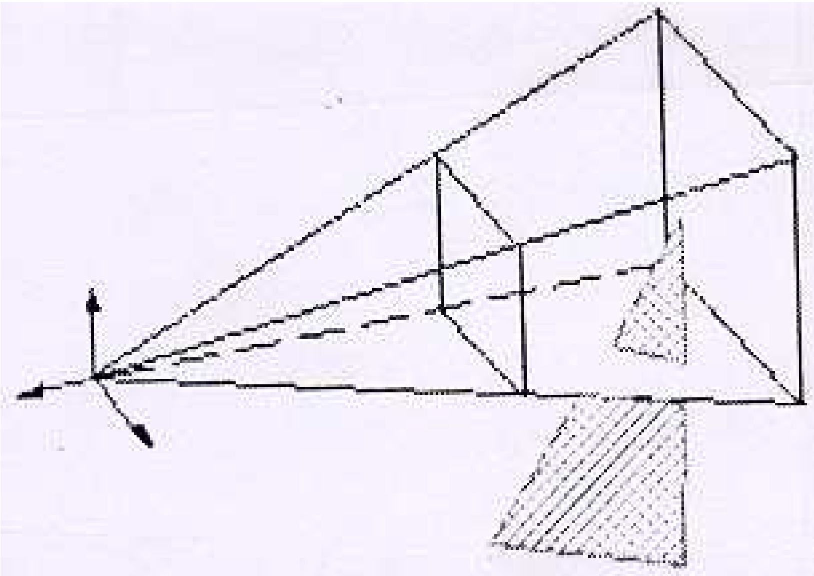
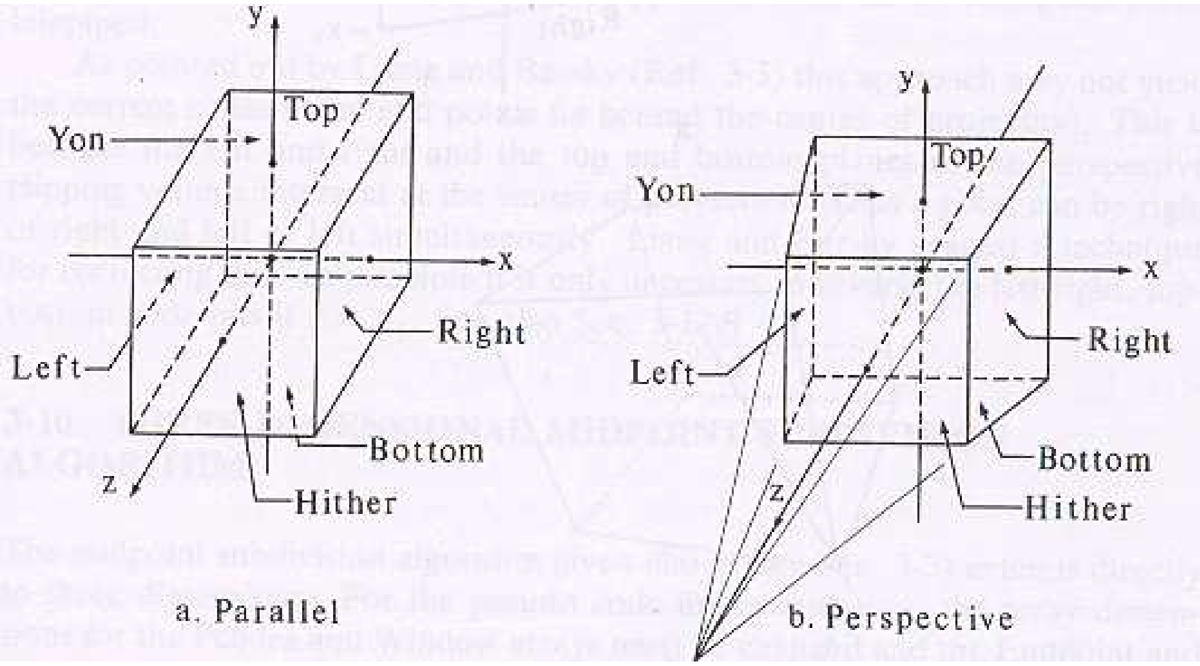
Clipping Against a Canonical View Volume



- Both the Cohen-Sutherland and the Cyrus-Beck line-clipping algorithms can be extended to 3D

- **Cohen-Sutherland 3D line-clipping**



Parallel

bit 1 - point is above the view volume (y>1)
bit 2 - point is below the view volume (y<-1)
bit 3 - point is right of view volume (x>1)
bit 4 - point is left of view volume (x<-1)
bit 5 - point is behind view volume (z<-1)
bit 6 - point is in front of view volume (z>0)

Perspective

bit 1 - point is above the view volume (y>-z)
bit 2 - point is below the view volume (y<z)
bit 3 - point is right of view volume (x>-z)
bit 4 - point is left of view volume (x<z)
bit 5 - point is behind view volume (z<-1)
bit 6 - point is in front of view volume (z>z_{min})

- Steps using the endpoint codes

1. If both endpoints have the code 000000, then the line is trivially accepted

2. If the logical AND of the codes is not zero, then the line can be trivially rejected

3. If the logical AND of the codes is zero, iterative clipping takes place

- **Parametric 3D line clipping**

- Parametric 3D line representation

(1) $x(t) = x_0 + t(x_1 - x_0)$

(2) $y(t) = y_0 + t(y_1 - y_0), \quad 0 \le t \le 1$

(3) $z(t) = z_0 + t(z_1 - z_0)$

- Calculate the intersection of a line with a plane

Let's consider the $y = 1$ plane <u>(case of parallel projection)</u>

(2) => $1 = y_0 + t(y_1 - y_0)$ or $t = \dfrac{1 - y_0}{y_1 - y_0}$

(1) => $x = x_0 + \dfrac{1 - y_0}{y_1 - y_0}(x_1 - x_0)$

(3) => $z = z_0 + \dfrac{1 - y_0}{y_1 - y_0}(z_1 - z_0)$

Let's consider the $y = z$ plane <u>(case of perspective projection)</u>

(2),(3) => $y_0 + t(y_1 - y_0) = z_0 + t(z_1 - z_0)$ or $t = \dfrac{z_0 - y_0}{(y_1 - y_0) - (z_1 - z_0)}$

(1) => $x = x_0 + \dfrac{(x_1 - x_0)(z_0 - y_0)}{(y_1 - y_0) - (z_1 - z_0)}$

(2) => $y = y_0 + \dfrac{(y_1 - y_0)(z_0 - y_0)}{(y_1 - y_0) - (z_1 - z_0)}$

(3) => $z = y$

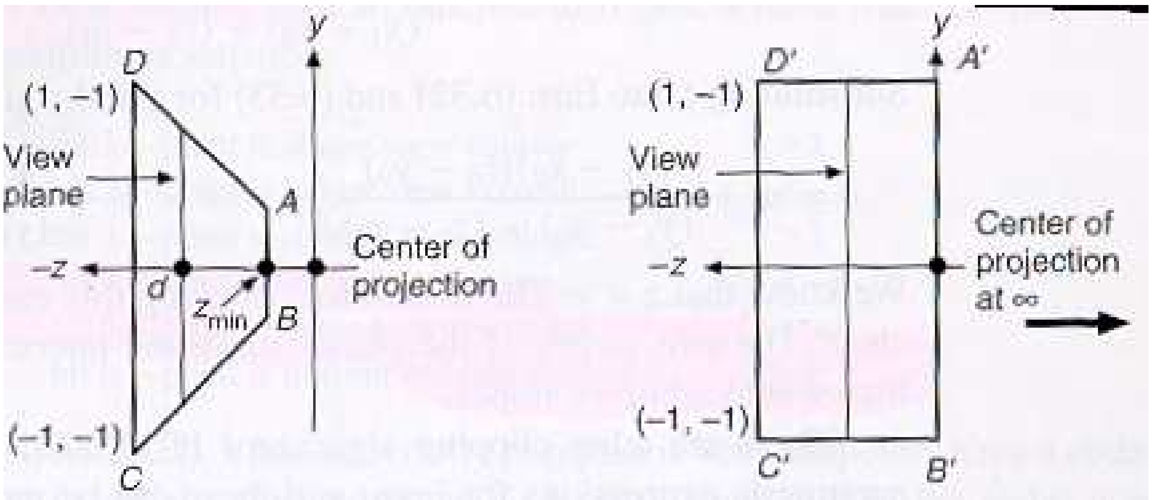
Clipping in Homogeneous Coordinates

- Why ?

  - Efficiency: Transform the perspective-projection canonical view volume into the parallel-projection view volume and use the same clip procedure for both cases

  - Homogenization can cause problems in clipping if $w < 0$

- Transform the perspective-projection canonical view volume to the parallel-projection view volume



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{1+z_{min}} & -\frac{z_{min}}{1+z_{min}} \\ 0 & 0 & -1 & 0 \end{bmatrix}, -1 < z_{min} < 0$$

$$P\begin{bmatrix} 0 \\ 0 \\ z_{min} \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -z_{min} \end{bmatrix}, \quad P\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{-z_{min}}{1+z_{min}} \\ 0 \end{bmatrix} \text{(note: } w = 0\text{)}$$

$$N'_{per} = PN_{per}\ S_{per}\ SH_{par}\ T(-PRP)\ R\ T(-VRP)$$

- **Defining the view volume in homogeneous coordinates**

$$-1 \le x \le 1 \Rightarrow -1 \le X/W \le 1 \Rightarrow -W \le X \le W$$

$$-1 \le y \le 1 \Rightarrow -1 \le Y/W \le 1 \Rightarrow -W \le Y \le W$$

$$-1 \le z \le 0 \Rightarrow -1 \le Z/W \le 0 \Rightarrow -W \le Z \le 0$$

- Constraints in homogeneous coordinates

Case 1: W>0 (see above)

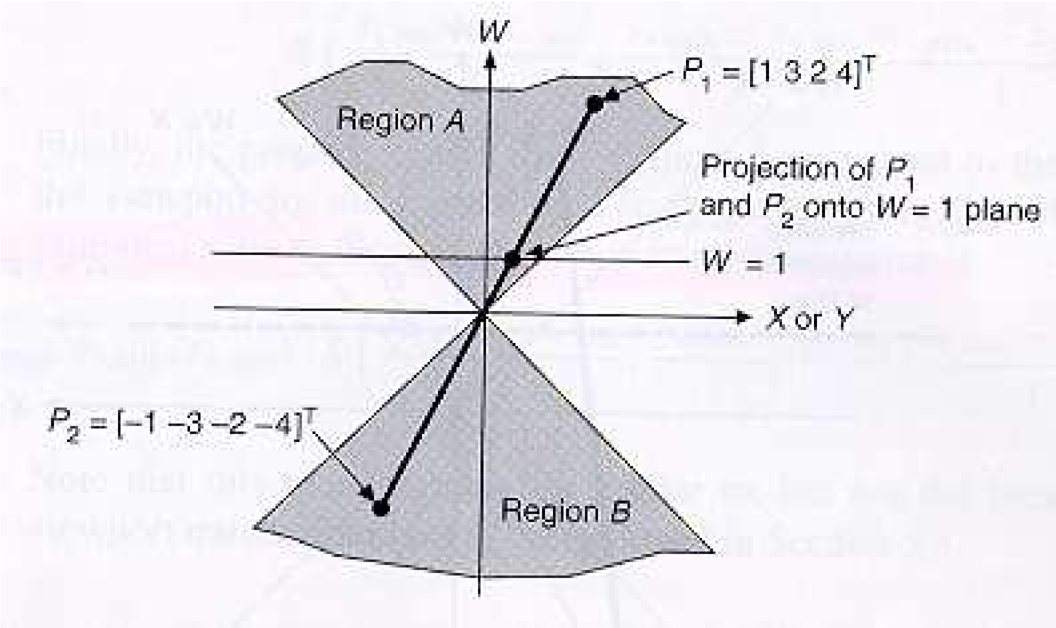
Case 2: W< 0

$$-W \ge X \ge W$$

$$-W \ge Y \ge W$$

$$-W \ge Z \ge 0$$

- **Problems**



- Clip twice (regions A,B)

- Negate coordinates of points with $W < 0$

$$P_1 = (1/4, 3/4, 2/4), \quad P_2 = (-1/-4, -3/-4, -2/-4) = P_1$$

- P_1, P_2 belong to the same 3D volume but into different 4D regions