Histogram Equalization

$$s_k = T(r_k) = \sum_{j=0}^{k} P_r(r_j) = \sum_{j=0}^{k} \frac{n_j}{n} \times (L-1)$$

Euler's Formula

$$e^{\pm j\theta} = cos(\theta) \pm jsin(\theta)$$

$$f_Y(y) = [f_X(x)\frac{dX}{dY}] \text{ evaluated at } x = T^{-1}(y)$$

$$\sin c(x) = \frac{\sin \pi x}{\pi x}$$

Inverse Formula

$$sin(\theta) = \frac{1}{2j}(e^{j\theta} - e^{-j\theta})$$

$$cos(\theta) = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$$

Bicubic Interpolation

$$I(x,y) = \sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij} x^i y^j$$

Bilinear Interpolation

$$\mathrm{I(x,y) = ax + by + cxy + d}$$

$$f''(x) = f'(x+1) - f'(x) = f(x+1) + f(x-1) - 2f(x)$$

$$f'(x) = f(x+1) - f(x)$$

$$(v.v_i) = x_1(v_1.v_i) + x_2(v_2.v_i) + \ldots + x_i(v_i.v_i) + \ldots + x_n(v_n.v_i) \quad x_i = \frac{v\ .v_i}{v_i.v_i}$$

$$\int_{t_1}^{t_2} \phi_i(t)\phi_j^*(t) = \begin{cases} 0, & i \neq j \\ \lambda_j, & i = j \end{cases} \qquad \int_{t_1}^{t_2} \phi_i(t)\phi_j(t) = \begin{cases} 0, & i \neq j \\ \lambda_j, & i = j \end{cases}$$

$$a_k = \frac{1}{\lambda_k}\int_{t_1}^{t_2} \phi_k^* \hat{x}(t)dt. \qquad x(t) \approx \hat{x}(t) = \sum_{i=0}^{N-1} a_i \phi_i(t).$$

Forward Transformation

$$f(x,y) = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} T(u,v)s(x,y,u,v) \quad x = 0,1,\ldots,M-1,\ y = 0,1,\ldots,N-1$$

Exponential Kernel

$$r(x,y,u,v) = e^{-j2\pi(\frac{ux+vy}{N})}$$

Inverse Transformation

$$T(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)r(x,y,u,v) \quad u = 0,1,\ldots,M-1,\ v = 0,1,\ldots,N-1$$

Trigonometric Fourier Series

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(\frac{2\pi nt}{T}) + \sum_{n=1}^{\infty} b_n \sin(\frac{2\pi nt}{T})$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos(\frac{2\pi nt}{T})dt \qquad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin(\frac{2\pi nt}{T})dt$$

Exponential Fourier Series

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi n}{T}t} \qquad j = \sqrt{-1}$$

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)e^{-j\frac{2\pi n}{T}t}dt \quad \text{for } n = 0, \pm 1, \pm 2, \ldots$$

$$c_0 = a_0$$

$$c_n = \frac{a_n}{2} - j\frac{b_n}{2}$$

$$c_{-n} = c_n^*$$

Delta Function

$$\delta(x) = \lim_{a \to 0} \frac{1}{a} rect(x/a) \qquad \delta(x) = \begin{cases} \to \infty\ as\ \alpha \to 0 & if\ x = 0 \\ \to 0\ as\ \alpha \to 0 & if\ x \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(x)dx = 1 \qquad \sum_{x=-\infty}^{\infty} \delta(x) = 1 \qquad \delta(x) = \begin{cases} 1 & if\ x = 0 \\ 0 & if\ x \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} f(t)\delta(t)dt = f(0) \qquad \sum_{x=-\infty}^{\infty} f(x)\delta(x) = f(0)$$

$$\sum_{x=-\infty}^{\infty} f(x)\delta(x - x_0) = f(x_0) \qquad \int_{-\infty}^{\infty} f(x)\delta(x - x_0)dx = f(x_0)$$

Fourier Transform

$$F(f(x)) = F(u) = \int_{-\infty}^{\infty} f(x)e^{-j2\pi ux}dx \qquad \mathbf{P(u)}=|F(u)|^2$$

$$F^{-1}(F(u)) = f(x) = \int_{-\infty}^{\infty} F(u)e^{j2\pi ux}du$$

$$\phi(F(u)) = \tan^{-1}(\frac{I(u)}{R(u)})$$

$$F(u) = R(u) + jI(u)$$

$$F(u) = |F(u)|e^{j\phi(u)}$$

$$|F(u)| = \sqrt{R^2(u) + I^2(u)}$$

2D Fourier Transform

$$F(f(x,y)) = F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi(ux+vy)}dxdy$$

$$F^{-1}(F(u,v)) = f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)e^{j2\pi(ux+vy)}dudv$$

2D DFT using 1D DFT

$$F(u,v) = \frac{1}{N}\sum_{x=0}^{N-1} e^{-j2\pi(\frac{ux}{N})} \sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(\frac{vy}{N})}$$

Spatial/frequency shifts (translation)

$f(x) \leftrightarrow F(u), \quad then$

(1) $f(x - x_0) \leftrightarrow e^{-j2\pi ux_0}F(u) \qquad \delta(x - x_0) \leftrightarrow e^{-j2\pi ux_0}$

(2) $f(x)e^{j2\pi u_0 x} \leftrightarrow F(u - u_0) \qquad e^{j2\pi u_0 x} \leftrightarrow \delta(u - u_0)$

DFT Properties: Add/Multi

$$F[f(x,y) + g(x,y)] = F[f(x,y)] + F[g(x,y)]$$

$$F[f(x,y)g(x,y)] \neq F[f(x,y)]F[g(x,y)]$$

DFT Properties: Translation

$$f(x - x_0, y - y_0) \longleftrightarrow F(u,v)e^{-j2\pi(\frac{ux_0 + vy_0}{N})}$$

$$f(x,y)e^{j2\pi\frac{(u_0x + v_0y)}{N}} \longleftrightarrow F(u - u_0, v - v_0)$$

$$f(x,y)(-1)^{x+y} \longleftrightarrow F(u - N/2, v - N/2)$$

DFT Prop.: Avg Val

$$\bar{f}(x,y) = \frac{1}{N}F(0,0)$$

2D Discrete Fourier Transform

$$F(u,v) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(\frac{ux}{M} + \frac{vy}{N})}$$

$u = 0,1,2,\ldots,M-1, v = 0,1,2,\ldots,N-1$

$F(u,v) \equiv F(u\Delta u, v\Delta v)$

$$f(x,y) = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{j2\pi(\frac{ux}{M} + \frac{vy}{N})}$$

$x = 0,1,2,\ldots,M-1, y = 0,1,2,\ldots,N-1$

$f(x,y) \equiv f(x\Delta x, y\Delta y)$

Discrete Fourier Transform $\Delta u = \frac{1}{N\Delta x}$

$$F(u) = \frac{1}{N}\sum_{x=0}^{N-1} f(x)e^{\frac{-j2\pi ux}{N}}, \quad u = 0,1,2,\ldots,N-1$$

$$f(x) \equiv f(x\Delta x),\ x = 0,1,2,\ldots,N-1$$

$$f(x) = \sum_{u=0}^{N-1} F(u)e^{\frac{j2\pi ux}{N}}, \quad x = 0,1,2,\ldots,N-1$$

$$F(u) \equiv F(u\Delta u),\ u = 0,1,2,\ldots,N-1$$

Fast Fourier Transform:

$$W_N = e^{\frac{-j2\pi}{N}} \qquad F(u) = \frac{1}{N}\sum_{x=0}^{N-1} f(x)W_N^{ux}$$

$$F(u) = \frac{1}{2}[F_{even}(u) + F_{odd}(u)W_{2M}^u],\ u = 0,1,\ldots,M-1$$

$$F(u) = \frac{1}{2}[F_{even}(u) - F_{odd}(u)W_{2M}^u],\ u = M, M+1,\ldots,2M-1$$

DFT Properties: Scale

$$af(x,y) <\!\!-\!\!-\!\!> aF(u,v)$$

Convolution

$$f(x) * g(x) = \int_{-\infty}^{\infty} f(a)g(x-a)da$$

Nyquist optimum sampling step

$$\Delta x = \frac{1}{2W}$$

FT of sin and cos functions

$$F(\cos(2\pi u_0 x)) = \frac{1}{2}[\delta(u-u_0)+\delta(u+u_0)]$$

$$F(\sin(2\pi u_0 x)) = \frac{1}{2j}[\delta(u-u_0)-\delta(u+u_0)]$$

$$L_{avg} = \sum_{k=0}^{Q-1} l(r_k)P(r_k)$$

Odd symmetry

$$h(x,y) = -h(N-x, M-y)$$

$$R_D = 1 - \frac{1}{C_R}$$

2D Butterworth LPF

$$H(u,v) = \frac{1}{1+[D(u,v)/D_0]^{2n}}$$

$$\text{where } D(u,v) = \sqrt{(u-N/2)^2+(v-N/2)^2}$$

2D ILPF

$$H(u,v) = \begin{cases} 1 & if \ D(u,v) \le D_0 \\ 0 & otherwise \end{cases}$$

$$\text{where } D(u,v) = \sqrt{(u-N/2)^2+(v-N/2)^2}$$

2D Gaussian LPF

$$H(u,v) = e^{-[D(u,v)]^2/2\sigma^2}$$

By letting $D_0 = \sigma$

$$H(u,v) = e^{-[D(u,v)]^2/2D_0^2}$$

$$\text{where } D(u,v) = \sqrt{u^2+v^2}$$

$$\text{or } D(u,v) = \sqrt{(u-N/2)^2+(v-N/2)^2}$$

$$H_{HP}(u,v) = 1 - H_{LP}(u,v)$$

$$h_{HP}(x,y) = \delta(x,y) - h_{LP}(x,y)$$

High-Frequency-Emphasis filter

$$G(u,v) = [k_1 + k_2 H_{HP}(u,v)]F(u,v)$$

$$H(u,v) = (\gamma_H - \gamma_L)[1-e^{-c([D(u,v)]^2/D_0^2)}] + \gamma_L$$

Difference of Gaussians filter

$$H(u) = Ae^{-u^2/2\sigma_1^2} - Be^{-u^2/2\sigma_2^2}$$

$$\text{where } A \ge B \text{ and } \sigma_1 \ge \sigma_2$$

$$h(x) = \sqrt{2\pi}\sigma_1 Ae^{-2\pi^2\sigma_1^2x^2} - \sqrt{2\pi}\sigma_2 Be^{-2\pi^2\sigma_2^2x^2}$$

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(a,b)\delta(x-a, y-b)\ da\ db$$

Band-reject filters

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \le D \le D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u,v) = \frac{1}{1+\left[\frac{DW}{D^2-D_0^2}\right]^{2n}}$	$H(u,v) = 1 - e^{-\left[\frac{D^2-D_0^2}{DW}\right]^2}$

Notch-reject filters

$$H(u,v) = \begin{cases} 0 & if \ D_1(u,v) \le D_0 \ or \ D_2(u,v) \le D_0 \\ 1 & otherwise \end{cases}$$

$$D_1(u,v) = \sqrt{(u-\frac{N}{2}-u_0)^2+(v-\frac{N}{2}-v_0)^2}$$

$$D_2(u,v) = \sqrt{(u-\frac{N}{2}+u_0)^2+(v-\frac{N}{2}+v_0)^2}$$

Mean Square Error (MSE)

$$\text{MSE} = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left[f(x,y)-\hat{f}(x,y)\right]^2$$

Signal to Noise Ratio (SNR)

$$\text{SNR} = \frac{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\hat{f}(x,y)^2}{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[f(x,y)-\hat{f}(x,y)]^2}$$

$$\hat{f}(x,y) = \frac{1}{mn}\sum_{(s,t)\in S_{x,y}} g(s,t)$$

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{x,y}} g(s,t)\right]^{\frac{1}{mn}}$$

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{x,y}}[g(s,t)]^{Q+1}}{\sum_{(s,t)\in S_{x,y}}[g(s,t)]^{Q}}$$

Wiener filter

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)}\ \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_n(u,v)/S_f(u,v)}\right]G(u,v)$$

$$S_n(u,v) = |N(u,v)|^2 \qquad S_f(u,v) = |F(u,v)|^2$$

$$H(u,v) = \int_0^T e^{-j2\pi(ux_0(t)+vy_0(t))}dt$$

$$H(u,v) = \frac{T}{\pi(u\alpha+vb)}\sin(\pi(ua+vb))e^{-j\pi(ua+vb)}$$

Entropy

$$H = -\sum_{k=0}^{L-1} P(r_k)log(P(r_k))$$

$$R = L_{avg} - H$$

CLS filter

$$p(x,y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2+\gamma|P(u,v)|^2}\right]G(u,v)$$

$$|H(u,v)|^2 = H(u,v)H^*(u,v)$$