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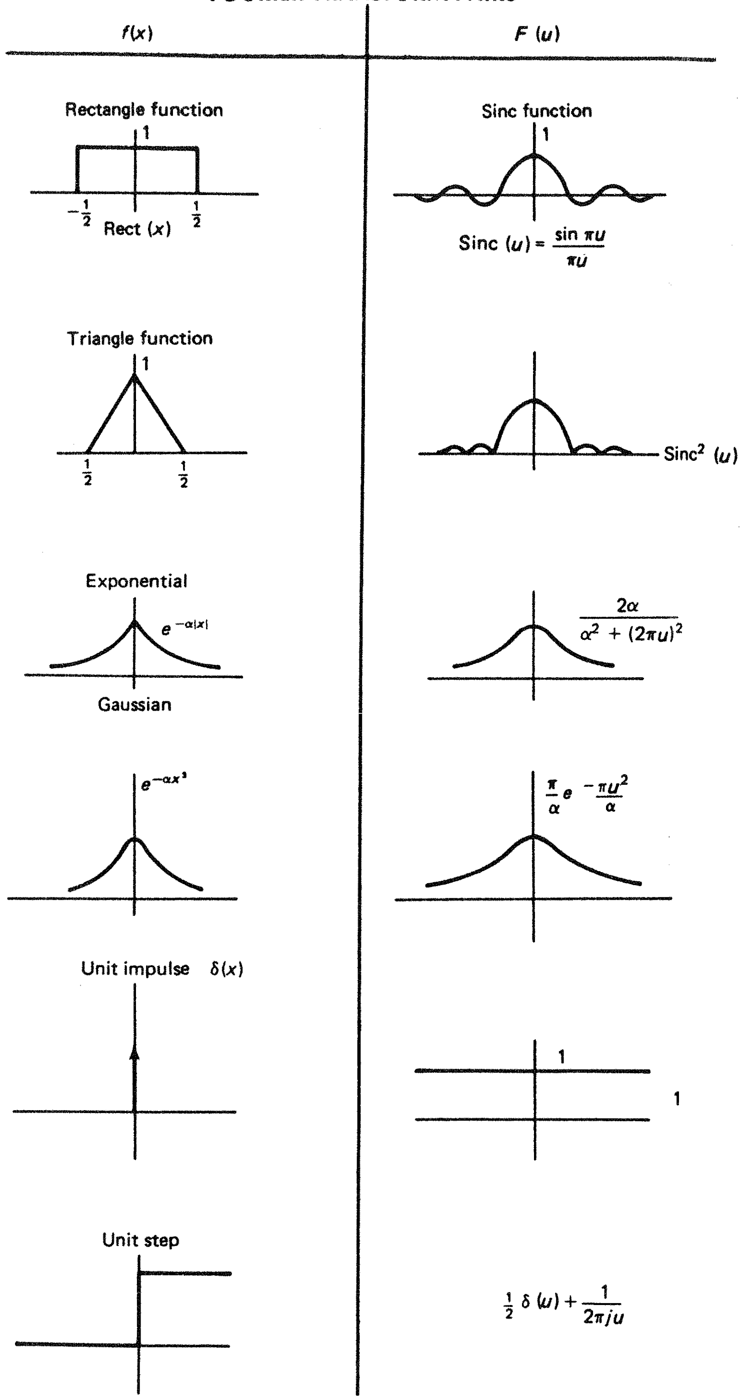
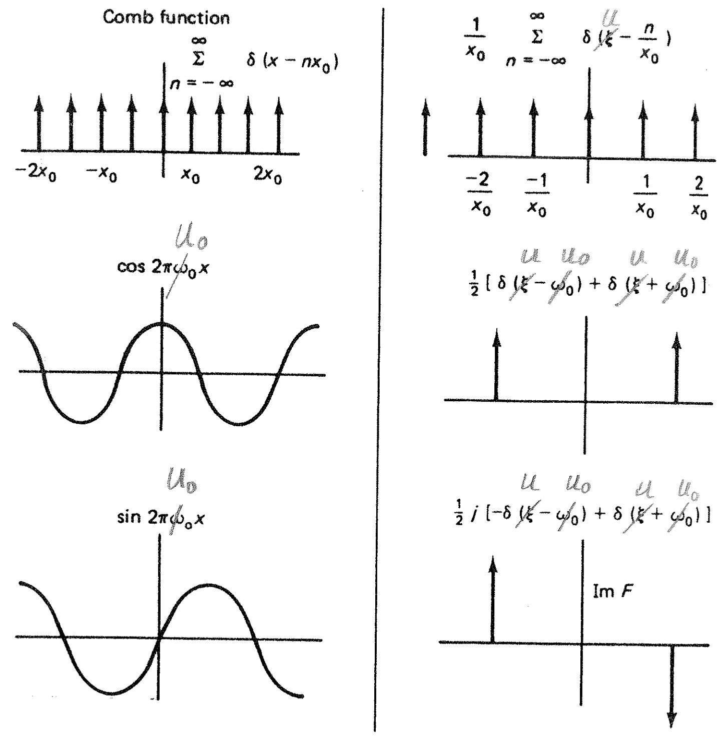
FOURIER TRANSFORM PAIRS





PROPERTIES OF THE FOURIER TRANSFORM

	Spatial Domain	Frequency Domain
	$f(x)$ $g(x)$	$F(u) = \mathfrak{F}[f(x)]$ $G(u) = \mathfrak{F}[g(x)]$
(1)	Linearity $c_1 f(x) + c_2 g(x)$ c_1, c_2 scalars	$c_1 F(u) + c_2 G(u)$
(2)	Scaling $f(ax)$	$\frac{1}{\lvert a\rvert} F\left(\frac{u}{a}\right)$
(3)	Shifting $f(x - x_0)$	$e^{-j2\pi x_0 u} F(u)$
(4)	Symmetry $F(x)$	$f(-u)$
(5)	Conjugation $f^*(x)$	$F^*(-u)$
(6)	Convolution $h(x) = f * g = \int_{-\infty}^{\infty} f(x') g(x - x')\, dx'$	$F(u) G(u)$
(7)	Differentiation $\frac{d^n f(x)}{dx^n}$	$(2\pi ju)^n F(u)$

Parseval's theorem:

$$\int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |F(\xi)|^2 d\xi$$

$$\int_{-\infty}^{\infty} f(x) g^*(x)\, dx = \int_{-\infty}^{\infty} F(\xi) G^*(\xi)\, d\xi$$

APPENDIX

A Selected Mathematical Tables

A.1 TRIGONOMETRIC IDENTITIES

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin A \sin B = \tfrac{1}{2}[\cos(A - B) - \cos(A + B)]$$

$$\cos A \cos B = \tfrac{1}{2}[\cos(A + B) + \cos(A - B)]$$

$$\sin A \cos B = \tfrac{1}{2}[\sin(A + B) + \sin(A - B)]$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A - \sin^2 A$$

$$\sin^2 A = \tfrac{1}{2}(1 - \cos 2A)$$

$$\cos^2 A = \tfrac{1}{2}(1 + \cos 2A)$$

$$\sin A = \frac{1}{2j}(e^{jA} - e^{-jA})$$

$$\cos A = \tfrac{1}{2}(e^{jA} + e^{-jA})$$

$$e^{\pm jA} = \cos A \pm j \sin A$$

A.2 COMPLEX FUNCTION IDENTITIES

$$z = x + jy = \sqrt{x^2 + y^2}\, e^{j \tan^{-1}(y/x)}$$

$$z^* = x - jy = \sqrt{x^2 + y^2}\, e^{-j \tan^{-1}(y/x)}$$

$$|z|^2 = zz^* = x^2 + y^2$$

$$\mathcal{R}e\{z\} = \tfrac{1}{2}[z + z^*]$$

$$\mathcal{I}m\{z\} = \frac{1}{2j}[z - z^*]$$

$$\mathcal{R}e\{z_1 z_2\} = \mathcal{R}e\{z_1\}\mathcal{R}e\{z_2\} - \mathcal{I}m\{z_1\}\mathcal{I}m\{z_2\}$$

$$\mathcal{I}m\{z_1 z_2\} = \mathcal{R}e\{z_1\}\mathcal{I}m\{z_2\} + \mathcal{I}m\{z_1\}\mathcal{R}e\{z_2\}$$

A.3 SERIES

Exponential:

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \cdots.$$

Trigonometric:

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \cdots.$$

$$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \cdots.$$

Binomial (for $x^2 < 1$):

$$(1 \pm x)^n = 1 \pm nx + \frac{1}{2!}n(n-1)x^2 \pm \frac{1}{3!}n(n-1)(n-2)x^3 + \cdots.$$

$$(1 \pm x)^{-n} = 1 \mp nx + \frac{1}{2!}n(n+1)x^2 \mp \frac{1}{3!}n(n+1)(n+2)x^3 + \cdots.$$

A.4 SUMMATIONS

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

A.5 INDEFINITE INTEGRALS

$$\int \sin ax\, dx = -\frac{1}{a}\cos ax$$

$$\int \cos ax\, dx = \frac{1}{a}\sin ax$$

$$\int \sin^2 ax\, dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

$$\int \cos^2 ax\, dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\int \sin ax \cos ax\, dx = \frac{1}{2a} \sin^2(ax)$$

$$\int x \sin ax\, dx = \frac{1}{a^2}(\sin ax - ax \cos ax)$$

$$\int x \cos ax\, dx = \frac{1}{a^2}(\cos ax + ax \sin ax)$$

$$\int x^2 \sin ax\, dx = \frac{1}{a^3}(2ax \sin ax + 2 \cos ax - a^2x^2 \cos ax)$$

$$\int x^2 \cos ax\, dx = \frac{1}{a^3}(2ax \cos ax - 2 \sin ax + a^2x^2 \sin ax)$$

$$\int \sin ax \sin bx\, dx = \frac{\sin (a-b)x}{2(a-b)} - \frac{\sin (a+b)x}{2(a+b)} \qquad a^2 \neq b^2$$

$$\int \cos ax \cos bx\, dx = \frac{\sin (a-b)x}{2(a-b)} + \frac{\sin (a+b)x}{2(a+b)} \qquad a^2 \neq b^2$$

$$\int \sin ax \cos bx\, dx = -\frac{\cos (a-b)x}{2(a-b)} - \frac{\cos (a+b)x}{2(a+b)} \qquad a^2 \neq b^2$$

$$\int e^{ax}\, dx = \frac{1}{a} e^{ax}$$

$$\int xe^{ax}\, dx = \frac{1}{a^2} e^{ax}(ax - 1)$$

$$\int x^2e^{ax}\, dx = \frac{1}{a^3} e^{ax}(a^2x^2 - 2ax + 2)$$

$$\int x^ne^{ax}\, dx = \frac{x^n}{a} e^{ax} - \frac{n}{a} \int x^{n-1}e^{ax}\, dx$$

$$\int e^{ax} \sin bx\, dx = \frac{1}{a^2 + b^2} e^{ax}(a \sin bx - b \cos bx)$$

$$\int e^{ax} \cos bx\, dx = \frac{1}{a^2 + b^2} e^{ax}(a \cos bx + b \sin bx)$$

$$\int \left[\frac{\sin ax}{x}\right]^2 dx = a \int \frac{\sin 2ax}{x} dx - \frac{\sin^2 ax}{x}$$

$$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab} \tan^{-1}\left(\frac{bx}{a}\right)$$

$$\int \frac{x^2\, dx}{a^2 + b^2x^2} = \frac{x}{b^2} - \frac{a}{b^3} \tan^{-1}\left(\frac{bx}{a}\right)$$

$$\int \frac{dx}{(a^2 + b^2x^2)^2} = \frac{x}{2a^2(a^2 + b^2x^2)} + \frac{1}{2a^3b}\tan^{-1}\left(\frac{bx}{a}\right)$$

$$\int \frac{x^2\,dx}{(a^2 + b^2x^2)^2} = \frac{-x}{2b^2(a^2 + b^2x^2)} + \frac{1}{2ab^3}\tan^{-1}\left(\frac{bx}{a}\right)$$

$$\int \frac{dx}{(a^2 + b^2x^2)^3} = \frac{x}{4a^2(a^2 + b^2x^2)^2} + \frac{3x}{8a^4(a^2 + b^2x^2)} + \frac{3}{8a^5b}\tan^{-1}\left(\frac{bx}{a}\right)$$

A.6 DEFINITE INTEGRALS

$$\int_0^\infty \frac{\sin ax}{x}\,dx = \begin{cases} \pi/2 & a > 0 \\ 0 & a = 0 \\ -\pi/2 & a < 0 \end{cases}$$

$$\int_0^x \frac{\sin u}{u}\,du \triangleq \text{Si}\,(x) \quad \text{(a tabulated integral as a function of } x\text{)}$$

$$\int_0^\infty \frac{\sin^2 ax}{x^2}\,dx = |a|\pi/2$$

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\pi/a}$$

$$\int_0^\infty xe^{-ax^2}\,dx = \frac{1}{2a}$$

$$\int_0^\infty x^2e^{-ax^2}\,dx = \frac{1}{4a}\sqrt{\pi/a}$$

$$\int_0^\infty \frac{dx}{(x^2 + a^2)(x^2 + b^2)} = \frac{\pi}{2ab(a + b)} \qquad a > 0, b > 0$$

$$\int_0^\infty \frac{dx}{ax^4 + b} = \frac{\pi}{2\sqrt{2b}}\left(\frac{b}{a}\right)^{1/4} \qquad ab > 0$$

$$\int_0^\infty \frac{dx}{ax^6 + b} = \frac{\pi}{3b}\left(\frac{b}{a}\right)^{1/6} \qquad ab > 0$$

$$\int_0^\infty \frac{dx}{1 + x^{2n}} = \frac{[\pi/(2n)]}{\sin\,[\pi/(2n)]} \qquad n \text{ an integer, } n > 0$$