CS 474/674 – Image Processing & Int.

Fall 2009 – Hw3

– Solutions –

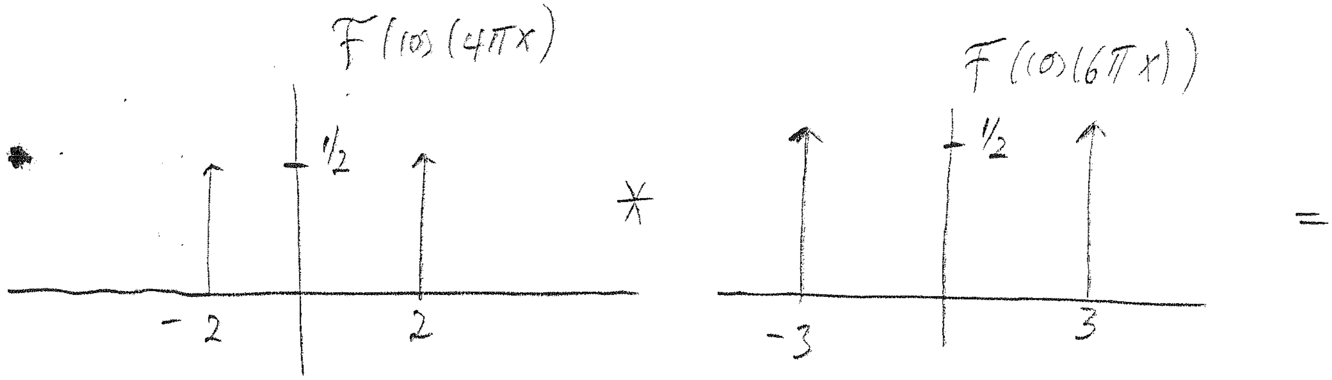
1)

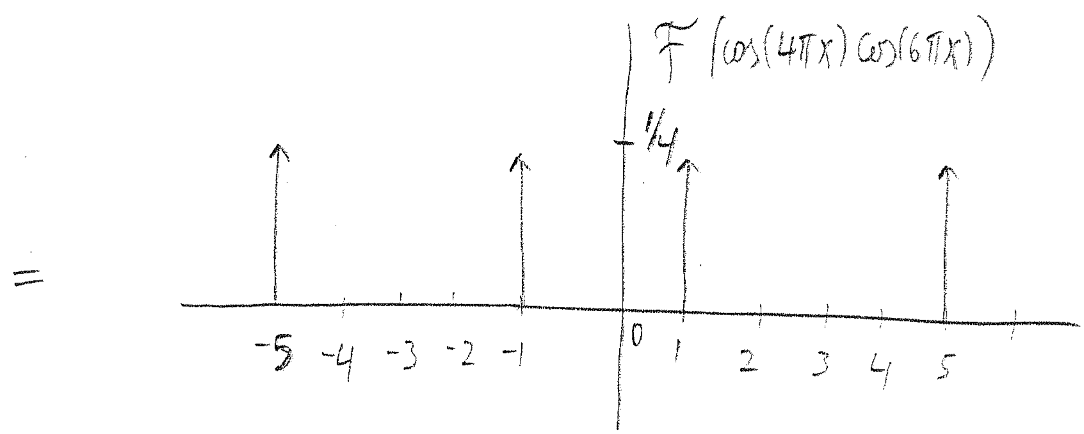
(a) from convolution theorem:

$$\mathcal{F}(\cos(4\pi x)\cos(6\pi x)) = \mathcal{F}(\cos(4\pi x)) * \mathcal{F}(\cos(6\pi x)).$$

$$\cos(4\pi x) = \cos(2\pi . 2 . x) = \cos(2\pi u_0 x) \rightarrow u_0 = 2.$$

$$\cos(6\pi x) = \cos(2\pi . 3 . x) = \cos(2\pi u_0 x) \Rightarrow u_0 = 3$$





(taking $\mathcal{F}^{-1}()$ will give us the result in the time domain)

– you don't need to compute $\mathcal{F}^{-1}()$ –

(b) From Nyquist theorem, we know that $\Delta x \leq \frac{1}{2W}$. In our case, $W=5$, so

$\Delta x \leq \frac{1}{2 \cdot 5} = 1/10$

3)

Problem 4.21

Recall that the reason for padding is to establish a "buffer" between the periods that are implicit in the DFT. Imagine the image on the left being duplicated infinitely many times to cover the xy-plane. The result would be a checkerboard, with each square being in the checkerboard being the image (and the black extensions). Now imagine doing the same thing to the image on the right. The results would be identical. Thus, either form of padding accomplishes the same separation between images, as desired.

4)

Problem 4.33

The complex conjugate simply changes j to $-j$ in the inverse transform, so the image on the right is given by

$$\begin{aligned}
\Im^{-1}[F^*(u,v)] &= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} F(u,v)e^{-j2\pi(ux/M+vy/N)} \\
&= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} F(u,v)e^{j2\pi(u(-x)/M+v(-y)/N)} \\
&= f(-x,-y)
\end{aligned}$$

which simply mirrors $f(x,y)$ about the origin, thus producing the image on the right.

5) Grad Student Only

(a) The averages of the two images are computed as follows:

$$\bar{f}(x,y) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)$$

and

$$
\begin{aligned}
\bar{f}_p(x,y) &= \frac{1}{PQ}\sum_{x=0}^{P-1}\sum_{y=0}^{Q-1} f_p(x,y) \\
&= \frac{1}{PQ}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y) \\
&= \frac{MN}{PQ}\bar{f}(x,y)
\end{aligned}
$$

where the second step is result of the fact that the image is padded with 0s. Thus, the ratio of the average values is

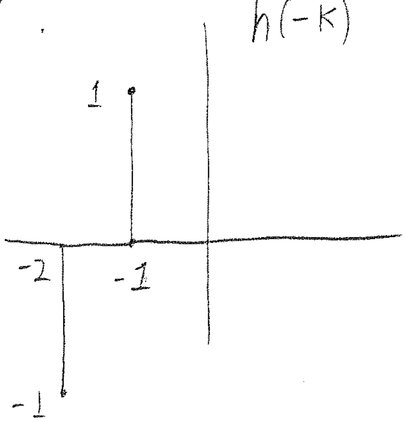
$$r = \frac{PQ}{MN}$$

Thus, we see that the ratio increases as a function of PQ, indicating that the average value of the padded image decreases as a function of PQ. This is as expected; padding an image with zeros decreases its average value.

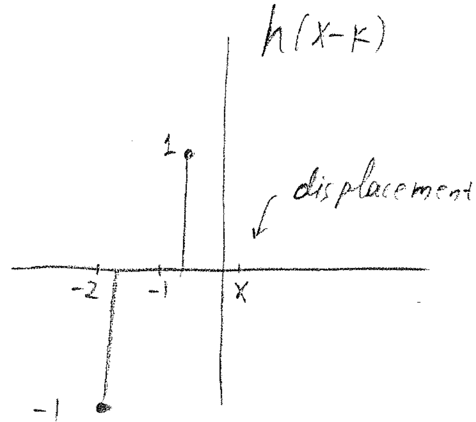
(b) Yes, they are equal. We know that $F(0,0) = MN\bar{f}(x,y)$ and $F_p(0,0) = PQ\bar{f}_p(x,y)$. And, from part (a), $\bar{f}_p(x,y) = MN\bar{f}(x,y)/PQ$. Then,

$$
\begin{aligned}
\frac{F_p(0,0)}{PQ} &= \frac{MN}{PQ}\frac{F(0,0)}{MN} \\
F_p(0,0) &= F(0,0).
\end{aligned}
$$

2)



$\Longrightarrow$



$$y(n) = x(n) * h(n) = \sum_{k=0}^{3} x(k)\, h(n-k)$$

$$n=0,\quad y(0) = \sum_{k=0}^{3} x(k)\, h(-k) = x(0)h(0) + x(1)h(-1) + x(2)h(-2) + x(3)h(-3) = 0$$

$$n=1,\quad y(1) = \sum_{k=0}^{3} x(k)\, h(1-k) = x(0)h(1) + x(1)h(0) + x(2)h(-1) + x(3)h(-2) = 1$$

$$n=2,\quad y(2) = \sum_{k=0}^{3} x(k)\, h(2-k) = x(0)h(2) + x(1)h(1) + x(2)h(0) + x(3)h(-1) = 0$$

$$n=3,\quad y(3) = \sum_{k=0}^{3} x(k)\, h(3-k) = x(0)h(3) + x(1)h(2) + x(2)h(1) + x(3)h(0) = 0$$

$$n=4,\quad y(4) = \sum_{k=0}^{3} x(k)\, h(4-k) = x(0)h(4) + x(1)h(3) + x(2)h(2) + x(3)h(1) = 0$$

$$n=5,\quad y(5) = \sum_{k=0}^{3} x(k)\, h(5-k) = x(0)h(5) + x(1)h(4) + x(2)h(3) + x(3)h(2) = -1$$