Camera Calibration

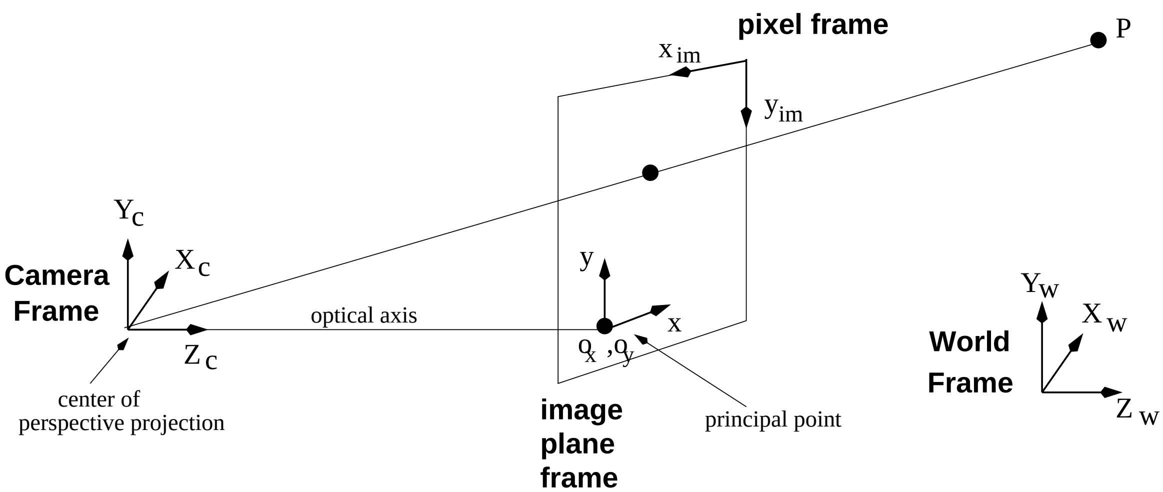
(Trucco, Chapter 6)

• What is the goal of camera calibration?

- To produce an estimate of the extrinsic and intrinsic camera parameters.

• Procedure

- Given the correspondences between a set of point features in the world (X_w, Y_w, Z_w) and their projections in an image (x_{im}, y_{im}), compute the intrinsic and extrinsic camera parameters.



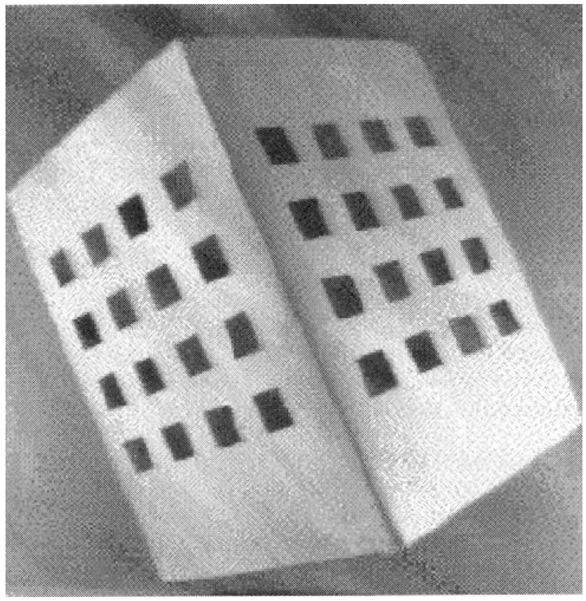
• Establishing the correspondences

- Calibration methods rely on one or more images of a calibration pattern:

(1) a 3D object of known geometry.

(2) it is located in a known position in space.

(3) it is generating image features which can be located accurately.



- Consider the above calibration pattern:

  * it consists of two orthogonal grids.

  * equally spaced black squares drawn on white, perpendicular planes.

  * assume that the world reference frame is centered at the lower left corner of the left grid, with axes parallel to the three directions identified by the calibration pattern.

  * given the size of the planes, their angle, the number of squares etc. (all known by construction), the coordinates of each vertex can be computed in the world reference frame using trigonometry.

  * the projection of the vertices on the image can be found by intersecting the edge lines of the corresponding square sides (or through corner detection).

• Methods

(1) Direct parameter calibration.

Direct recovery of the intrinsic and extrinsic camera parameters.

(2) Camera parameters through the projection matrix

$$M = M_{in} \ M_{ex} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix}$$

(2.1) Estimate the elements of the projection matrix.

(2.2) Compute the intrinsic/extrinsic as closed-form functions of the entries of the projection matrix.

Method 1: Direct Parameter Calibration

- We assume that the world reference frame is known (e.g., the origin is the middle lower corner of the calibration pattern).

• Review of basic equations

- From world coordinates to camera coordinates (note that we have changed the order of rotation/translation):

$$P_c = R(P_w - T) \text{ or } P_c = RP_w - RT \text{ or } P_c = RP_w - T'$$

- In the rest of this discussion, I will replace T' with T:

$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}$$

- From camera coordinates to pixel coordinates:

$$x_{im} = - x/s_x + o_x = - \frac{f}{s_x} \frac{X_c}{Z_c} + o_x$$

$$y_{im} = - y/s_y + o_y = - \frac{f}{s_y} \frac{Y_c}{Z_c} + o_y$$

- Relating world coordinates to pixel coordinates:

$$x_{im} - o_x = - f/s_x \frac{r_{11}X_w + r_{12}Y_w + r_{13}Z_w + T_x}{r_{31}X_w + r_{32}Y_w + r_{33}Z_w + T_z}$$

$$y_{im} - o_y = - f/s_y \frac{r_{21}X_w + r_{22}Y_w + r_{23}Z_w + T_y}{r_{31}X_w + r_{32}Y_w + r_{33}Z_w + T_z}$$

• Independent intrinsic parameters

- The five intrinsic parameters f, s_x, s_y, o_x, o_y are not independent.

- We can define the following four independent parameters:

$f_x = f/s_x$, the focal length in horizontal pixels
$\alpha = s_y/s_x$ (or $\alpha = f_x/f_y$), aspect ratio
(o_x, o_y), image center coordinates

• Method 1: main steps

(1) Assuming that o_x and o_y are known, estimate all the remaining parameters.

(2) Estimate o_x and o_y

• Step 1: estimate f_x, α, R, and T

- To simplify notation, consider $(x_{im} - o_x, y_{im} - o_y) = (x, y)$

$$x = -f_x \frac{r_{11}X_w + r_{12}Y_w + r_{13}Z_w + T_x}{r_{31}X_w + r_{32}Y_w + r_{33}Z_w + T_z}$$

$$y = -f_y \frac{r_{21}X_w + r_{22}Y_w + r_{23}Z_w + T_y}{r_{31}X_w + r_{32}Y_w + r_{33}Z_w + T_z}$$

- Using the fact that the above two equations have the same denominator, we get the following equation:

$$x\ f_y(r_{21}X_w + r_{22}Y_w + r_{23}Z_w + T_y) = y\ f_x(r_{11}X_w + r_{12}Y_w + r_{13}Z_w + T_x)$$

Problem Statement

Assuming that o_x and o_y are known, compute f_x, α, R, and T from N corresponding pairs of points $(X_i^w, Y_i^w, Z_i^w), (x_i, y_i)$, $i = 1, \ldots, N$.

Derive a system of equations

- Each pair of corresponding points leads to an equation:

$$x_i \ f_y(r_{21}X_i^w + r_{22}Y_i^w + r_{23}Z_i^w + T_y) = y_i \ f_x(r_{11}X_i^w + r_{12}Y_i^w + r_{13}Z_i^w + T_x)$$

- Rewrite the above equation as follows (i.e., divide by f_y):

$$x_i X_i^w v_1 + x_i Y_i^w v_2 + x_i Z_i^w v_3 + x_i v_4 - y_i X_i^w v_5 - y_i Y_i^w v_6 - y_i Z_i^w v_7 - y_i v_8 = 0$$

where

$$\begin{array}{ll} v_1 = r_{21} & v_5 = \alpha r_{11} \\ v_2 = r_{22} & v_6 = \alpha r_{12} \\ v_3 = r_{23} & v_7 = \alpha r_{13} \\ v_4 = T_y & v_8 = \alpha T_x \end{array}$$

- N corresponding points lead to a homogeneous system of N equations with 8 unknowns:

$$Av = 0 \quad \text{where:}$$

$$A = \begin{bmatrix} x_1 X_1^w & x_1 Y_1^w & x_1 Z_1^w & x_1 & -y_1 X_1^w & -y_1 Y_1^w & -y_1 Z_1^w & -y_1 \\ x_2 X_2^w & x_2 Y_2^w & x_2 Z_2^w & x_2 & -y_2 X_2^w & -y_2 Y_2^w & -y_2 Z_2^w & -y_2 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ x_N X_N^w & x_N Y_N^w & x_N Z_N^w & x_N & -y_N X_N^w & -y_N Y_N^w & -y_N Z_N^w & -y_N \end{bmatrix}$$

Solving the system

- It can be shown that if $N \geq 7$, then A has rank 7.

- If $A = UDV^T$, we have discussed in class that the system has a nontrivial solution v which is proportional to the column of V corresponding to the smallest singular value of A (i.e., the last column of V which we denote as $\bar{v}$):

$$v = \kappa \bar{v} \ (\kappa \text{ is the scale factor}) \quad \text{or} \quad \bar{v} = \gamma v \ (\gamma = 1/\kappa)$$

- Using the components of v and $\bar{v}$:

$$(\bar{v}_1, \bar{v}_2, \bar{v}_3, \bar{v}_4, \bar{v}_5, \bar{v}_6, \bar{v}_7, \bar{v}_8) = \gamma(r_{21}, r_{22}, r_{23}, T_y, \alpha r_{11}, \alpha r_{12}, \alpha r_{13}, \alpha T_x)$$

Determine α and $|\gamma|$

$$\sqrt{\bar{v}_1^2 + \bar{v}_2^2 + \bar{v}_3^2} = \sqrt{\gamma^2 (r_{21}^2 + r_{22}^2 + r_{23}^2)} = |\gamma|$$

$$(r_{21}^2 + r_{22}^2 + r_{23}^2 = 1)$$

$$\sqrt{\bar{v}_5^2 + \bar{v}_6^2 + \bar{v}_7^2} = \sqrt{\gamma^2 \alpha^2 (r_{11}^2 + r_{12}^2 + r_{13}^2)} = \alpha|\gamma|$$

$$(r_{11}^2 + r_{12}^2 + r_{13}^2 = 1 \text{ and } \alpha > 0)$$

Determine $r_{21}, r_{22}, r_{23}, r_{11}, r_{12}, r_{13}, T_y, T_x$

- We can determine the above parameters, up to an unknown common sign.

$$\begin{array}{ll} r_{21} = 1/|\gamma| \; \bar{v}_1 & r_{11} = 1/\alpha|\gamma| \; \bar{v}_5 \\ r_{22} = 1/|\gamma| \; \bar{v}_2 & r_{12} = 1/\alpha|\gamma| \; \bar{v}_6 \\ r_{23} = 1/|\gamma| \; \bar{v}_3 & r_{13} = 1/\alpha|\gamma| \; \bar{v}_7 \\ T_y = 1/|\gamma| \; \bar{v}_4 & T_x = 1/\alpha|\gamma| \; \bar{v}_8 \end{array}$$

Determine r_{31}, r_{32}, r_{33}

- Can be estimated as the cross product of R_1 and R_2:

$$R_3 = R_1 \; x \; R_2$$

- The sign of R_3 is already fixed (the entries of R_3 remain unchanged if the signs of all the entries of R_1 and R_2 are reversed).

Ensuring the orthogonality of R

- The computation of R does not take into account explicitly the orthogonality constraints.

- The estimate $\hat{R}$ of R cannot be expected to be orthogonal (e.g., $\hat{R}\hat{R}^T = I$).

- We can "enforce" the orthogonality on $\hat{R}$ by using its SVD: $\hat{R} = UDV^T$

- Replace D with I, e.g., $\hat{R}' = UIV^T$ $(\hat{R}'\hat{R}'^T = I)$

Determine the sign of γ

- Consider the following equations again:

$$x = -f/s_x \frac{r_{11}X_w + r_{12}Y_w + r_{13}Z_w + T_x}{r_{31}X_w + r_{32}Y_w + r_{33}Z_w + T_z} = -f/s_x \frac{X_c}{Z_c}$$

$$y = -f/s_y \frac{r_{21}X_w + r_{22}Y_w + r_{23}Z_w + T_y}{r_{31}X_w + r_{32}r_w + r_{33}Z_w + T_z} = -f/s_y \frac{Y_c}{Z_c}$$

- If Z_c>0, then x and $r_{11}X_w + r_{12}Y_w + r_{13}Z_w + T_x$ must have opposite signs (it is sufficient to check the sign for one of the points).

if $x(r_{11}X_w + r_{12}Y_w + r_{13}Z_w + T_x) > 0$, then
  reverse the signs of r_{11}, r_{12}, r_{13}, and T_x
else
  no further action is required

- Similarly, if Z_c>0, then y and $r_{21}X_w + r_{22}Y_w + r_{23}Z_w + T_x$ must have opposite signs (it is sufficient to check the sign for one of the points).

if $y(r_{21}X_w + r_{22}Y_w + r_{23}Z_w + T_y) > 0$, then
  reverse the signs of r_{21}, r_{22}, r_{23}, and T_y
else
  no further action is required

Determine T_z and f_x:

- Consider the equation:

$$x = -f/s_x \frac{r_{11}X_w + r_{12}Y_w + r_{13}Z_w + T_x}{r_{31}X_w + r_{32}Y_w + r_{33}Z_w + T_z}$$

- Let's rewrite it in the form:

$$x(r_{31}X_w + r_{32}Y_w + r_{33}Z_w + T_z) = -f/s_x(r_{11}X_w + r_{12}Y_w + r_{13}Z_w + T_x)$$

- We can obtain T_z and f_x by solving a system of equations like the above, written for N points:

$$A\begin{bmatrix} T_z \\ f_x \end{bmatrix} = b \quad \text{where}$$

$$A = \begin{bmatrix} x_1 & (r_{11}X_1^w + r_{12}Y_1^w + r_{13}Z_1^w + T_x) \\ x_2 & (r_{11}X_2^w + r_{12}Y_2^w + r_{13}Z_2^w + T_x) \\ \cdots & \cdots \\ x_N & (r_{11}X_N^w + r_{12}Y_N^w + r_{13}Z_N^w + T_x) \end{bmatrix} \quad b = \begin{bmatrix} -x_1(r_{31}X_1^w + r_{32}Y_1^w + r_{33}Z_1^w + T_x) \\ -x_2(r_{31}X_2^w + r_{32}Y_2^w + r_{33}Z_2^w + T_x) \\ \cdots \\ -x_N(r_{31}X_N^w + r_{32}Y_N^w + r_{33}Z_N^w + T_x) \end{bmatrix}$$

- Using SVD, the (least-squares) solution is:

$$\begin{bmatrix} T_z \\ f_x \end{bmatrix} = (A^T A)^{-1} A^T b$$

Determine f_y:

- From $f_x = f/s_x$ and $f_y = f/s_y$ we have:

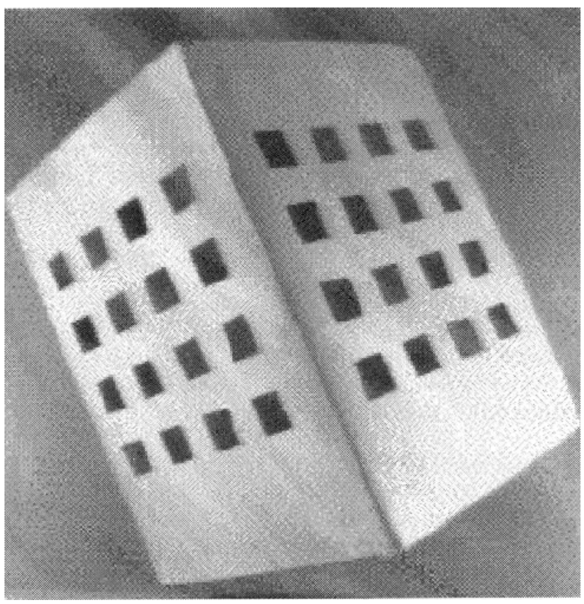
$$f_y = f_x/\alpha$$

• **Step 2: estimate o_x and o_y**

- The computation of o_x and o_y will be based on the following theorem:

Orthocenter Theorem: Let T be the triangle on the image plane defined by the three vanishing points of three mutually orthogonal sets of parallel lines in space. The image center (o_x, o_y) is the orthocenter of T.

- We can use the same calibration pattern to compute three vanishing points (use three pairs of parallel lines defined by the sides of the planes).



Note 1: it is important that the calibration pattern is imaged from a viewpoint guaranteeing that none of the three mutually orthogonal directions will be near parallel to the image plane !

Note 2: to improve the accuracy of the image center computation, it is a good idea to estimate the center using several views of the calibration pattern and average the results.

Method 2: Camera parameters through the projection matrix

• **Review of basic equations**

$$\begin{bmatrix} x_h \\ y_h \\ w \end{bmatrix} = M_{in}\ M_{ex} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} = M \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$x = \frac{x_h}{w} = \frac{m_{11}X_w + m_{12}Y_w + m_{13}Z_w + m_{14}}{m_{31}X_w + m_{32}Y_w + m_{33}Z_w + m_{34}}$$

$$y = \frac{y_h}{w} = \frac{m_{21}X_w + m_{22}Y_w + m_{23}Z_w + m_{24}}{m_{31}X_w + m_{32}Y_w + m_{33}Z_w + m_{34}}$$

(*Note:* I have replaced x_{im} with x and y_{im} with y for simplicity)

• **Step 1: solve for m_{ij}s**

- The matrix M has 11 independent entries (e.g., divide every entry by m_{11}).

- We would need at least N=6 world-image point correspondences to solve for the entries of M.

$$m_{11}X_i^w + m_{12}Y_i^w + m_{13}Z_i^w + m_{14} - m_{31}x_iX_i^w - m_{32}x_iY_i^w - m_{33}x_iZ_i^w + m_{34} = 0$$

$$m_{21}X_i^w + m_{22}Y_i^w + m_{23}Z_i^w + m_{24} - m_{31}y_iX_i^w - m_{32}y_iY_i^w - m_{33}y_iZ_i^w + m_{34} = 0$$

- These equations will lead to a homogeneous system of equations:

$$Am = 0 \quad \text{where}$$

$$A = \begin{bmatrix} X_1^w & Y_1^w & Z_1^w & 1 & 0 & 0 & 0 & 0 & -x_1X_1^w & -x_1Y_1^w & -x_1Z_1^w & -x_1 \\ 0 & 0 & 0 & 0 & X_1^w & Y_1^w & Z_1^w & 1 & -y_1X_1^w & -y_1Y_1^w & -y_1Z_1^w & -y_1 \\ X_2^w & Y_2^w & Z_2^w & 1 & 0 & 0 & 0 & 0 & -x_2X_2^w & -x_2Y_2^w & -x_2Z_2^w & -x_2 \\ 0 & 0 & 0 & 0 & X_2^w & Y_2^w & Z_2^w & 1 & -y_2X_2^w & -y_2Y_2^w & -y_2Z_2^w & -y_2 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ X_N^w & Y_N^w & Z_N^w & 1 & 0 & 0 & 0 & 0 & -x_NX_N^w & -x_NY_N^w & -x_NZ_N^w & -x_N \\ 0 & 0 & 0 & 0 & X_N^w & Y_N^w & Z_N^w & 1 & -y_NX_N^w & -y_NY_N^w & -y_NZ_N^w & -y_N \end{bmatrix}$$

- It can be shown that A has rank 11 (for $N \geq 11$).

- If $A = UDV^T$, the system has a nontrivial solution m which is proportional to the column of V corresponding to the smallest singular value of A (i.e., the last column of V denoted here as $\bar{m}$):

$$m = \kappa \bar{m} \ (\kappa \text{ is the scale factor}) \quad \text{or} \quad \bar{m} = \gamma m \ (\gamma = 1/\kappa)$$

• Step 2: find the intrinsic/extrinsic parameters using m_{ij}s

- The full expression for M is as follows:

$$M = \begin{bmatrix} -f_x r_{11} + o_x r_{31} & -f_x r_{12} + o_x r_{32} & -f_x r_{13} + o_x r_{33} & -f_x T_x + o_x T_z \\ -f_y r_{21} + o_y r_{31} & -f_y r_{22} + o_y r_{32} & -f_y r_{23} + o_y r_{33} & -f_y T_y + o_y T_z \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

- Let's define the following vectors:

$$q_1 = (m_{11}, m_{12}, m_{13})^T$$

$$q_2 = (m_{21}, m_{22}, m_{23})^T$$

$$q_3 = (m_{31}, m_{32}, m_{33})^T$$

$$q_4 = (m_{14}, m_{24}, m_{34})^T$$

- The solutions are as follows (see book for details):

$$o_x = q_1^T q_3 \qquad o_y = q_2^T q_3$$

$$f_x = \sqrt{q_1^T q_1 - o_x^2} \qquad f_y = \sqrt{q_2^T q_2 - o_y^2}$$

- The rest parameters are easily computed ....

<u>Question:</u> how would you estimate the accuracy of a calibration algorithm?

• Some comments

- The precision of calibration depends on how accurately the world and image points are located.

- Studying how localization errors "propagate" to the estimates of the camera parameters is very important.

- Although the two methods described here should produce the same results (at least theoretically), we usually obtain different solutions due to different error propagations.

- Method 2 is simpler and should be preferred if we do not need to compute the intrinsic/extrinsic camera parameters explicitly.