CS474/674 Image Processing and Interpretation
Sample Midterm Exam - Solutions

Name:____

1. **[25 points]** True/False Questions – To get credit, **you must give brief reasons for each answer!**

T F The filter shown below is a smoothing filter.

$$\begin{matrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{matrix}$$

No negative weights.

T F Assuming an NxN image, the complexity of 2D FFT is $O(N^2 \log N)$.

1D FFT: $N\log(N)$
N rows: $N^2\log(N)$
N columns $N^2\log(N)$

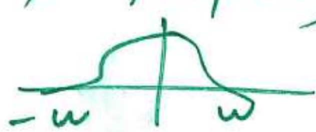
Overall $N^2\log(N)$

T F The magnitude of the FT carries more information than its phase.

The reconstruction using phase preserves more information.

T F The Nyquist theorem holds true for band-limited functions only.

The max frequency w is finite



T F Unsharp masking is a special case of high boost filtering.

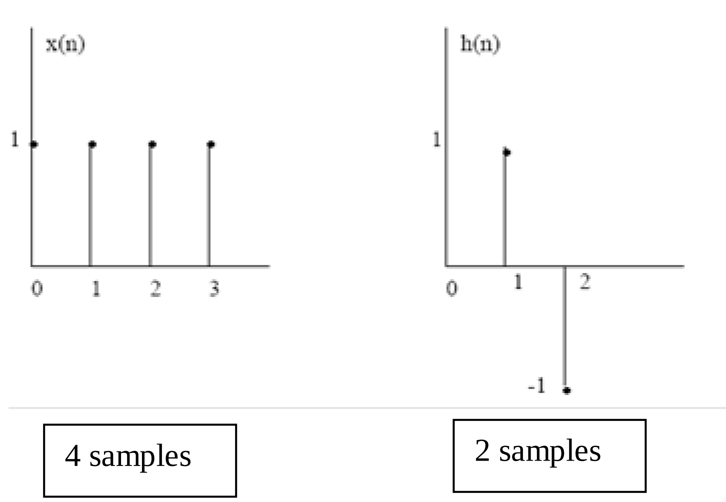
If A=1 then unsharp masking.

$$g(x,y) = (A-1)f(x,y) + \underbrace{(f(x,y) - f_{lp}(x,y))}_{\text{unsharp masking}}$$

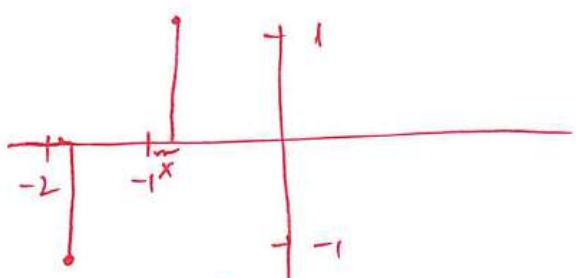
2. [15 points] State and prove the convolution theorem in the continuous case. For simplicity, assume 1-D functions.

See notes

3. [15 points] Find and plot the discrete convolution of the following discrete sequences:



$h(x-n)$



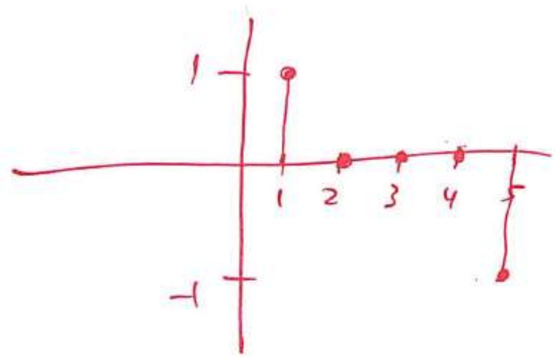
if $-1+x=0 \Rightarrow x=1$ $\quad f*g=1$

if $-1+x=1 \Rightarrow x=2$ $\quad f*g=0$

$x=3$ $\quad f*g=0$

$x=4$ $\quad f*g=0$

$x=5$ $\quad f*g=-1$



4. [20 points]. A 3 bits/pixel image of size 5x5 is given below. Find the following: (a) the output of a 3x3 averaging filter at (1,1), (b) the output of a 3x3 median filter at (1,1) and (c) the gradient magnitude at (1,1) using the Sobel masks shown below.

IMAGE

y \ x=	0	1	2	3	4
0	3	7	6	2	0
1	2	4	6	1	1
2	4	7	2	5	4
3	3	0	6	2	1
4	5	7	5	1	2

G_y

−1	−2	−1
0	0	0
1	2	1

G_x

−1	0	1
−2	0	2
−1	0	1

a) $\dfrac{3+7+6+2+4+6+4+7+2}{9} = 41/9 \approx 4.55 = 5$

b) 2 2 3 4 4 6 6 7 7 $\rightarrow$ 4

c) $G_y = (-3-14-6)+(4+14+2) = -3$

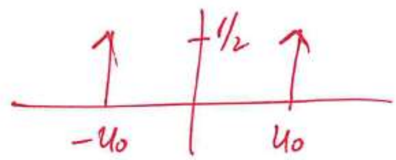
$G_x = (-3-4-4)+(6+12+2) = 9$

magnitude $= \sqrt{G_x^2 + G_y^2} = \sqrt{(-3)^2+9^2} = \sqrt{90} \cong 9.48 \approx 9$

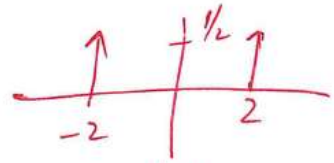
4. [15 points] What is the FT of cos(4πx)+cos(10πx)? How many samples should we obtain according to the Nyquist theorem in order to avoid aliasing?

we know that $\cos(2\pi u_0 x)$ has the following

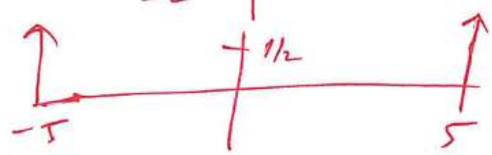
FT:



$\cos(4\pi x) = \cos(2.\pi\, 2x)$

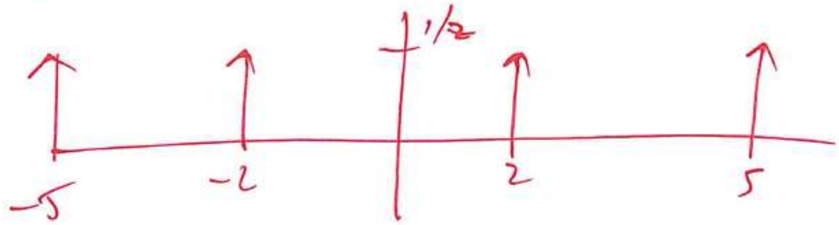


$\cos(10\pi x) = \cos(2\pi\, 5x)$



$\mathcal{F}\left(\cos(4\pi x) + \cos(10\pi x)\right) =$

$\mathcal{F}\left(\cos(4\pi x)\right) + \mathcal{F}\left(\cos(10\pi x)\right)$



max freq $W = 5$ so $\Delta x \le \frac{1}{2W} = \frac{1}{10}$

at least 10 samples

5. [10 points] Given the 3x3 image shown below, compute the histogram equalized image (assume that the gray-levels are in the range [0..7]). Show all the steps.

3 1 1
1 7 6
0 2 1

r	0	1	2	3	4	5	6	7
n_r	1	4	1	1	0	0	1	1
	~~0.11~~	~~0.44~~	~~0.11~~	~~0.11~~	~~0~~	~~0~~	0.11	0.11
	1/9	4/9	1/9	1/9	0	0	1/9	1/9

$$S_k = T(r_k) = \sum_{j=0}^{k} P_r(r_j) \qquad L=8$$

$$S_0 = \frac{1}{9} \times (L-1) = 7/9 \simeq 1$$

$$S_1 = \left(\frac{1}{9}+\frac{4}{9}\right) \times 7 \simeq \frac{35}{9} \simeq 4$$

$$S_2 = (1/9 + 4/9 + 1/9) \times 7 = \frac{42}{9} \simeq 5$$

$$S_3 = (1/9 + 4/9 + 1/9 + 1/9) \times 7 = \frac{49}{9} \simeq 5$$

$$S_4 = S_5 = 5$$

$$S_6 = \frac{56}{9} \approx 6$$

$$S_7 = 7$$

r	0	1	2	3	4	5	6	7
s	1	4	5	5	5	5	6	7

Equalized image:

5 4 4
4 7 6
1 5 4

7. Graduate Students Only [10 points] The pixel intensity values of a gray level image have the probability density function $p_r(r)$ given by $p_r(r)=2(1-r)$, for $0 \le r \le 1$, and zero otherwise. It is desired to transform the gray levels of the image so that they have the probability density function $p_z(z)=2z$, for $0 \le z \le 1$, and zero otherwise. Assume that r and z are continuous random variables. Find the transformation that accomplishes that.

$$P_r(r) = \begin{cases} 2(1-r) & 0 \le r \le 1 \\ 0 & \text{otherwise} \end{cases} \qquad P_z(z) = \begin{cases} 2z & 0 \le z \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$S = T(r) = \int_0^r 2(1-w)\,dw = 2\left[\int_0^r dw - \int_0^r w\,dw\right] =$$

$$= 2\left[w\Big|_0^r - \frac{1}{2}w^2\Big|_0^r\right] = 2\left(r - \frac{1}{2}r^2\right) =$$

$$= 2r - r^2 = r(2-r)$$

$$V = G(z) = \int_0^z 2w\,dw = 2\int_0^z w\,dw = 2\,\frac{1}{2}w^2\Big|_0^z$$

$$= z^2 \qquad \underline{z = \sqrt{v} = G^{-1}(v)}$$

$$Q(r) = G^{-1}(T(r)) = \sqrt{r(2-r)}$$