1. Image \( f(x,y) \) with \( f_{\text{min}} = 6, \ f_{\text{max}} = 18 \)
   Image \( g(x,y) \) with \( g_{\text{min}} = 10, \ g_{\text{max}} = 50 \)

   \[ T(m) = ma + b = n \] where \( T(m) \): a linear transformation function
   \( m \): a gray level in image \( f(x,y) \)
   \( n \): the corresponding gray level in image \( g(x,y) \)

   Solving the equation for \( m \): \( f_{\text{min}} = 6 \) and \( n = g_{\text{min}} = 10 \)
   \[ 6a + b = 10 \quad b = 10 - 4a \]

   Solving the equation for \( m \): \( f_{\text{max}} = 18 \) and \( n = g_{\text{max}} = 50 \)
   \[ 18a + b = 50 \quad b = 50 - 18a \]

   Then,
   \[ 50 - 18a = 10 - 4a \quad 14a = 40 \quad a = \frac{20}{7} \quad b = 10 - \frac{80}{7} = -\frac{10}{7} \]

   Transformation function is:
   \[ T(m) = \frac{20}{7}m - \frac{10}{7} \]

2. Let's first draw the histogram of the original image

\[ p(0) = \frac{2}{18}, \quad p(1) = \frac{9}{18}, \quad p(2) = \frac{3}{18}, \quad p(3) = 0, \quad p(4) = \frac{2}{18}, \quad p(5) = 0, \quad p(6) = \frac{1}{18} \]
The transformation to be used is:

\[ s_k = T(r_k) = (L-1) \sum_{j=0}^{k} p(r_j) \]

where \( L = 8 \) (eight possible grey level)

Applying the transformation, we get:

- \( s_0 = 0 \cdot p(0) = 0 \cdot \frac{1}{18} = 0 \)
- \( s_1 = 0 \cdot (p(0) + p(1)) = 0 \cdot \frac{3}{18} = 0 \)
- \( s_2 = 0 \cdot \sum_{j=0}^{1} p(r_j) = 0 \cdot \frac{3}{18} = 0 \)
- \( s_3 = 0 \cdot \sum_{j=0}^{2} p(r_j) = 0 \cdot \frac{9}{18} = 0 \)
- \( s_4 = 0 \cdot \sum_{j=0}^{3} p(r_j) = 0 \cdot \frac{15}{18} = 0 \)
- \( s_5 = 0 \cdot \sum_{j=0}^{4} p(r_j) = 0 \cdot \frac{21}{18} = 0 \)
- \( s_6 = 0 \cdot \sum_{j=0}^{5} p(r_j) = 0 \cdot \frac{27}{18} = 0 \)
- \( s_7 = 0 \cdot \sum_{j=0}^{6} p(r_j) = 0 \cdot \frac{33}{18} = 0 \)
- \( s_8 = 0 \cdot \sum_{j=0}^{7} p(r_j) = 0 \cdot \frac{39}{18} = 0 \)

Finally, calculating the distribution values:

- \( p(0) = 0 \)
- \( p(1) = \frac{1}{18} \)
- \( p(2) = 0 \)
- \( p(3) = 0 \)
- \( p(4) = \frac{8}{18} \)
- \( p(5) = \frac{14}{18} \)
- \( p(6) = \frac{20}{18} \)
- \( p(7) = \frac{26}{18} \)
- \( p(8) = \frac{32}{18} \)

The histogram of the equalized image:

\[ p(c) = r_5 = \frac{1}{18} \]
\[ p(c) = r_6 + r_7 = \frac{2}{18} \]

The equalized image is given below:

\[
\begin{array}{cccccc}
4 & 5 & 4 & 4 & 5 & 1 \\
1 & 4 & 6 & 4 & 1 & 4 \\
4 & 4 & 7 & 4 & 5 & 5
\end{array}
\]
3. Problem 3.11 (page 194)

First, we obtain the histogram equalization transformation:

\[ s = T(r) = \int_0^r p_r(w) \, dw = \int_0^r (-2w + 2) \, dw = -r^2 + 2r. \]

Next we find

\[ \nu = G(z) = \int_0^z p_z(w) \, dw = \int_0^z 2w \, dw = z^2. \]

Finally,

\[ z = G^{-1}(\nu) = \pm \sqrt{\nu}. \]

But only positive intensity levels are allowed, so \( z = \sqrt{\nu} \). Then, we replace \( \nu \) with \( s \), which in turn is \(-r^2 + 2r\), and we have

\[ z = \sqrt{-r^2 + 2r}. \]

5. Problem 3.7 (page 194)

Let \( n = MN \) be the total number of pixels and let \( n_{r_j} \) be the number of pixels in the input image with intensity value \( r_j \). Then, the histogram equalization transformation is

\[ s_k = T(r_k) = \sum_{j=0}^k n_{r_j}/n = \frac{1}{n} \sum_{j=0}^k n_{r_j}. \]

Because every pixel (and no others) with value \( r_k \) is mapped to value \( s_k \), it follows that \( n_{s_k} = n_{r_k} \). A second pass of histogram equalization would produce values \( \nu_k \) according to the transformation

\[ \nu_k = T(s_k) = \frac{1}{n} \sum_{j=0}^k n_{s_j}, \]

But, \( n_{s_j} = n_{r_j} \), so

\[ \nu_k = T(s_k) = \frac{1}{n} \sum_{j=0}^k n_{r_j} = s_k \]

which shows that a second pass of histogram equalization would yield the same result as the first pass. We have assumed negligible round-off errors.
(i) Zero-order interpolation is the nearest neighbour interpolation.

\[ I_a(1.5, 1.8) = 7 \]

(ii) First-order interpolation using average

\[ I_1(1.5, 1.8) = \frac{1 + 5 + 7 + 15}{4} = 7 \]

(iii) First-order interpolation using a bilinear function:

\[ I_1(x, y) = ax + by + cy + d \]

To determine the constants, we put the nearest neighbours into the equation.

(a) \[ I_1(1.1) = a + b + c + d = 1 \]
(b) \[ I_1(2.2) = 2a + b + 2c + d = 5 \]
(c) \[ I_1(1.2) = a + 2b + 2c + d = 7 \]
(d) \[ I_1(2.2) = 2a + 2b + 4c + d = 15 \]

(e) \[ a + c = 4 \] (from a & b)
(f) \[ a + 2c = 8 \] (from a & d)
(g) \[ c = 4, a = 0 \] (from c & f)
(h) \[ b + d = 3 \] (from a & g)
(i) \[ 2b + d = 1 \] (from c & g)
(j) \[ b = 2, d = 3 \] (from h & i)

\[ I_1(x, y) = 2y + 4xy - 5 \]

\[ I_1(1.5, 1.8) = 3.6 + 3.36 - 5 = 7.36 \approx 8 \]