Name:_______________________________________________

1. [40 points] To get credit, you must justify your answers!

T F There are four main steps in edge detection.

T F While we cannot estimate the direction of an edge using the Laplacian, we can still estimate its strength.

T F Using lens allows us duplicate the pinhole geometry without having to use very small apertures.

T F Convolving a Gaussian with itself yields another Gaussian with double the standard deviation of the original Gaussian.

T F Radial distortion affects mostly the pixels close to the center of an image.
**T  F** Focusing in the human eye is achieved by varying the distance between the lens and the retina.

\[
A = \begin{bmatrix}
0.96 & 1.72 \\
0.8 & 0.96
\end{bmatrix}
\]

**T  F** The rank of the matrix whose SVD is shown below, is 2.

\[
\begin{bmatrix}
0.6 & -0.8 \\
0.8 & 0.6
\end{bmatrix}
\begin{bmatrix}
3 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
0.8 & 0.6 \\
0.6 & -0.8
\end{bmatrix}
\]

**T  F** Ramp edges can be better localized using the gradient than the Laplacian.

**T  F** If an \( n \times n \) matrix is diagonalizable, then its columns form a basis in \( \mathbb{R}^n \)

**T  F** Each point in Cartesian coordinates has a unique representation in Homogeneous coordinates.
2. [30 points] Short answer questions.

(a) What is the difference between convolution and correlation? Under what condition is convolution equivalent to correlation?

(b) Explain how the Moravec interest operator works. What are its main weaknesses?
(c) What is the separability property of Gaussian convolution? Why would one want a convolution filter to be separable?

(d) Define the terms “depth of field” and “field of view”. What are the parameters that affect them and how?
(e) What are the main steps of the Canny edge detector? Describe each step.

(f) In many applications, an image is smoothed by applying Gaussian filters of several sizes. Why would one want to smooth an image using different parameters of the Gaussian?
3. **[10 pts]** Consider the vector $(7, 3, 2)$; what are its coordinates in a new coordinate system which has been obtained by applying the following transformations on the original system: translation by $(4, -3, 7)$, followed by rotation around Z axis by $90^\circ$.
4. [10 points] Using SVD decomposition, we can compute more efficiently the solution of $Ax=b$, where $b\neq 0$. Show how.
5. (a) [5 points] How do we show that a set of vectors $x_1, x_2, \ldots, x_n$ forms a basis?

(b) [5 points] Assuming that $x_1, x_2, \ldots, x_n$ form a basis, show how to find the expansion of another vector $y$ onto this basis. Is the expansion unique?
6. Graduate Students Only [20 points] How does the Harris interest operator improve the Moravec interest operator? Derive the “auto-correlation” matrix (i.e., give the proof). What are the properties of the auto-correlation matrix? How is it useful for detecting interest points?