Histogram Equalization $s_k = T(r_k) = \sum_{j=0}^{k} P_r(r_j) = \sum_{j=0}^{k} \frac{n_j}{n} \times (L-1)$

$f_Y(y) = [f_X(x)\frac{dX}{dY}]$ evaluated at $x = T^{-1}(y)$

$\sin c(x) = \frac{\sin \pi x}{\pi x}$

Bilinear Interpolation $I(x,y) = ax + by + cxy + d$

$f''(x) = f'(x+1) - f'(x) = f(x+1) + f(x-1) - 2f(x)$

$f'(x) = f(x+1) - f(x)$

$(v.v_i) = x_1(v_1.v_i) + x_2(v_2.v_i) + \ldots + x_i(v_i.v_i) + \ldots + x_n(v_n.v_i) \quad x_i = \frac{v\ .v_i}{v_i.v_i}$

Euler's Formula $e^{\pm j\theta} = cos(\theta) \pm jsin(\theta)$

Inverse Formula

$sin(\theta) = \frac{1}{2j}(e^{j\theta} - e^{-j\theta})$

$cos(\theta) = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$

Bicubic Interpolation

$$I(x,y) = \sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij} x^i y^j$$

$$\int_{t_1}^{t_2} \phi_i(t)\phi_j^*(t) = \begin{cases} 0, & i \neq j \\ \lambda_j, & i = j \end{cases} \qquad \int_{t_1}^{t_2} \phi_i(t)\phi_j(t) = \begin{cases} 0, & i \neq j \\ \lambda_j, & i = j \end{cases}$$

$$a_k = \frac{1}{\lambda_k}\int_{t_1}^{t_2} \phi_k^* \hat{x}(t)dt. \qquad x(t) \approx \hat{x}(t) = \sum_{i=0}^{N-1} a_i\phi_i(t).$$

Forward Transformation

$f(x,y) = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} T(u,v)s(x,y,u,v) \quad x = 0,1,\ldots,M-1, \ y = 0,1,\ldots,N-1$

Exponential Kernel

$r(x,y,u,v) = e^{-j2\pi(\frac{ux+vy}{N})}$

Inverse Transformation

$T(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)r(x,y,u,v) \quad u = 0,1,\ldots,M-1, \ v = 0,1,\ldots,N-1$

Trigonometric Fourier Series $f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(\frac{2\pi nt}{T}) + \sum_{n=1}^{\infty} b_n \sin(\frac{2\pi nt}{T})$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos(\frac{2\pi nt}{T})dt \qquad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin(\frac{2\pi nt}{T})dt$

Exponential Fourier Series $f(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi n}{T}t}$

$c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)e^{-j\frac{2\pi n}{T}t}dt$ for $n = 0, \pm 1, \pm 2, \ldots$

$j = \sqrt{-1}$

$c_0 = a_0$

$c_n = \frac{a_n}{2} - j\frac{b_n}{2}$

$c_{-n} = c_n^*$

Delta Function $\delta(x) = \lim_{a \to 0} \frac{1}{a} rect(x/a) \qquad \delta(x) = \begin{cases} \to \infty \ as\ \alpha \to 0 & if\ x = 0 \\ \to 0\ as\ \alpha \to 0 & if\ x \neq 0 \end{cases}$

$\int_{-\infty}^{\infty} \delta(x)dx = 1 \qquad \sum_{x=-\infty}^{\infty} \delta(x) = 1 \qquad \delta(x) = \begin{cases} 1 & if\ x = 0 \\ 0 & if\ x \neq 0 \end{cases}$

$\int_{-\infty}^{\infty} f(t)\delta(t)dt = f(0) \qquad \sum_{x=-\infty}^{\infty} f(x)\delta(x) = f(0)$

$\sum_{x=-\infty}^{\infty} f(x)\delta(x-x_0) = f(x_0) \qquad \int_{-\infty}^{\infty} f(x)\delta(x-x_0)dx = f(x_0)$

Fourier Transform $F(f(x)) = F(u) = \int_{-\infty}^{\infty} f(x)e^{-j2\pi ux}dx$ **$P(u)=|F(u)|^2$**

$F^{-1}(F(u)) = f(x) = \int_{-\infty}^{\infty} F(u)e^{j2\pi ux}du \qquad \phi(F(u)) = \tan^{-1}(\frac{I(u)}{R(u)})$

$F(u) = R(u) + jI(u) \qquad F(u) = |F(u)|e^{j\phi(u)}$

$|F(u)| = \sqrt{R^2(u) + I^2(u)}$

Spatial/frequency shifts (translation)

$f(x) \leftrightarrow F(u), \quad then$

(1) $f(x - x_0) \leftrightarrow e^{-j2\pi ux_0}F(u) \qquad \delta(x - x_0) \leftrightarrow e^{-j2\pi ux_0}$

(2) $f(x)e^{j2\pi u_0 x} \leftrightarrow F(u - u_0) \qquad e^{j2\pi u_0 x} \leftrightarrow \delta(u - u_0)$

2D Fourier Transform

$F(f(x,y)) = F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi(ux+vy)}dxdy$

$F^{-1}(F(u,v)) = f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)e^{j2\pi(ux+uy)}dudv$

2D DFT using 1D DFT

$F(u,v) = \frac{1}{N}\sum_{x=0}^{N-1} e^{-j2\pi(\frac{ux}{N})}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(\frac{vy}{N})}$

DFT Properties: Add/Multi

$F[f(x,y) + g(x,y)] = F[f(x,y)] + F[g(x,y)]$

$F[f(x,y)g(x,y)] \neq F[f(x,y)]F[g(x,y)]$

DFT Properties: Translation

$f(x - x_0, y - y_0) \longleftrightarrow F(u,v)e^{-j2\pi(\frac{ux_0 + vy_0}{N})}$

$f(x,y)e^{j2\pi\frac{(u_0x + v_0y)}{N}} \longleftrightarrow F(u - u_0, v - v_0)$

$f(x,y)(-1)^{x+y} \longleftrightarrow F(u - N/2, v - N/2)$

DFT Prop.: Avg Val

$\bar{f}(x,y) = \frac{1}{N}F(0,0)$

2D Discrete Fourier Transform

$F(u,v) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(\frac{ux}{M} + \frac{vy}{N})}$

$u = 0,1,2,\ldots,M-1, v = 0,1,2,\ldots,N-1$

$F(u,v) \equiv F(u\Delta u, v\Delta v)$

$f(x,y) = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{j2\pi(\frac{ux}{M} + \frac{vy}{N})}$

$x = 0,1,2,\ldots,M-1, y = 0,1,2,\ldots,N-1$

$f(x,y) \equiv f(x\Delta x, y\Delta y)$

Discrete Fourier Transform $\Delta u = \frac{1}{N\Delta x}$

$F(u) = \frac{1}{N}\sum_{x=0}^{N-1} f(x)e^{\frac{-j2\pi ux}{N}}, \quad u = 0,1,2,\ldots,N-1$

$f(x) \equiv f(x\Delta x), \ x = 0,1,2,\ldots,N-1$

$f(x) = \sum_{u=0}^{N-1} F(u)e^{\frac{j2\pi ux}{N}}, \quad x = 0,1,2,\ldots,N-1$

$F(u) \equiv F(u\Delta u), \ u = 0,1,2,\ldots,N-1$

Fast Fourier Transform: $W_N = e^{\frac{-j2\pi}{N}} \quad F(u) = \frac{1}{N}\sum_{x=0}^{N-1} f(x)W_N^{ux}$

$F(u) = \frac{1}{2}[F_{even}(u) + F_{odd}(u)W_{2M}^u], \ u = 0,1,\ldots,M-1$

$F(u) = \frac{1}{2}[F_{even}(u) - F_{odd}(u)W_{2M}^u], \ u = M, M+1,\ldots,2M-1$

DFT Properties: Scale

$af(x,y) <---> aF(u,v)$

Convolution

$$f(x) * g(x) = \int_{-\infty}^{\infty} f(a)g(x-a)da$$

FT of sin and cos functions

$$F(cos(2\pi u_0 x)) = \frac{1}{2}[\delta(u-u_0) + \delta(u+u_0)]$$

$$F(\sin(2\pi u_0 x)) = \frac{1}{2j}[\delta(u-u_0) - \delta(u+u_0)]$$