

# CS365 – Final Exam Review

## Spring 2007

### Sequences and Summations

- Sequences/Summations Notation
- Useful summation manipulations
  - Take constants out of the summation
  - Split into two summations
  - Index shifting
  - Order reversal
  - Grouping

$$\sum_x cf(x) = c \sum_x f(x)$$

$$\sum_x f(x) + g(x) = \left( \sum_x f(x) \right) + \sum_x g(x)$$

$$\sum_{i=j}^k f(i) = \sum_{l=j+n}^{k+n} f(l-n)$$

$$\sum_{i=j}^k f(i) = \left( \sum_{i=j}^m f(i) \right) + \sum_{i=m+1}^k f(i) \quad \text{if } j \leq m < k$$

$$\sum_{i=j}^k f(i) = \sum_{l=0}^{k-j} f(k-l)$$

$$\sum_{i=0}^n f(i) = \sum_{i=0}^n f(n-i)$$

$$\sum_{i=0}^{2k} f(i) = \sum_{i=0}^k f(2i) + f(2i+1)$$

- Important series
  - Arithmetic series (know proof)
  - Geometric series (know proof – both finite and infinite)

$$\sum_{k=0}^n ar^k = a(r^{n+1} - 1) / (r - 1), r \neq 1$$

$$\sum_{k=1}^n k = n(n+1) / 2$$

$$\sum_{k=0}^{\infty} x^k = 1/(1-x), |x| < 1$$

$$\sum_{k=1}^{\infty} kx^{k-1} = 1/(1-x)^2, |x| < 1$$

## Algorithms

- Analysis of algorithms (goal/objectives, how, why?)
- Linear Search/Binary search (example)
- Order of growth – VERY IMPORTANT!
  - Understand very well, both intuitively and mathematically.
  - What does it mean that two algorithms have the same rate of growth?
  - Running time of various statements (while-loop, for-loop, if-then-else, block of statements).
- big-O, big-Ω, Θ, small-o, small-ω (properties, relations)
  - Understand their relation VERY WELL!
  - Need to use mathematical definitions in proofs.
- Common orders of magnitude
  - Understand their relation VERY WELL!
- Algorithmic/Problem complexity
  - Know how to analyze the complexity of simple algorithms (e.g., linear search and binary search).
  - What is the complexity of a problem?
- Tractable/Intractable problems (know the definitions)
- P/NP problems (know the definitions)

## Matrices

- Matrix notation
- Matrix properties
  - Equality
  - Sums
  - Products
  - Inverse
  - Transpose
  - Symmetry

## Mathematical Induction (Study VERY WELL!)

- Predicate-logic inference rule

$$\begin{array}{l} P(0) \\ \forall n \geq 0 (P(n) \rightarrow P(n+1)) \\ \hline \therefore \forall n \geq 0 P(n) \end{array}$$

- Why is induction valid? (i.e., proofs)
- Main steps of induction
- Weak vs Strong induction (know both!)
- Do as many examples as you can!

## Combinatorics

- Main rules
  - Sum rule
  - Product rule
  - Combinations of Sum and Product Rules
- Inclusion-Exclusion Principle
- Pigeonhole Principle
- Generalized Pigeonhole Principle
- Permutations (with or without repetitions)
- Combinations (with or without repetitions)
- Permutations (assuming indistinguishable Objects)
- Distributing distinguishable Objects into distinguishable Boxes