \in V} P_{tx,ij} f_{ij} - P_{rx} \sum_{j \in W} f_{ji} \leq e_i, \forall i \in W \quad (3)$$

$$f_{ij} \leq M a_{ij}, \forall (i, j) \in A \quad (4)$$

$$\sum_{j \in V} a_{ij} \leq L_{OUT}, \forall i \in W \quad (5)$$

$$\sum_{j \in W} a_{ji} \leq L_{IN}, \forall i \in W \quad (6)$$

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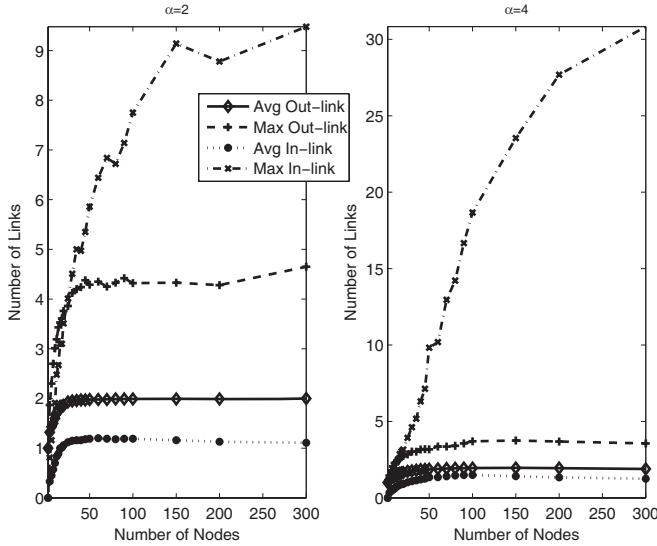


Fig. 1. Number of links as a function of number of nodes when the numbers of incoming and outgoing links are unlimited. Area per node is  $1000 \text{ m}^2$ .

First constraint states that all flows are non-negative. Second constraint models the conservation of raw data flow. Third constraint states that the energy dissipation at node- $i$  is limited by the energy budget of node- $i$  ( $e_i$ ). Energy dissipation on transmission and reception at node- $i$  are given by the first and second terms on the left hand side of the inequality, respectively. Fourth constraint is used to link the continuous variables  $f_{ij}$ 's with the discrete variables  $a_{ij}$ 's. Note that  $a_{ij}$  is zero if there is no data flow on arc  $(i, j)$  and  $a_{ij}$  is unity if there is non-zero flow on arc  $(i, j)$ .  $M$  is a large constant. Fifth constraint enables us to limit the number of outgoing links from sensor nodes. Sixth constraint is to limit the number of incoming links (*i.e.*,  $L_{OUT}$  and  $L_{IN}$  are the limits on the number of outgoing and incoming links, respectively). Note that constraints 5 and 6 are used to limit the number of links - not the amount of incoming/outgoing data (limiting the number of links does not lead to data reduction).

### III. RESULTS

The communication parameters are chosen as  $\rho = 50 \text{ nJ}$  and  $\varepsilon = 100 \text{ pJ}$ , same as the ones in [2]. We use a disk network consisting of  $N$  sensor nodes which are randomly placed within the disk and a base station at the center. The results are averaged over 100 independent runs (*i.e.*, 100 randomly generated topologies). There are no limitations on the communication ranges of the nodes. All nodes create raw data at a uniform rate (1 bps). We use  $NN_x$  notation to denote a network with  $x$  nodes.

We first solve the optimization problem when there is no limit on the numbers of links ( $L_{IN} \rightarrow \infty$ ,  $L_{OUT} \rightarrow \infty$ ). The number of links as a function of number of nodes in the network is presented in Fig. 1. We use the framework presented in Section II to obtain the results in Fig. 1.

For  $\alpha = 2$  average numbers of outgoing and incoming links are roughly limited by 2 and 1, respectively whereas for  $\alpha = 4$  average numbers of outgoing and incoming links are roughly limited by 2 and 1.5, respectively. Maximum numbers of incoming and outgoing links are considerably

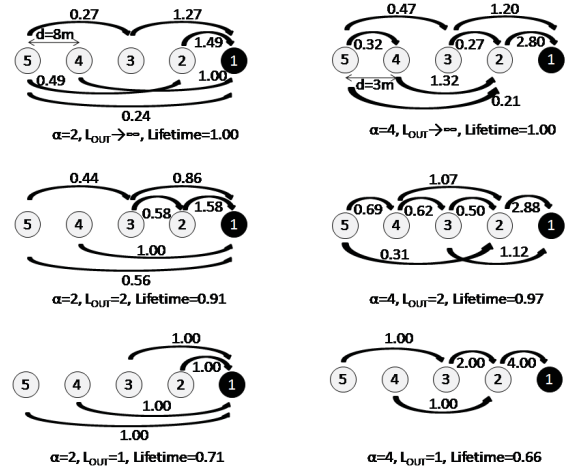


Fig. 2. Optimal routes for the selected topologies with the parameter sets indicated. Lifetime values for the second and third rows are normalized with the lifetimes obtained for the configurations at the top row. Node-1 is designated as the base station. The numbers on each arc denotes the amount of data flow. Amount of data generated at each node is normalized to 1.

larger than the averages. The example topology presented in Fig. 2 (top row right column) is helpful in understanding this fact. The average numbers of incoming and outgoing links for sensor nodes (excluding the base station - node-1) are 1.75 and 1.25, respectively. However, node-5 (the node farthest from the bases station) has 3 outgoing links (the maximum number of outgoing links) and node-2 (the closest node to the base station) has 3 incoming links (the maximum number of incoming links). Hence, the numbers of outgoing links of nodes farther from the base station are more than average number of outgoing links and the numbers of incoming links of nodes closer to the base station are more than average number of incoming links. Maximum number of incoming links increases sharply as the network gets larger to balance the energy dissipation within the network (*i.e.*, as the number of nodes increases nodes closer to the base station receive data from an increased number of nodes located farther from the base station - some nodes closer to the base station become hot spots).

To illustrate graphically the effects of limiting the number of links, we solve the optimization problem using example topologies presented in Fig. 2. We use both  $\alpha = 2$  (left column) and  $\alpha = 4$  (right column) to explore the dependency on the propagation model. When the number of outgoing links is limited by 2, network lifetimes are 9.0 % ( $\alpha = 2$ ) and 3.0 % ( $\alpha = 4$ ) lower than the lifetimes obtained with no limit on the number of outgoing links. The decrease in the network lifetime reaches 29.0 % for  $\alpha = 2$  and 34.0 % for  $\alpha = 4$  when the number of outgoing links is limited by one. Note that when there is no limit on the number of outgoing links the optimal routing decision in  $\alpha = 2$  case for node-5 is to split its generated data in three parts, transmit 24.0 % of the data directly to the base station, forward 49.0 % of data to node-2, and forward 27.0 % of the data to node-3. However, node-5 transmits all its data directly to the base station when the number of outgoing links is limited by one. Limiting the number of links affects network lifetime due to elimination of

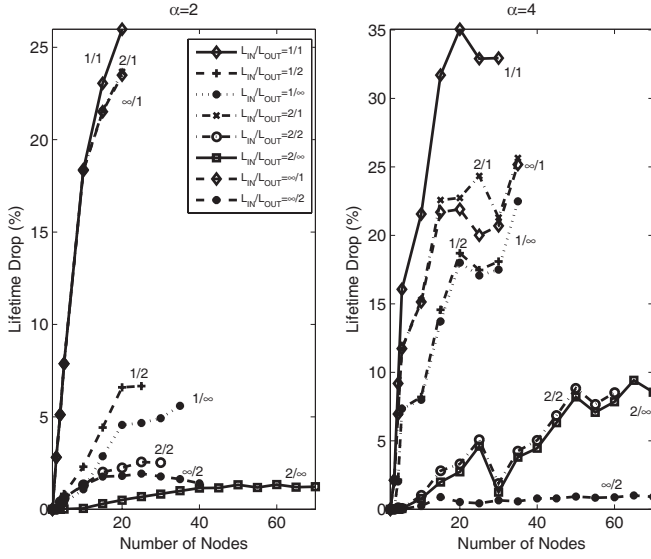


Fig. 3. Percentage lifetime drop with respect to unlimited incoming/outgoing links case as a function of number of nodes. Area per node is  $1000 \text{ m}^2$ .

optimal data splitting and routing decisions.

In Fig. 3 lifetime decreases (with respect to the lifetime obtained in an unconstrained incoming/outgoing links network) due to the limitations on  $L_{IN}$  and/or  $L_{OUT}$  are presented as a function of number of nodes. The data points presented in Fig. 3 are within 1.0 % neighborhood of the optimal value. Generally speaking, MBLP problems are more computationally intensive than pure LP problems, thus to solve a problem in reasonable time we terminate computations once 1.0 % neighborhood is achieved. Furthermore, it is not possible to obtain data for certain parameter sets. Computations do not converge to 1.0 % neighborhood of the optimal value in reasonable time (*i.e.*, weeks). Therefore some of  $L_{IN}/L_{OUT}$  curves are presented only for small networks.

The largest lifetime decrease occurs when both  $L_{IN}$  and/or  $L_{OUT}$  are limited by one because the nodes are so restricted that they cannot balance their energy dissipation and inefficient data flows have to be carried out (*i.e.*, nodes cannot split their data). We observe a decrease of 26.0 % (1/1,  $NN_{20}$ ) for  $\alpha = 2$  case and 33.0 % (1/1,  $NN_{30}$ ) for  $\alpha = 4$  case. If both incoming and outgoing links are limited by two then the maximum lifetime decreases are 2.6 % (2/2,  $NN_{25}$ ) for  $\alpha = 2$  case and 8.8 % (2/2,  $NN_{50}$ ) for  $\alpha = 4$  case. If there is no limit on the incoming links and the limit on the outgoing links is two (*i.e.*,  $L_{IN} \rightarrow \infty$ ,  $L_{OUT} \geq 2$ ) then the lifetime decrease is at most 2.0 %. In general, limiting  $L_{IN}$  rather than  $L_{OUT}$  has a more severe impact on the network lifetime. The decrease in the network lifetime is less than 1.0 % (not shown in the figure) if the numbers of both incoming and outgoing links are limited by at least three (*i.e.*,  $L_{IN} \geq 3$ ,  $L_{OUT} \geq 3$ ). This is not a surprising result because the average numbers of incoming and outgoing links are less than three in unrestricted networks (Fig. 1). We conjecture that allowing three incoming/outgoing links is good enough to balance the energy dissipation.

In Fig. 4 lifetime decreases are presented as a function of area per node. There is no decrease in lifetime for small area networks because the optimum routing decision for all nodes is to transmit all their data directly to the base station. When the

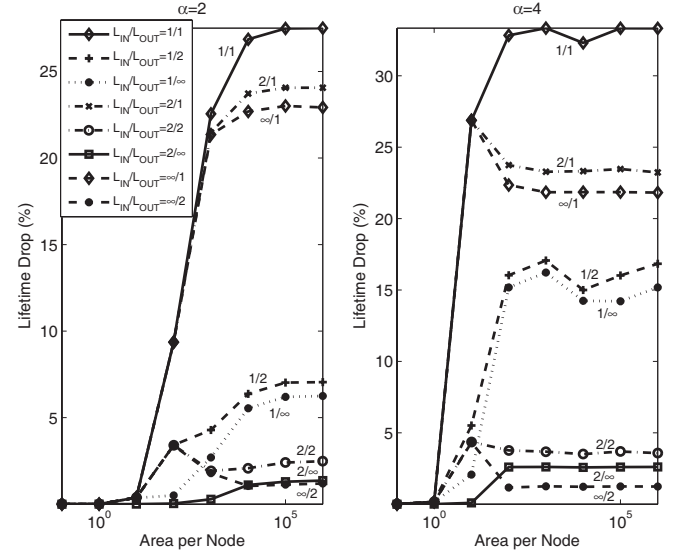


Fig. 4. Percentage lifetime drop with respect to unlimited incoming/outgoing links case as a function of number of nodes. Number of nodes is 15.

area per node value is above a threshold value lifetime drop characteristics stays relatively constant because the optimal routing patterns do not change significantly for large inter-node distances.

#### IV. CONCLUSION

In this letter we investigated the impact of limiting the numbers of incoming and outgoing links on the network lifetime through an MBLP framework. Our results show that if the numbers of incoming and outgoing links are upper-limited by one, then the network lifetime can decrease more than 30.0 %. However, if the numbers of incoming and outgoing links are limited by at least three then the decrease in network lifetime is negligible (less than 1.0 %). We conclude that splitting the flows into three parts is good enough to balance the energy consumption among the sensor nodes.

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