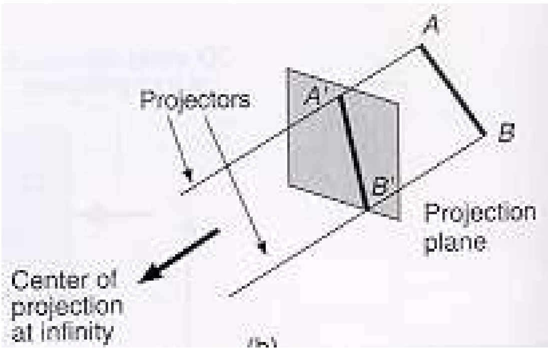
Parallel Projection

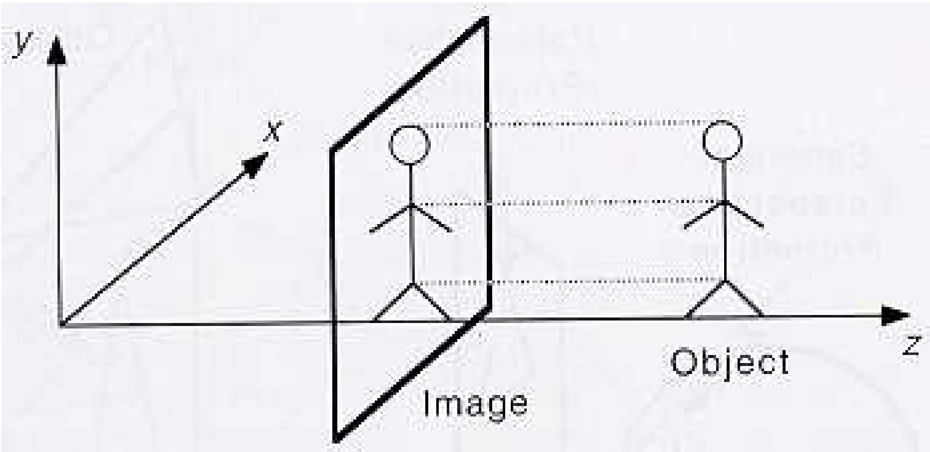
- The projection of a 3D scene onto a plane by a set of parallel lines having a direction, called the direction of projection (e.g., the center of projection is at infinity)



- **Orthographic projection**

  - The direction of projection is perpendicular to the projection plane

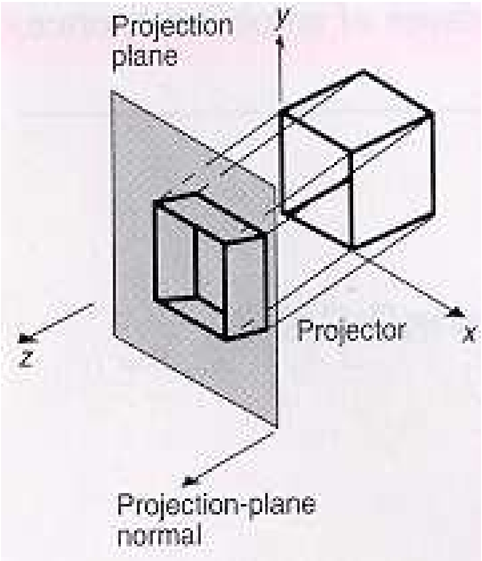
  (The direction of projection and the direction of the projection-plane normal are the same)



- **Oblique projection**

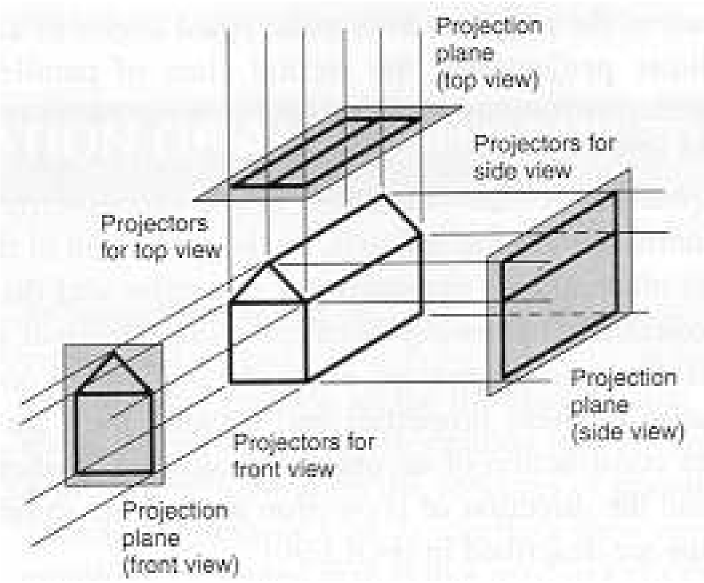
  - The direction of projection is not perpendicular to the projection plane

  (The direction of projection and the direction of the projection-plane normal are not the same)



- **Categories of orthographic projection**

  - Multiview: *front*, *side*, and *top* views (*elevations*)



* Projection plane is parallel to one of the principal planes

* Illustrate only one face of the object

* 3D apperance is difficult to be illustrated

* Lengths and angles are accurately depicted and can be measured from the drawings

- Axonometric: *isometric*, *dimetric*, *trimetric*

  * Projection plane is not parallel to any of the principal planes

  - Describe several faces (usually three) of an object

  * 3D appearance can be illustrated

  * Resembles perspective projection but now foreshortening is uniform

  * Scaling factors can be different for each principal axis

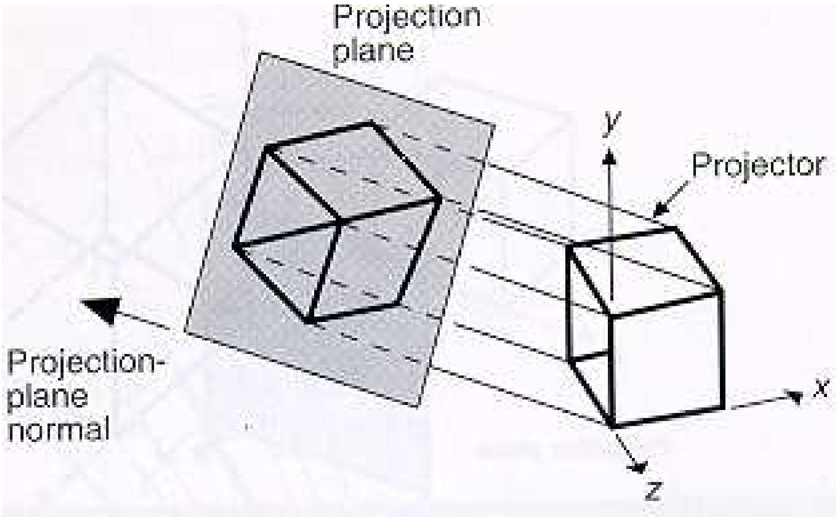
- **Categories of axonometric projection**

  - They are classified according to the orientation of the projection plane (i.e., the projection-plane normal) and the coordinate axes

  - Isometric: the projection-plane normal makes equal anges with all principal axes

  - The projection-plane normal is parallel to one of the following vectors: $(\pm x, \pm y, \pm z)$ (8 possible isometric views)

  - Scaling factors are the same for all coordinate axes (same foreshortening for all axes)



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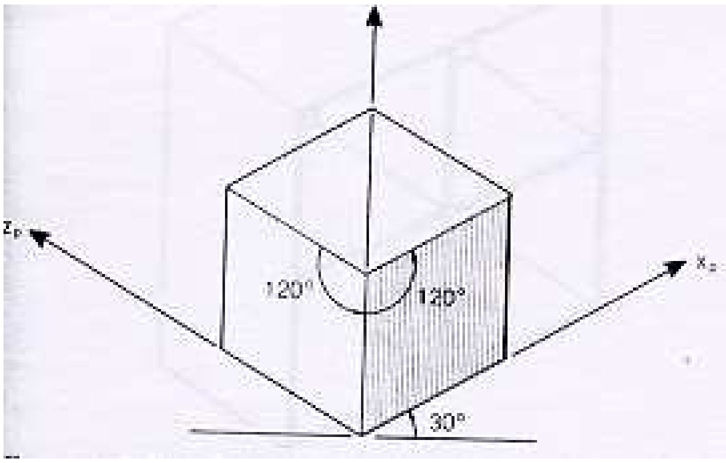
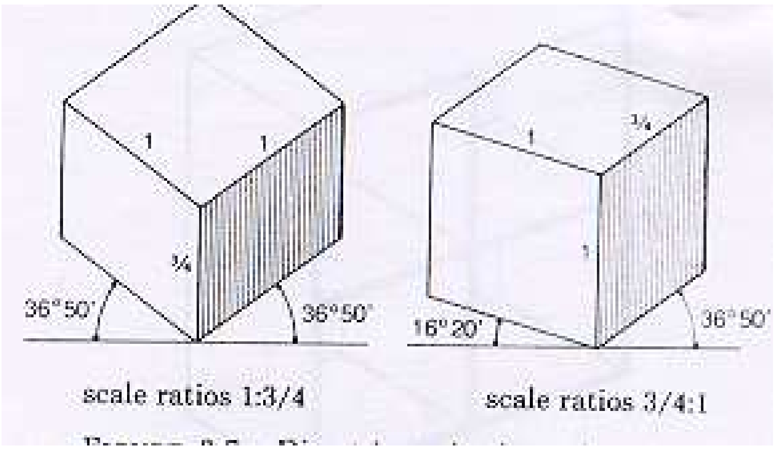
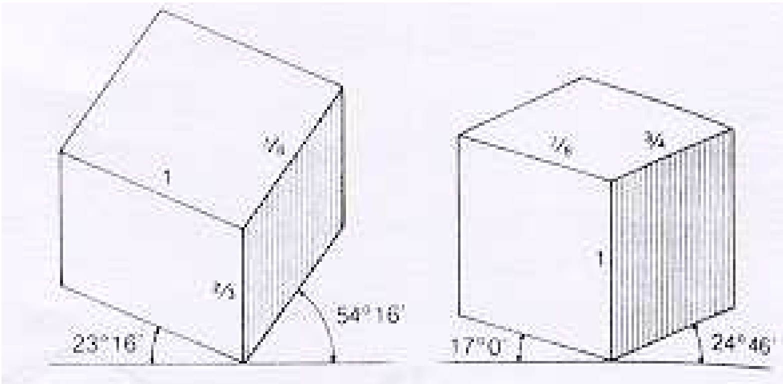


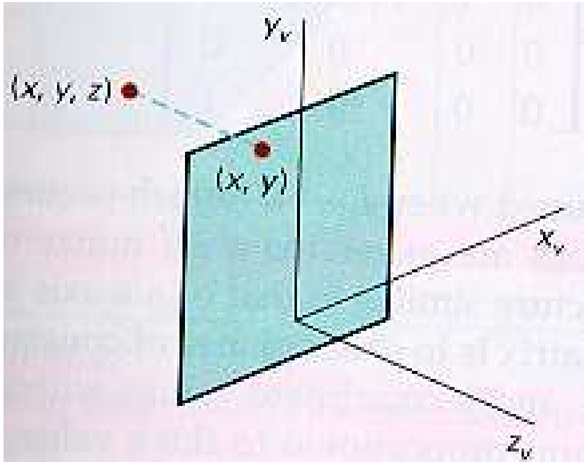
- <u>Dimetric:</u> the projection-plane normal makes equal anges with only two principal axes



- <u>Trimetric:</u> the projection-plane normal makes unequal anges with all principal axes



- **Mathematics of orthographic projection**



$x_p = x$, $y_p = y$, $z_p = 0$ (projection plane at z=0)

- **Using matrix notation to express orthographic transformations**

$$M_{orth} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **Properties of orthographic projection**

  - Parallel lines remain parallel

  - In the case of isometric projection, distances are preserved

  - In general, distances can be measured along each principal axis using different scale factors

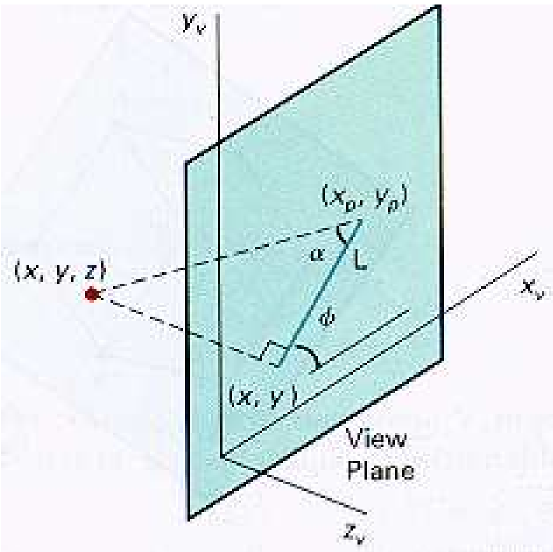
- **Oblique projections**

  - The direction of projection is different from the direction of the projection-plane normal

  - The projection plane is parallel to a principal plane

  - Combine the properties of multiview and axonometric:

- **Mathematics of oblique projections**



$$x_p = x + L\cos\phi,\ y_p = y + L\sin\phi,\ tan(a) = \frac{z}{L}$$

- **Using matrix notation to express oblique transformations**

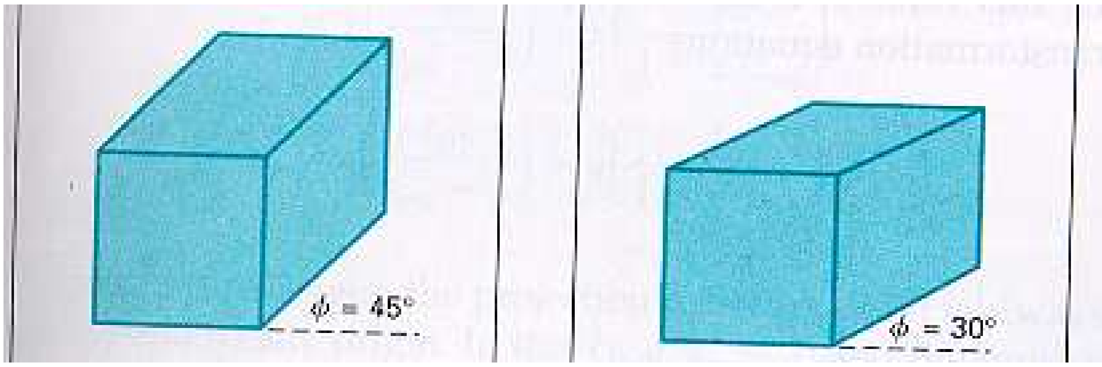
$$M_{par} = \begin{bmatrix} 1 & 0 & 0 & Lcos\phi \\ 0 & 1 & 0 & Lsin\phi \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

* Present the exact face for one face of an object

* Illustrate the 3D structure of the object

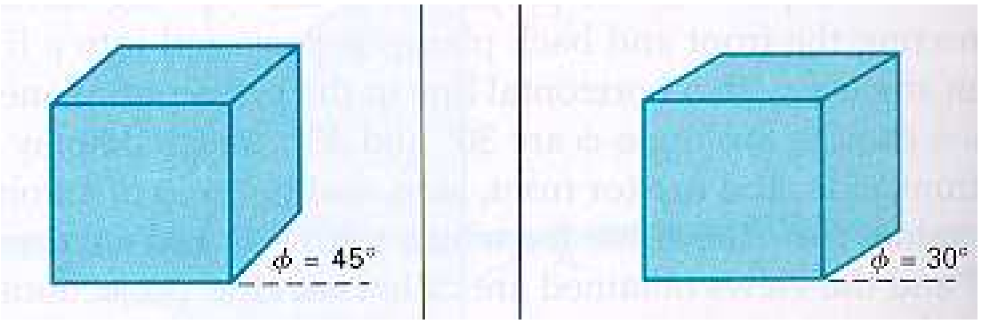
- **Categories of oblique projections**

- Cavalier: $\tan\alpha = 1$ or $\alpha = 45^o$



* Lines perpendicular to the projection plane are projected with no change in length

- Cabinet: $\tan\alpha = 2$ or $\alpha = 63.4^o$



* Lines perpendicular to the projection plane are projected ate one-half of their length