CS365 Mathematics of Computer Science
Fall 2006 – Dr. George Bebis
Midterm Exam
1:00-2:15 PM

Name:____

1. **(25 points)** True/False Questions – To get credit, **you must give brief reasons for each answer!**

   T F The statement *"1 + 2=4"* is a valid proposition.

   T F The proposition $q \vee p \vee (p \rightarrow q)$ is a tautology.

   T F $\forall x \, \exists y \, (y^2 = x)$

   T F $\exists x \, (P(x) \wedge Q(x)) \Leftrightarrow (\exists x \, P(x)) \wedge (\exists x \, Q(x))$

   T F For any two sets A, B, we have $A - B = A \cap \overline{B}$

2. **(15 points)** Using symbolic derivations (i.e., not truth tables), show that the proposition below is a tautology.

$$(p \wedge q) \rightarrow (p \vee q)$$

3. **(20 points)** Construct an argument using rules of inference to show that the hypotheses "Linda, a student in this class, owns a red convertible", "Everyone who owns a red convertible has gotten at least on speeding ticket", imply the conclusion "Someone in this class has gotten a speeding ticket". Indicate clearly which inference rule you use at each step of your derivation.

4. **(25 points)** Prove the equality below **without** using Venn diagrams or membership tables.

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

5. **(15 points)** Prove the following statement:

   "The sum of two odd integers is even"