CS485/685 Computer Vision

Spring 2010 – Dr. George Bebis

Homework 3 - Due Date: 3/11/2010

1 Find a decomposition (i.e., write A as $P\Lambda P^{-1}$) for the following matrix:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

2. Consider the 3D point $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$. What would be coordinates of the point after applying the following composite transformation: (i) rotation of 90 degrees about the x-axis, (ii) translation by $d_x = -2$, $d_y = 1$, $d_z = 1$ and, (iii) scaling by $s_x = 1$, $s_y = 2$ and $s_z = 0.5$. Show your calculations clearly.

3. Prove that the following matrix represents a rigid transformation ($a = \frac{\sqrt{2}}{2}$).

$$\begin{bmatrix} a & -a & 0 \\ a & a & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4 It is easy to show that the unit vectors $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ form a basis in R^3. Prove that $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ form also a basis of R^3. What is the representation of vector $\begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$ in this basis?

Graduate Students Only

5 Suppose A is a real mxn matrix. Prove that the squares of the singular values of A are the eigenvalues of $A^T A$. (*hint*: if A is a symetric matrix, it can be written as $A = P\Lambda P^T$ where the columns of P are the eigenvectors of A and Λ is a diagonal matrix with diagonal elements equal to the eigenvalues of A).