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Analysis of Algorithms

Midterm Examination

Duration: 1 hour

Name: **Student Number:**

1. (a) Let *f(n)* and *g(n)* be asymptotically nonnegative functions. Prove that $\Theta(f(n)+g(n)) = max(f(n),g(n))$.

(b) Prove (formally) that the worst-case running time of quicksort is $\Theta(n^2)$

(c) Briefly describe what we mean by a randomized algorithm. Why are we using randomization in quicksort ?

2. Prove or disprove the following conjecture: $f(n)=O(g(n))$ *implies* $2^{f(n)}=O(2^{g(n)})$

3. (a) Determine an asymptotic bound for the following recurrence: $T(n)=T(n-1)+n$.

(b) Use recursion trees to determine a tight asymptotic bound for the following recursion: $T(n)=T(n/10)+T(9n/10)+n$.

4. The following "elegant" sorting algorithm has been proposed:

STOOGE-SORT(A,i,j)
if A[i] > A[j] then
  exchange A[i] <--> A[j]
if i+1 ≥ j then return / returns nothing ... */*
k <-- $\lfloor (j-i+1)/3 \rfloor$ */* round down */*
STOOGE-SORT(A,i,j-k) / first two-thirds */*
STOOGE-SORT(A,i+k,j) / last two-thirds */*
STOOGE-SORT(A,i,j-k) / first two-thirds again */*

(a) Give a recurrence for the worst-case running time of STOOGE-SORT.

(b) Give a tight asymptotic (Θ-notation) bound on the worst-case running time by solving the recurrence.

5. Describe an efficient algorithm that, given *n* integers in the range of *1* to *k*, preprocess the input and then answers any query about how many of the *n* integers fall into the range *[a..b]* in O(1) time.