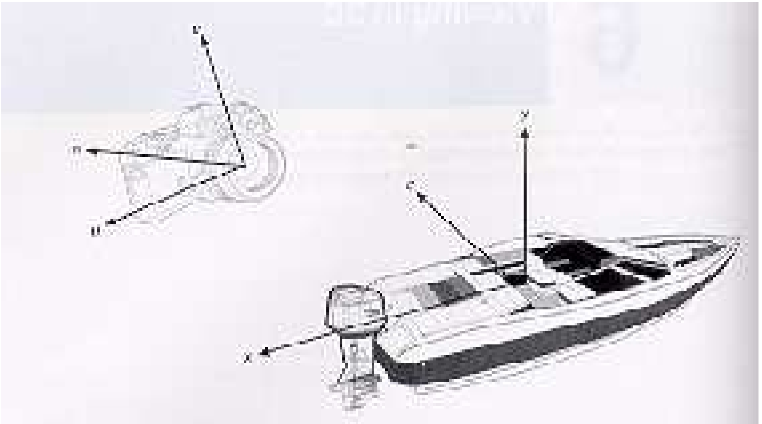
Viewing in 3D

- More complex than 2D viewing process

- 3D objects must be displayed on a 2D view plane (screen)

• **Synthetic camera**



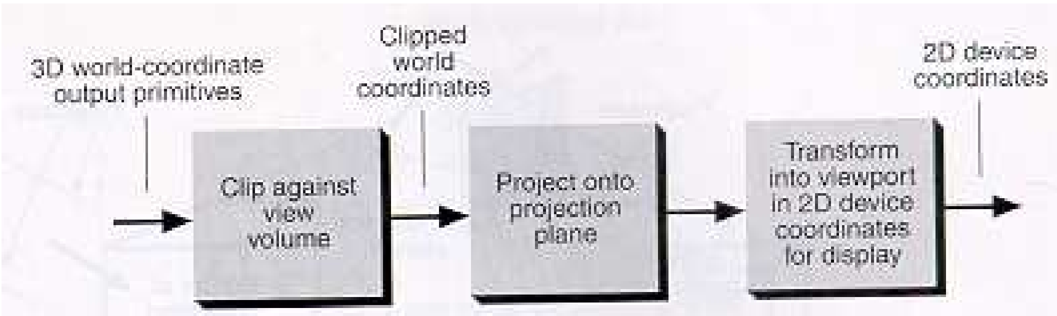
- Position the camera at a particular point in space

- Decide on the camera orientation

- The scene is cropped to the size of the window of the camera

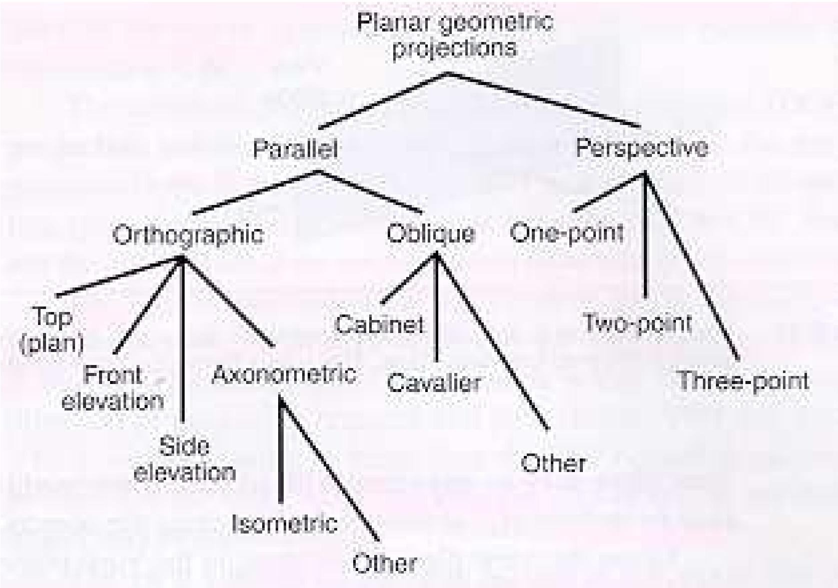
- The light from the visible surfaces is projected onto the camera film

• **Conceptual model of the 3D viewing process**



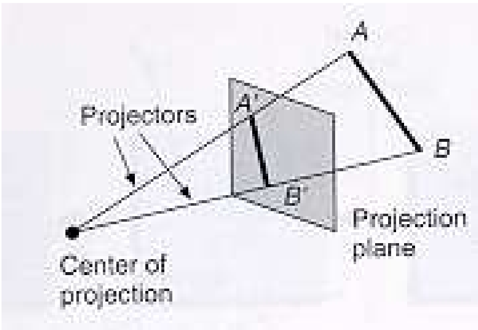
- **Projections**

  - A transformation that maps points in a coordinate system of dimension n into points in a coordinate system of dimension m, where $m < n$

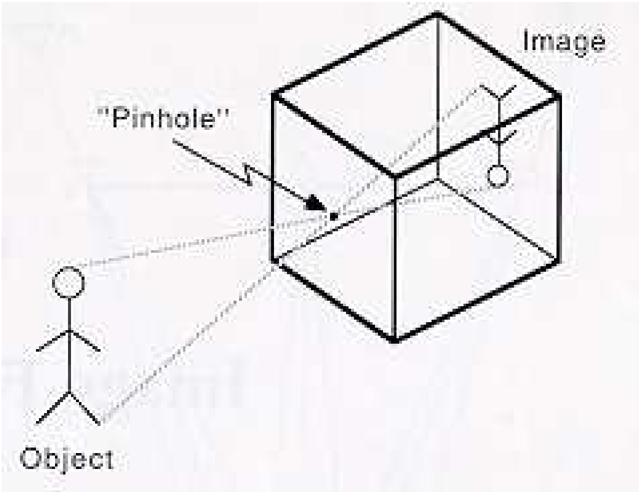


Perspective Projection

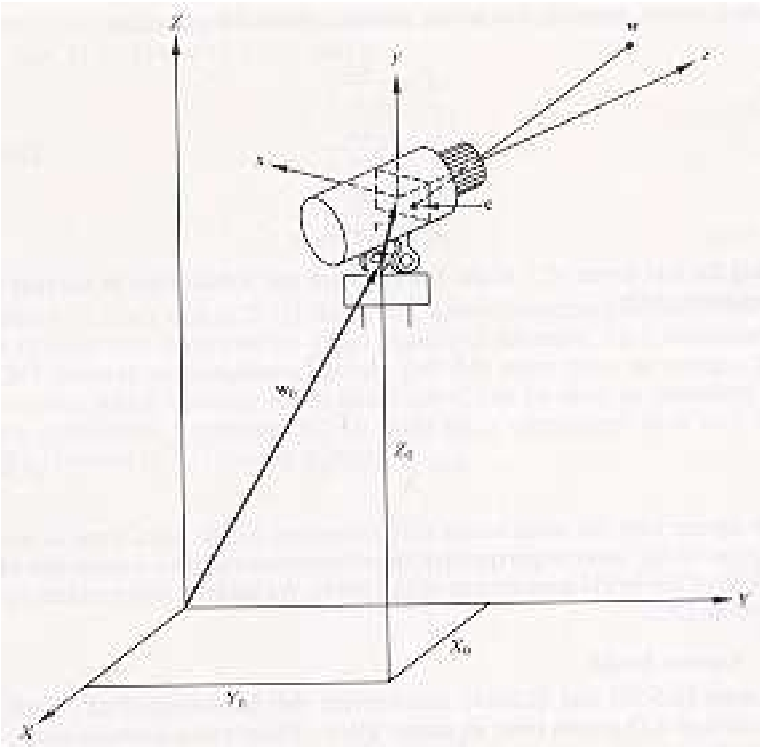
- The projection of a 3D scene onto a plane by straight lines that pass through a single point, called <u>the center of projection</u>



- **Pinhole camera model**



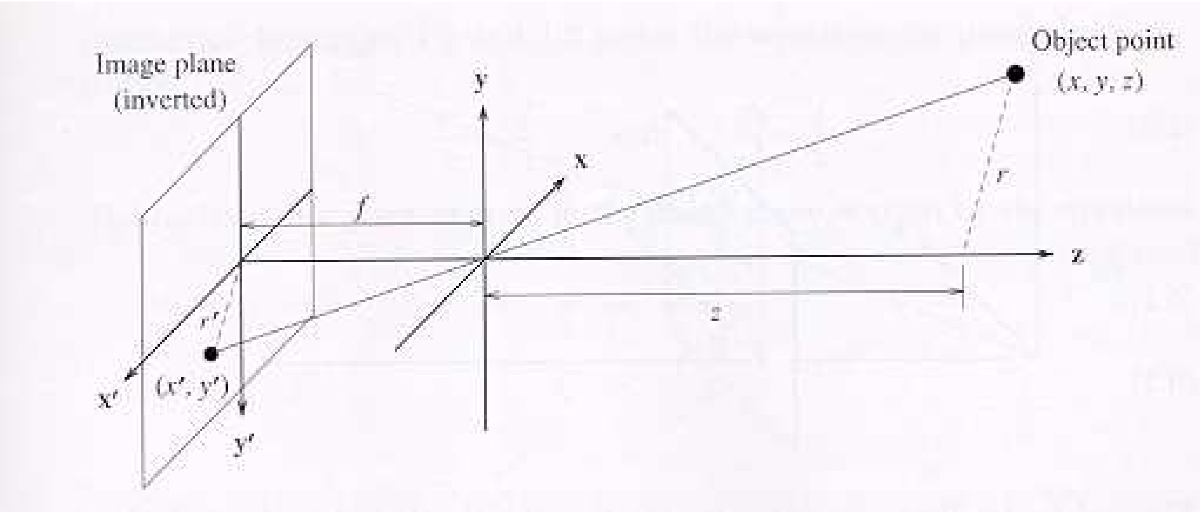
- **Camera's position vs world coordinate system**



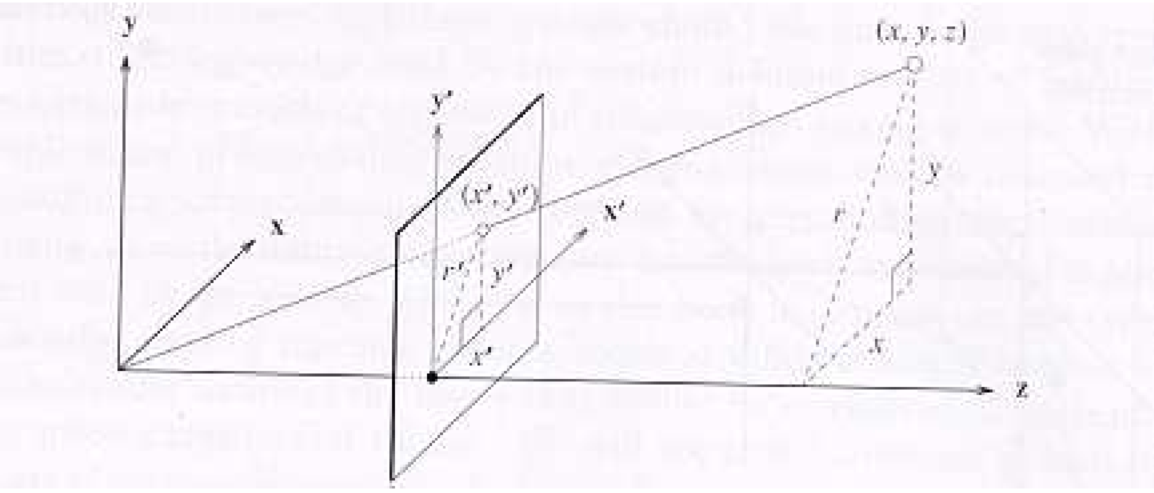
- **Simpler arrangement**

  - Center of projection coincides with the origin of the world coordinate system

  - The camera axes are aligned with the world coordinate system



  - Avoid image inversion by assuming that the image plane is in front of the center of projection



- **Mathematics of perspective projection**

  - From $OA'B'$ and OAB similar triangles: $\dfrac{d}{z} = \dfrac{r_p}{r}$

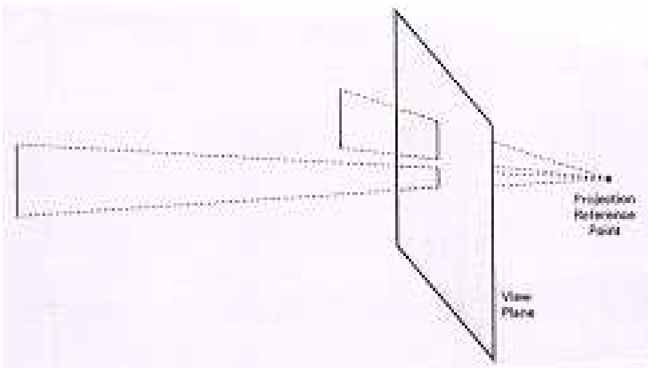
  - From $A'B'C'$ and ABC similar triangles: $\dfrac{x_p}{x} = \dfrac{y_p}{y} = \dfrac{r_p}{r}$

$$\frac{x_p}{x} = \frac{d}{z} \Rightarrow x_p = \frac{xd}{z} \quad (z \neq 0)$$

$$\frac{y_p}{y} = \frac{d}{z} \Rightarrow y_p = \frac{yd}{z} \quad (z \neq 0)$$

- **Perspective foreshortening**

  - Objects far away from the camera appear smaller, while objects close to the camera appear bigger



- **Using matrix notation to express perspective transformations**

$$M_{per} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$$

$$M_{per}\ P = M_{per} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$

$$\text{homogenize: } x_p = \frac{x}{z/d} = \frac{dx}{z}$$

$$\text{homogenize: } y_p = \frac{y}{z/d} = \frac{dy}{z}$$

$$\text{(homogenize: } z_p = \frac{z}{z/d} = d\text{, position of the image plane)}$$

- If image plane is at $z = -d$

$$M'_{per} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/d & 1 \end{bmatrix}$$

$$x_p = \frac{dx}{z+d}$$

$$y_p = \frac{dy}{z+d}$$

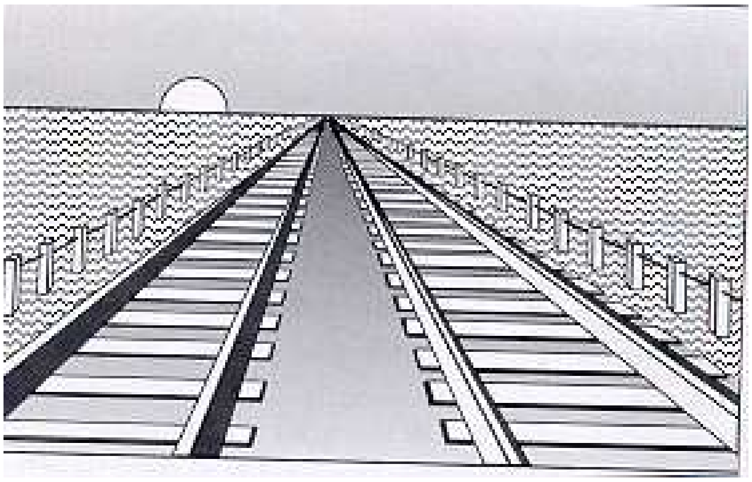
- **Properties of perspective projection**

  - Perspective projections of objects are very realistic

  - Distances and angles are not preserved in general (except for the faces parallel to the projection plane)

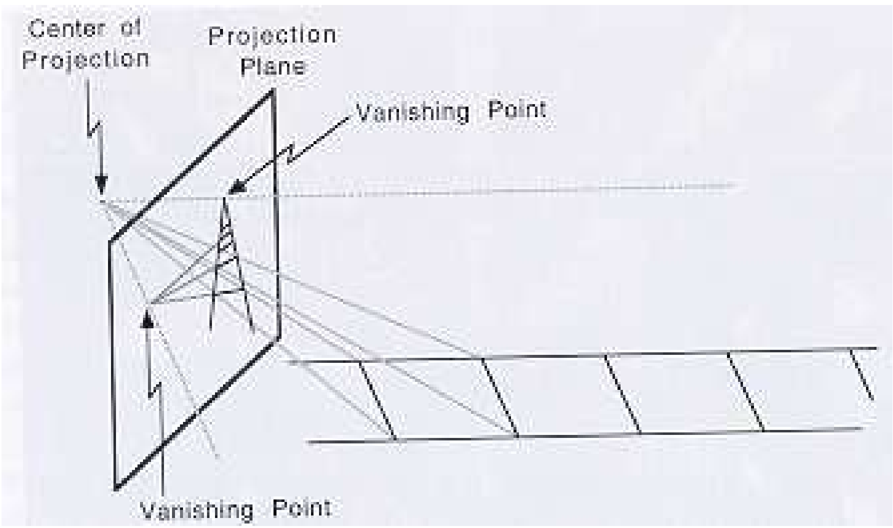
  - Parallel lines do not in general project as parallel lines (except for the faces parallel to the projection plane)

  Parallel lines which are not parallel to the projection plane converge to a single point, called a vanishing point



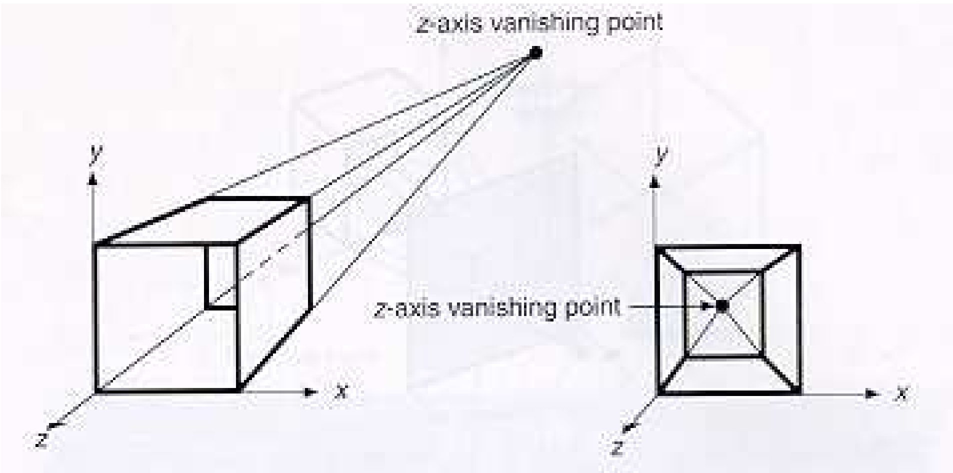
  - The vanishing point of a set of parallel lines is the point where a line, parallel to the set of parallel lines and passing through the center of projection, intersects the projection plane

  - Since parallel lines meet only at infinity, vanishing points are also called points at infinity



- A vanishing point corresponding to a set of lines which are parallel to one of the principal axes is called an axis vanishing point

- If the projection plane is parallel to the *xy* plane, then there is only one axis vanishing point (z-axis vanishing point)



- **Classification of perspective projections**

  - One point perspective: projection plane must be parallel to a principal plane

  - Two point perspective: projection plane must be parallel to a principal axis

  - Three point perspective: projection plane is not parallel to any principal axis