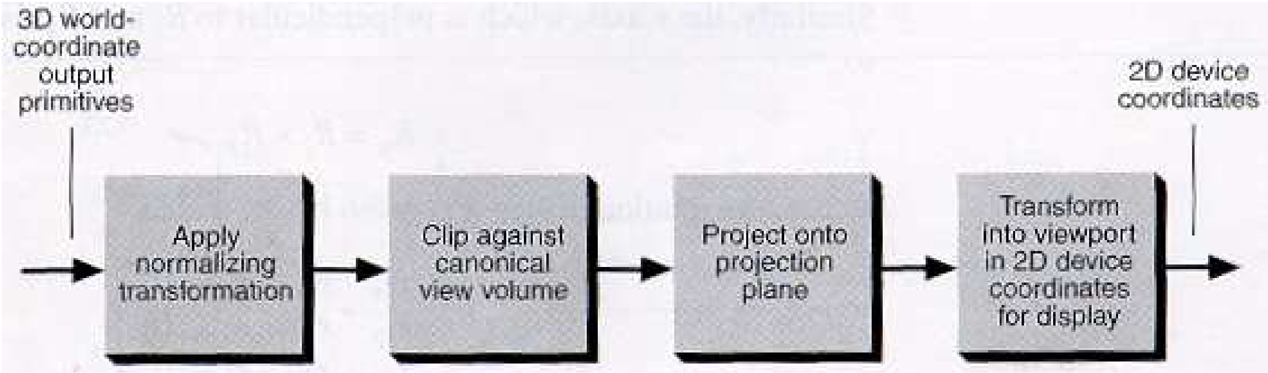
Implementation of 3D Viewing



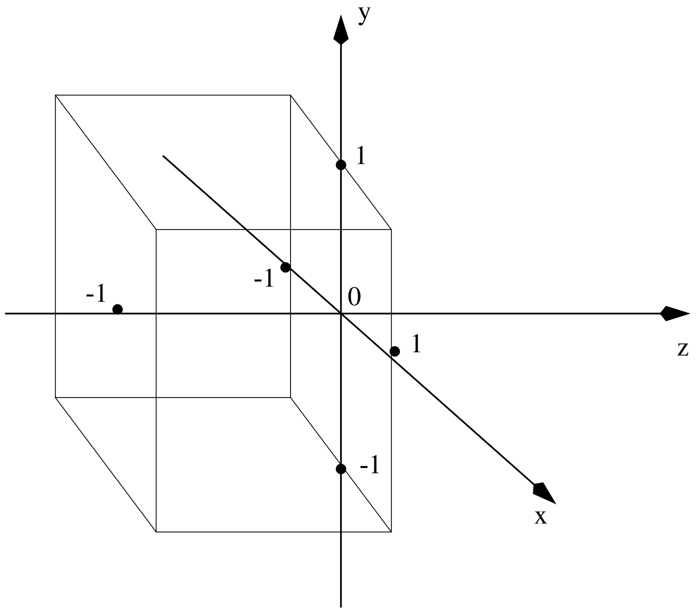
- **Canonical View Volumes**

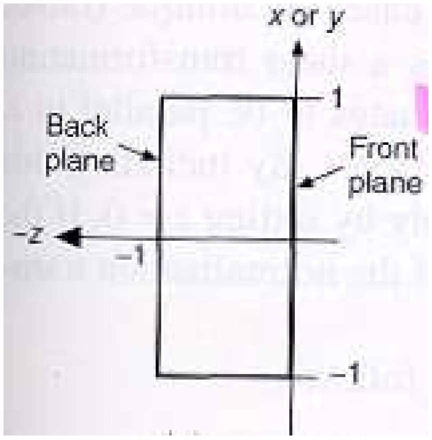
  - It is easier to apply clipping using certain view volumes

  - Idea: map the view volume to a canonical view volume (normalizing transformation)

- **Parallel projection canonical view volume**

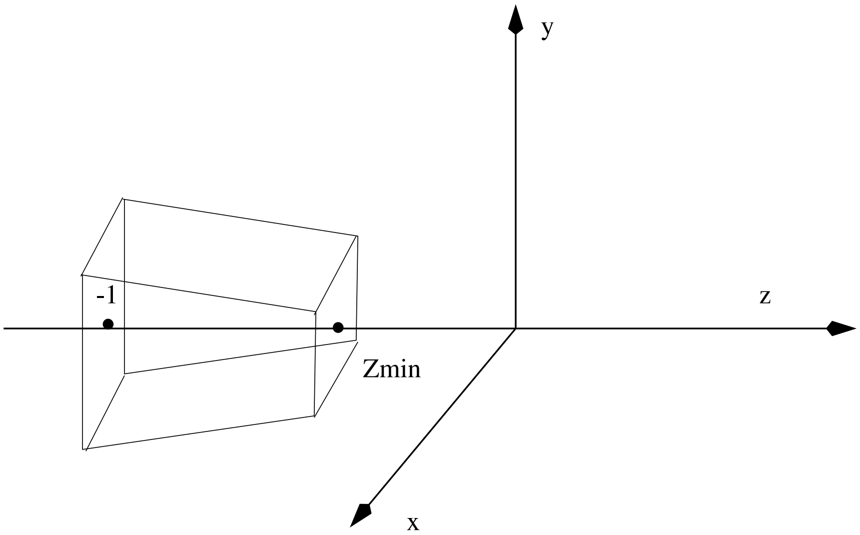
$$x = -1,\ x = 1,\ y = -1,\ y = 1,\ z = 0,\ z = -1$$

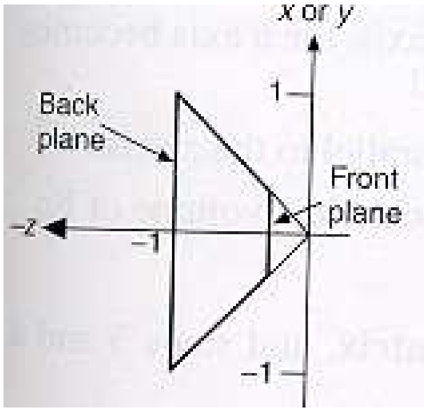




- **Perspective projection canonical view volume**

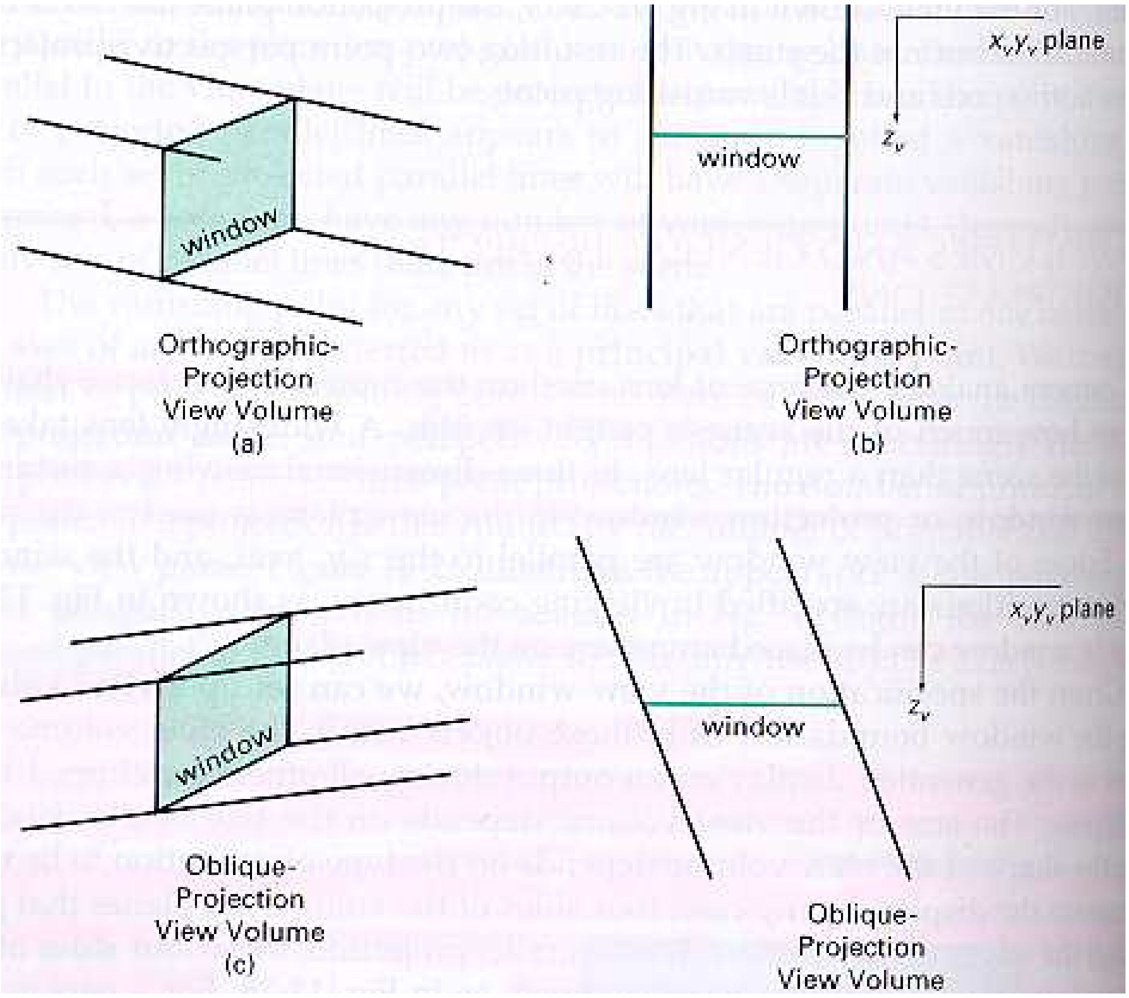
$$x = z,\ x = -z,\ y = z,\ y = z,\ z = -z_{\min},\ z = -1$$



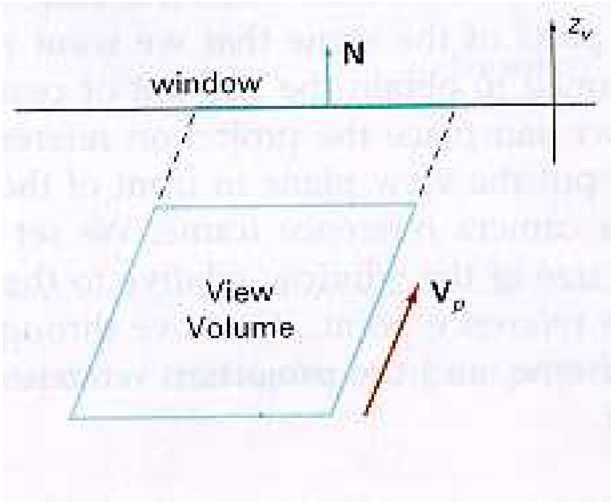


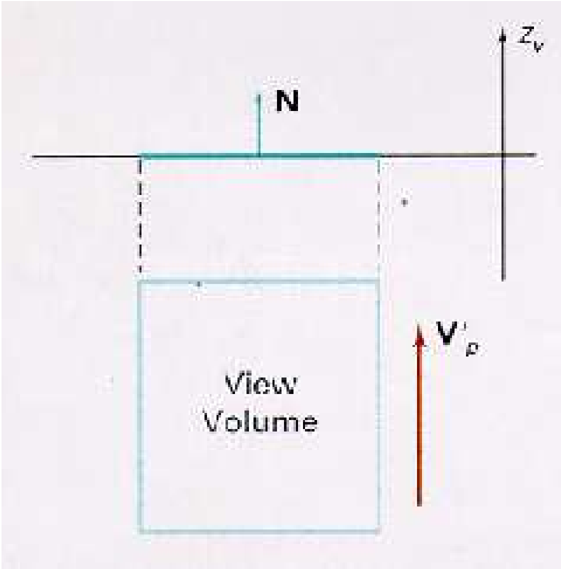
- **The parallel projection case**

  - Consider the most general case: oblique projection



- <u>Idea</u>





- <u>Steps to derive N_{par}</u>

1. Align the VCS with the WCS (*view orientation matrix*)

2. Shear such that the direction of projection (DOP=CW-PRP) becomes parallel with VPN

$$CW = \begin{bmatrix} \dfrac{u_{\max}+u_{\min}}{2} \\ \dfrac{v_{\max}+v_{\min}}{2} \\ 0 \\ 1 \end{bmatrix}, \quad PRP = \begin{bmatrix} prp_u \\ prp_v \\ prp_n \\ 1 \end{bmatrix}, \quad DOP = \begin{bmatrix} \dfrac{u_{\max}+u_{\min}}{2} - prp_u \\ \dfrac{v_{\max}+v_{\min}}{2} - prp_v \\ -prp_n \\ 1 \end{bmatrix} = \begin{bmatrix} dop_x \\ dop_y \\ dop_z \\ 1 \end{bmatrix}$$

$$DOP' = SH_{par}\ DOP \Rightarrow \begin{bmatrix} 0 \\ 0 \\ dop_z \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & sh_{xpar} & 0 \\ 0 & 1 & sh_{ypar} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} dop_x \\ dop_y \\ dop_z \\ 1 \end{bmatrix}$$

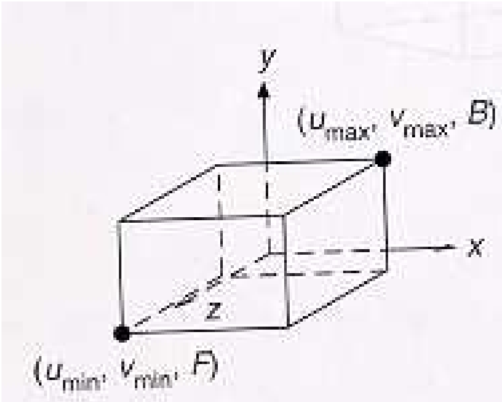
$$sh_{xpar} = \frac{dop_x}{dop_z}, \quad sh_{ypar} = \frac{dop_y}{dop_z}$$

View volume after step 2:

$$u_{\min} \le x \le u_{\max}$$

$$v_{\min} \le y \le v_{\max}$$

$$B \le z \le F$$



3. Translate and scale into the parallel-projection canonical view volume

* Translate the front center of the view volume (point P) to (0,0,0)

* Scale down to the size of the canonical view volume (2x2x1)

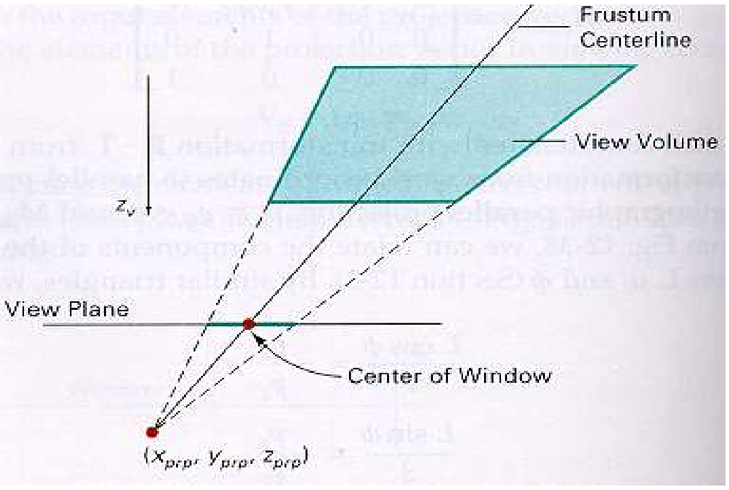
$$T_{par} = T\left(-\frac{u_{\max} + u_{\min}}{2}, -\frac{v_{\max} + v_{\min}}{2}, -F\right)$$

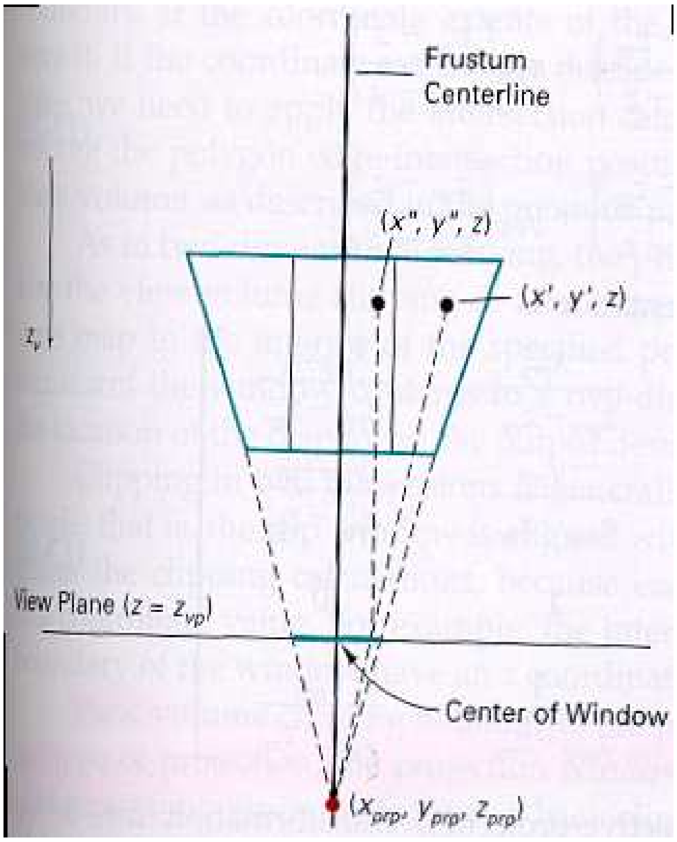
$$S_{par} = S\left(\frac{2}{u_{\max} - u_{\min}}, \frac{2}{v_{\max} - v_{\min}}, \frac{1}{F - B}\right)$$

$$\boxed{N_{par} = S_{par}\ T_{par}\ SH_{par}\ R\ T(-VRP)}$$

- **The perspective projection case**

- Idea





- Steps to derive N_{per}

1. Align the VSC with the WCS (*view orientation matrix*)

2. Translate such that the center of projection (COP=PRP), is at the origin

$$COP = PRP = (prp_u, prp_v, prp_n), \text{ thus: } T(-PRP)$$

3. Shear such that the center line of the view volume becomes the z axis

$$SH_{per} = SH_{par}$$

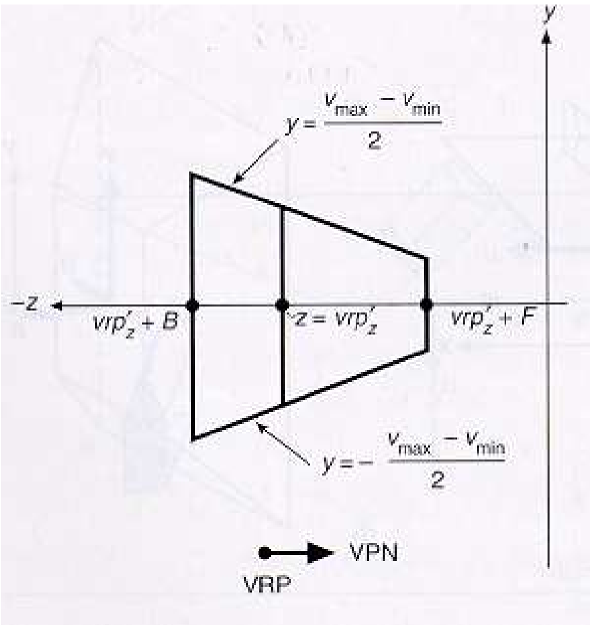
(the center line of the view volume has the same direction as that of parallel projection, DOP=CW-PRP)

<u>View volume after step 3:</u>

$$-\frac{u_{\max}-u_{\min}}{2vrp'_z}\, z \le x \le \frac{u_{\max}-u_{\min}}{2vrp'_z}\, z$$

$$-\frac{v_{\max}-v_{\min}}{2vrp'_z}\, z \le y \le \frac{v_{\max}-v_{\min}}{2vrp'_z}\, z$$

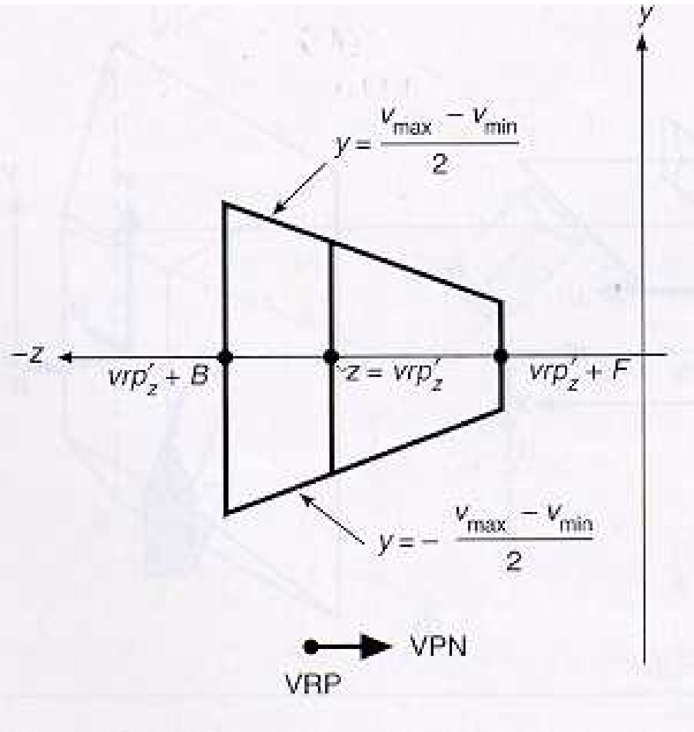
$$vrp'_z + B \le z \le vrp'_z + F$$

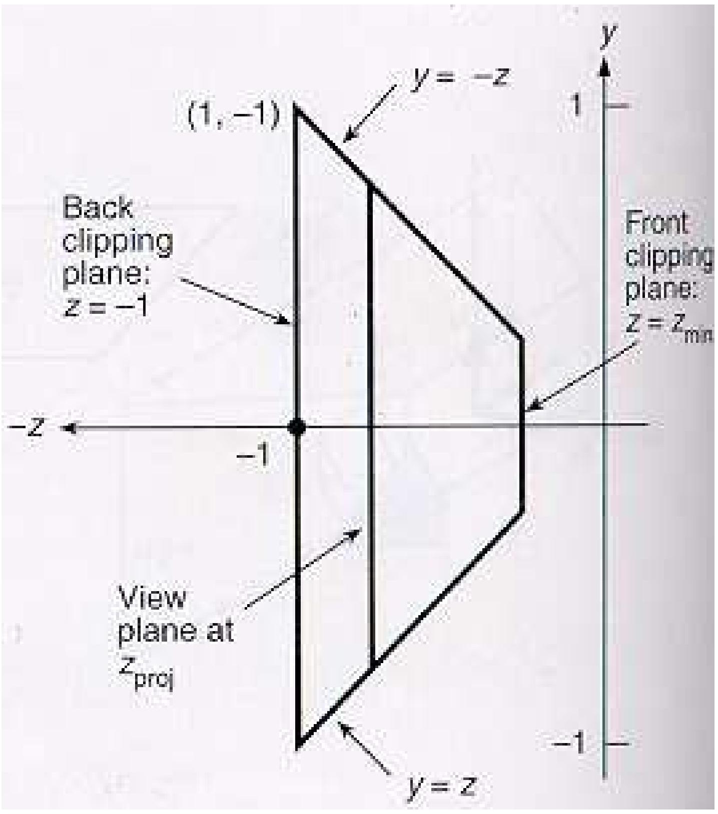


$$VRP' = SH_{par}\; T(-PRP)\begin{bmatrix}0\\0\\0\\1\end{bmatrix} = \begin{bmatrix}1 & 0 & \frac{dop_x}{dop_z} & 0\\ 0 & 1 & \frac{dop_x}{dop_z} & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & -prp_u\\ 0 & 1 & 0 & -prp_v\\ 0 & 0 & 1 & -prp_n\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

$$VRP' = \begin{bmatrix}-prp_u - prp_n\,\frac{dop_x}{dop_z}\\ -prp_v - prp_n\,\frac{dop_y}{dop_z}\\ -prp_n\\ 1\end{bmatrix} = \begin{bmatrix}vrp'_x\\ vrp'_y\\ vrp'_z\\ 1\end{bmatrix}$$

4. Scale down to the size of the canonical view volume





Find the scale transformation such that $P'_i = SP_i$, $i = 1, 2, \ldots$

* $\underline{P'_1 = SP_1}$

$$\begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ vrp'_z + B \\ 1 \end{bmatrix} \Rightarrow s_z = -\frac{1}{vrp'_z + B}$$

* $\underline{P'_2 = SP_2}$

$$\begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \frac{v_{max} - v_{min}}{2vrp'_z}(vrp'_z + B) \\ vrp'_z + B \\ 1 \end{bmatrix} \Rightarrow s_y = \frac{2vrp'_z}{(v_{max} - v_{min})(vrp'_z + B)}$$

* Similarly, $s_x = \frac{2vrp'_z}{(u_{max} - u_{min})(vrp'_z + B)}$

Thus, $S_{per} = S(s_x, s_y, s_z)$

$$* \; z_{proj} = S_{per} \begin{bmatrix} 0 \\ 0 \\ 0 \\ vrp'_z \end{bmatrix} = -\frac{vrp'_z}{vrp'_z + B}$$

$$* \; z_{\min} = S_{per} \begin{bmatrix} 0 \\ 0 \\ 0 \\ vrp'_z + F \end{bmatrix} = -\frac{vrp'_z + F}{vrp'_z + B}$$

$$* \; z_{\max} = S_{per} \begin{bmatrix} 0 \\ 0 \\ 0 \\ vrp'_z + B \end{bmatrix} = -\frac{vrp'_z + B}{vrp'_z + B} = -1$$

$$\boxed{N_{per} = S_{per} \; SH_{par} \; T(-PRP) \; R \; T(-VRP)}$$