1. [30 points – 3pts each] True/False Questions – To get credit, you must give brief reasons for each answer!

T  F The Laplacian-of-Gaussian (LoG) operator performs both smoothing and differentiation.  

Corrections:
- Uses a Gaussian for smoothing and the Laplacian for differentiation.

T  F The lower the frequency of a sinusoidal, the more samples must be taken to gain an accurate representation of the wave.  

Corrections:
- The higher the frequency, the more samples (from Nyquist theorem: $A \times \frac{1}{2f}$).

T  F The Laplacian operator can be used for estimating edge direction.

Corrections:
- No, it uses one mask only, so direction cannot be estimated.

T  F The Hough transform of a point in the image space is a point in the Hough space.

Corrections:
- Point $\rightarrow$ line or sinusoidal.

T  F Otsu's method makes no assumptions about histogram shape.

Corrections:
- It assumes bimodal histogram.
T F The Nyquist theorem applies to band-limited functions only.

\[ \Delta x = \frac{1}{2w} \]

T F A gradient magnitude of zero at a given pixel implies that the Laplacian at this pixel is also zero.

- Zero gradient means that the image is constant!
- That means that the second derivative will be zero as well!

T F Median filtering is a sharpening filter.

Smoothing filter

T F The magnitude of the FT carries more information than its phase.

The phase does not correspond to actual frequency components of the image. Shift from (0, 0)

T F The first step in edge detection is differentiation (i.e., enhancement).

Smoothing
2. [10 points] Assume that an image has the histogram shown below, where \( p_1(z) = \frac{z}{2} \) corresponds to the intensity of the objects and \( p_2(z) = (z-1)/2 \) corresponds to the intensity of the background. Assuming that \( P_1 = P_2 = 1/2 \) (where \( P_1 \) and \( P_2 \) are the a priori probabilities of object and background pixels) find the optimal threshold between object and background pixels.

\[
E_1 = \int_1^T \frac{T}{2} x - \frac{T}{2} \, dx = \frac{1}{4} \frac{1}{2} \left( \frac{T^2}{2} - \frac{1}{4} \right) - \frac{1}{2} (T-1)
\]

\[
E_2 = \int_T^2 \frac{T}{2} x \, dx = \frac{1}{4} \frac{1}{2} \left( \frac{T^2}{2} - \frac{1}{4} \right)
\]

\[
E(T) = P_2 E_1 + P_1 E_2 = \frac{1}{2} \left( \frac{1}{4} (T^2 - 1) - \frac{1}{2} (T-1) \right) + \frac{1}{4} (4-T^2)
\]

\[
E(T) = \frac{1}{2} \left( -\frac{1}{2} T + \frac{5}{4} \right) = \frac{5}{8} - \frac{1}{4} T
\]

\[
E'(T) = -\frac{1}{4} < 0 \text{ decreasing function}
\]

\[
T = 1 \quad E(1) = \frac{5}{8} - \frac{1}{4} = \frac{5}{8} - \frac{2}{8} = \frac{3}{8}
\]

\[
T = 2 \quad E(2) = \frac{5}{8} - \frac{4}{8} = \frac{1}{8}
\]
3. [15 pts] The following image is given:

<table>
<thead>
<tr>
<th>Gray level</th>
<th>Count</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>12</td>
<td>(\frac{3}{8})</td>
</tr>
<tr>
<td>95</td>
<td>4</td>
<td>(\frac{1}{3})</td>
</tr>
<tr>
<td>169</td>
<td>4</td>
<td>(\frac{1}{3})</td>
</tr>
<tr>
<td>243</td>
<td>12</td>
<td>(\frac{3}{8})</td>
</tr>
</tbody>
</table>

(a) Compute its first order entropy.

\[
H = - \sum_{i=0}^{3} P(r_i) \log(P(r_i)) = \frac{1.81}{b/15/pixel}
\]

(b) Construct the Huffman code for this image.

(c) What is the average number of bits/pixel in this case? Is this a good code?

\[
\text{Long} = e(0.21) + e(2.43) + e(1.5) + e(1.69)
\]

\[
= 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8} + 3 \times \frac{1}{8} = \frac{3}{8} + \frac{6}{8} + \frac{3}{8} + \frac{3}{8}
\]

\[
= 15/8 = 1.875 \text{ bit/pixel}
\]

Yes, it is a good encoding; very close to \(H\)!
4. **[10 points]** Suppose that you process an image with a filter \( h(i, j) \). Let us assume that the filter averages the four nearest neighbors of a pixel \((i, j)\), excluding itself.

(a) Determine the filter in the frequency domain.

\[
    h(x, y) = \frac{1}{4} \left( f(x-1, y) + f(x+1, y) + f(x, y-1) + f(x, y+1) \right)
\]

\[
    \mathcal{F}(X, Y) = e^{-j2\pi NU}
\]

\[
    H(u, v) = \frac{1}{4} \left[ e^{-2\pi ju} + e^{2\pi ju} + e^{-2\pi jv} + e^{2\pi jv} \right]
\]

\[
    = \frac{1}{2} \cos 2\pi nu + \frac{1}{2} \cos 2\pi nv
\]

(b) Characterize the filter (i.e., determine if it is a lowpass, highpass, or bandpass). Justify your answer.

*Low pass filter*
5. [15 points] JPEG is a popular image compression technique.

(a) Describe the steps of the JPEG image compression algorithm. Indicate what kind of redundancy (i.e., coding, interpixel, psychovisual) is addressed by each step.

1. Divide image into 8x8 subimages
2. Shift levels in the range [-128, 127]
3. Apply DCT — interpixel redundancy
4. Quantize coefficients — psychovisual redundancy
5. Apply zig-zag ordering
6. Encode coefficients — coding redundancy

(b) Why using DCT instead of some other transform, for example, DFT?

more compact!

faster

(c) What is the purpose of zig-zag ordering? Which step causes information loss?

to facilitate run-length encoding!

(long runs of 0s)
6. [10 points] State and prove the convolution theorem in the continuous case. For simplicity, assume 1-D functions.

\[
\mathcal{F}\left((f(x) * g(x))\right) = F(u) \cdot G(u)
\]

\[
= \iint f(a) \cdot g(x-a) \cdot da \cdot e^{-j2\pi au} \cdot dx
\]

\[
= \int f(a) \cdot \int g(x-a) \cdot e^{-j2\pi au} \cdot dx \cdot da
\]

\[
= \int f(a) \cdot G(u) \cdot e^{-j2\pi au} \cdot da = G(u) \cdot \int f(a) \cdot e^{-j2\pi au} \cdot da
\]

\[
= G(u) \cdot F(u)
\]
7. (Undergraduate Students Only) [10 points] When using region merging for segmentation, we need to determine the similarity between pairs of regions. Two methods discussed in class on this issue are: (1) hypothesis testing and (ii) weak boundaries. Explain how each of them works.

X  N/A
7. [10 points] This problem relates to image restoration.

(a) Explain how the inverse filtering technique works.

Let's assume that $G(u,v)$ corresponds to the spectrum of the degraded image. Then,

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

where $H(u,v)$ is the degradation function spectrum.

(b) What assumptions does inverse filtering make?

It works well when $N(u,v) = 0$ (noise).

(c) What is the main issue associated with inverse filtering? How do we deal with it?

The assumption that $N(u,v) = 0$ is not realistic.

As a result,

$$F(u,v) = \frac{G(u,v)}{H(u,v)} + \frac{N(u,v)}{H(u,v)}$$

When $H(u,v) \to 0$, then $rac{N(u,v)}{H(u,v)} \to \infty$.

In practice, we carry out

$$\frac{G(u,v)}{H(u,v)}$$

in a limited region of $H(u,v) \to$ radius parameter.