1. [25 points] True/False Questions – To get credit, you must give brief reasons for each answer!

T F The filter shown below is a smoothing filter.

1 2 1
2 1 2
1 2 1

T F Assuming an NxN image, the complexity of 2D FFT is $O(N^2 \log N)$.

T F The magnitude of the FT carries more information than its phase.

T F The Nyquist theorem assumes band-limited functions only.

T F Unsharp masking is a special case of high boost filtering.
2. [15 points] State and prove the convolution theorem in the continuous case. For simplicity, assume 1-D functions.
3. **[15 points]** Find and plot the discrete convolution of the following discrete sequences:

![Graph showing discrete sequences](image)

- **x(n)**: 4 samples
- **h(n)**: 2 samples
4. **[20 points]**. A 3 bits/pixel image of size 5x5 is given below. Find the following: (a) the output of a 3x3 averaging filter at (1,1), (b) the output of a 3x3 median filter at (1,1) and (c) the gradient magnitude at (1,1) using the Sobel masks shown below.

<table>
<thead>
<tr>
<th>y</th>
<th>x=</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
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<td>3</td>
<td>7</td>
<td>6</td>
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</table>
4. **[15 points]** What is the FT of \( \cos(4\pi x) + \cos(10\pi x) \)? How many samples should we obtain according to the Nyquist theorem in order to avoid aliasing?
5. [10 points] Given the 3x3 image shown below, compute the histogram equalized image (assume that the gray-levels are in the range [0..7]). Show all the steps.

```
3 1 1
1 7 6
0 2 1
```
7. Graduate Students Only [10 points] The pixel intensity values of a gray level image have the probability density function \( p_r(r) \) given by \( p_r(r) = 2(1 - r) \), for \( 0 \leq r \leq 1 \), and zero otherwise. It is desired to transform the gray levels of the image so that they have the probability density function \( p_z(z) = 2z \), for \( 0 \leq z \leq 1 \), and zero otherwise. Assume that \( r \) and \( z \) are continuous random variables. Find the transformation that accomplishes that.