CS 4/791E Computer Vision

Spring 2004 - Dr. George Bebis

Homework 4

Due date: 4/22/04

1. As we have discussed in class, the solution of an over-determined system $Ax = b$ (A is mxn with $m > n$) can be computed as follows: $x = (A^T A)^{-1} A^T b$ where A^+ is the pseudo-inverse of A. Show that this is equivalent to computing $x = V D_0^{-1} U^T b$ where $A = UDV^T$ is the SVD decomposition of A and

$$D_0^{-1} = \begin{cases} 1/\sigma_i & \text{if } \sigma_i > t \\ 0 & otherwise \end{cases}$$

2. Consider the 3D point $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$. What would be coordinates of the point after applying the following composite transformation: (i) rotation of 90 degrees about the x-axis, (ii) translation by $d_x = -2$, $d_y = 1$, $d_z = 1$ and, (iii) scaling by $s_x = 1$, $s_y = 2$ and $s_z = 0.5$. Show your calculations clearly.

3. Prove that the following matrix represents a rigid transformation ($a = \frac{\sqrt{2}}{2}$).

$$\begin{bmatrix} a & -a & 0 \\ a & a & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4. Suppose A is a real mxn matrix. Prove that the squares of the singular values of A are the eigenvalues of $A^T A$. (*hint*: if A is a symetric matrix, it can be written as $A = P \Lambda P^T$ where the columns of P are the eigenvectors of A and Λ is a diagonal matrix with diagonal elements equal to the eigenvalues of A).