CS474/674 Image Processing and Interpretation

Fall 2009 – Dr. George Bebis

Homework 5 - Solutions

1. Problem 5.11 (page 390)

The key to understanding the behavior of the contra-harmonic filter is to think of the pixels in the neighborhood surrounding a noise impulse as being constant, with the impulse noise point being in the center of the neighborhood. For the noise spike to be visible, its value must be considerably larger than the value of its neighbors. Also keep in mind that the power in the numerator is 1 plus the power in the denominator.

(**a**) By definition, pepper noise is a low value (really 0). It is most visible when surrounded by light values. The center pixel (the pepper noise), will have little influence in the sums. If the area spanned by the filter is approximately constant, the ratio will approach the value of the pixels in the neighborhood—thus reducing the effect of the low-value pixel. For example, here are some values of the filter for a dark point of value 1 in a 3×3 region with pixels of value 100: For $Q = 0.5$, filter = 98.78; for $Q = 1$, filter = 99.88, for $Q = 2$, filter = 99.99; and for $Q = 5$, filter = 100.00.

(**b**) The reverse happens when the center point is large and its neighbors are small. The center pixel will now be the largest. However, the exponent is now negative, so the small numbers will dominate the result. The numerator can then be thought of a constant raised to the power $Q + 1$ and the denominator as a the same constant raised to the power Q. That constant is the value of the pixels in the neighborhood. So the ratio is just that value.

(**c**) When the wrong polarity is used, the large numbers in the case of the salt noise will be raised to a positive power, thus the noise will overpower the result. For salt noise the image will become very light. The opposite is true for pepper noise—the image will become dark.

(**d**) When $Q = -1$, the value of the numerator at any location is equal to the number of pixels in the neighborhood (mn). The terms of the sum in the denominator are 1 divided by individual pixel values in the neighborhood. For example, for a 3×3 enighborhoood, the response of the filter when $Q = -1$ is: $9/[1/p1 + 1/p2 + \cdots 1/p9]$ where the p's are the pixel values in the neighborhood. Thus, low pixel values will tend to produce low filter responses, and vice versa. If, for example, the filter is centered on a large spike surrounded by zeros, the response will be a low output, thus reducing the effect of the spike.

(**e**) In a constant area, the filter returns the value of the pixels in the area, independently of the value of Q.

2. Problem 5.17 (page 391)

Following the image coordinate convention in the book, vertical motion is in the x-direction and horizontal motion is in the y-direction. Then, the components of motion are as follows:

$$x_0(t) = \begin{cases} \frac{at}{T_1} & 0 \leq t \leq T_1 \\ a & T_1 < t \leq T_1 + T_2 \end{cases}$$

and

$$y_0(t) = \begin{cases} 0 & 0 \leq t \leq T_1 \\ \frac{b(t-T_1)}{T_1} & T_1 < t \leq T_1 + T_2. \end{cases}$$

Then, substituting these components of motion into Eq. (5.6-8) yields

$$\begin{aligned} H(u,v) &= \int_0^{T_1} e^{-j2\pi[uat/T_1]} dt + \int_{T_1}^{T_1+T_2} e^{-j2\pi[ua+vb(t-T_1)/T_2)} dt \\ &= \frac{T_1}{\pi ua} \sin(\pi ua) e^{-j\pi ua} + e^{-j2\pi ua} \int_{T_1}^{T_1+T_2} e^{-j2\pi vb(t-T_1)/T_2} dt \\ &= \frac{T_1}{\pi ua} \sin(\pi ua) e^{-j\pi ua} + e^{-j2\pi ua} \int_0^{T_2} e^{-j2\pi vb\tau/T_2} d\tau \\ &= \frac{T_1}{\pi ua} \sin(\pi ua) e^{-j\pi ua} + e^{-j2\pi ua} \frac{T_2}{\pi vb} \sin(\pi vb) e^{-j\pi vb} \end{aligned}$$

where in the third line we made the change of variables $\tau = t - T_1$. The blurred image is then

$$g(x,y) = \Im^{-1}\left[H(u,v)F(u,v)\right]$$

where $F(u,v)$ is the Fourier transform of the input image.

3. **(Graduate Students Only)** Problem 5.26 (page 392)

One possible solution: (1) Perform image averaging to reduce noise. (2) Obtain a blurred image of a bright, single star to simulate an impulse (the star should be as small as possible in the field of view of the telescope to simulate an impulse as closely as possible. (3) The Fourier transform of this image will give $H(u,v)$. (4) Use a Wiener filter and vary K until the sharpest image possible is obtained.