1. (25 points) True/False Questions – To get credit, you must give brief reasons for each answer!

T    F   The statement "1 + 2=4" is a valid proposition.

T    F   The proposition $q \lor p \lor (p \rightarrow q)$ is a tautology.

T    F   $\forall x \exists y (y^2=x)$

T    F   $\exists x (P(x) \land Q(x)) \leftrightarrow (\exists x P(x)) \land (\exists x Q(x))$

T    F   For any two sets A, B, we have $A - B = A \cap \overline{B}$
2. **(15 points)** Using symbolic derivations (i.e., not truth tables), show that the proposition below is a tautology.

\[(p \land q) \rightarrow (p \lor q)\]

3. **(20 points)** Construct an argument using rules of inference to show that the hypotheses "Linda, a student in this class, owns a red convertible", "Everyone who owns a red convertible has gotten at least one speeding ticket", imply the conclusion "Someone in this class has gotten a speeding ticket". Indicate clearly which inference rule you use at each step of your derivation.
4. (25 points) Prove the equality below without using Venn diagrams or membership tables.

\[ A \cup B = \overline{A} \cap \overline{B} \]

5. (15 points) Prove the following statement:

"The sum of two odd integers is even"