Region Merging

(Jain et al., section 3.4.1, 3.4.2)

- Region merging operations eliminate false boundaries and spurious regions by merging adjacent regions that belong to the same object.

- Merging schemes begin with a partition satisfying condition (4) (e.g., regions produced using thresholding).

$$(4)\ P(R_i) = \text{True}$$

- Then, they proceed to fulfill condition (5) by gradually merging adjacent image regions.

$$(5)\ P(R_i \cup R_j) = \text{False}$$

(1) Form initial regions in the image.

(2) Build a regions adjacency graph (RAG).

(3) For each region do:

(3.1) Consider its adjacent region and test to see if they are similar.

(3.2) For regions that are similar (i.e., $P(R_i \cup R_j)$=True), merge them and modify the RAG.

(4) Repeat step 3 until no regions are merged.

• How to determine region similarity?

(1) Based on the gray values of the regions.

* Compare their mean intensities.

* Use surface fitting to determine whether the regions may be approximated by one surface.

* Use hypothesis testing to judge the similarity of adjacent regions (assumes that the intensity values are drawn from a probability distribution).

(2) Based on the weakness of boundaries between the regions.

• Region merging using hypothesis testing

- This approach considers whether or not to merge adjacent regions based on the probability that they will have the same statistical distribution of intensity values.

- Assume that the gray-level values in an image region are drawn from Gaussian distributions.

$$p(g_i) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(g_i-\mu)^2}{2\sigma^2}}$$

- We can estimate the parameters of the Gaussian using Maximum-Likelihood:

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} g_i, \quad \hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (g_i - \hat{\mu})^2$$

- Given two regions R_1 and R_2 with m_1 and m_2 pixels respectively, there are two possible hypotheses:

H$_0$: Both regions belong to the same object. The intensities are all drawn from a single Gaussian distribution $N(\mu_0, \sigma_0)$:

H$_1$: The regions belong to different objects. The intensities of each region are drawn from separate Gaussian distributions $N(\mu_1, \sigma_1)$ and $N(\mu_2, \sigma_2)$:

- The joint probability density under H_0, assuming all pixels are independently drawn, is given by:

$$p(g_1, g_2, \ldots, g_{m_1+m_2} | H_0) = \prod_{i=1}^{m_1+m_2} p(g_i | H_0) = \frac{1}{(\sqrt{2\pi\sigma_0})^{m_1+m_2}} \, e^{-\frac{(m_1+m_2)}{2}}$$

The joint probability density under H_1 is given by:

$$p(g_1, g_2, \ldots, g_{m_1+m_2} | H_1) = \frac{1}{(\sqrt{2\pi\sigma_1})^{m_1}} \, e^{-\frac{m_1}{2}} \quad \frac{1}{(\sqrt{2\pi\sigma_2})^{m_2}} \, e^{-\frac{m_2}{2}}$$

- The likelihood ratio is defined as the ration of the probability densities under the two hypotheses:

$$L = \frac{p(g_1, g_2, \ldots, g_{m_1+m_2} | H_1)}{p(g_1, g_2, \ldots, g_{m_1+m_2} | H_0)} = \frac{\sigma_0^{m_1+m_2}}{\sigma_1^{m_1} \; \sigma_2^{m_2}}$$

- If the likelihood ratio is below a threshold value, there is strong evidence that there is only one region and the two regions may be merged.

• Region merging by removing weak edges

- The idea is to combine two regions if the boundary between them is weak.

- A weak boundary is one for which the intensities on either side differ by less than some threshold T_1.

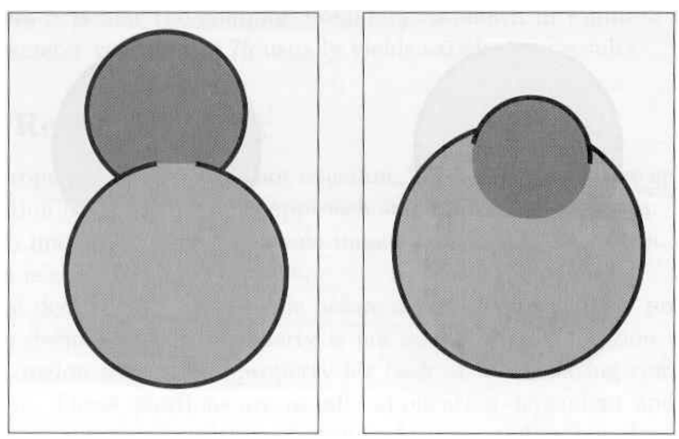
- The relative lengths between the weak boundary and the region boundaries must be also considered.

<u>Approach 1</u>

Merge adjacent regions R_1 and R_2 if

$$\frac{W}{S} > T_2$$

where W is the length of the weak part of the boundary, and $S = min(S_1, S_2)$ is the minimum of the perimeter of the two regions.

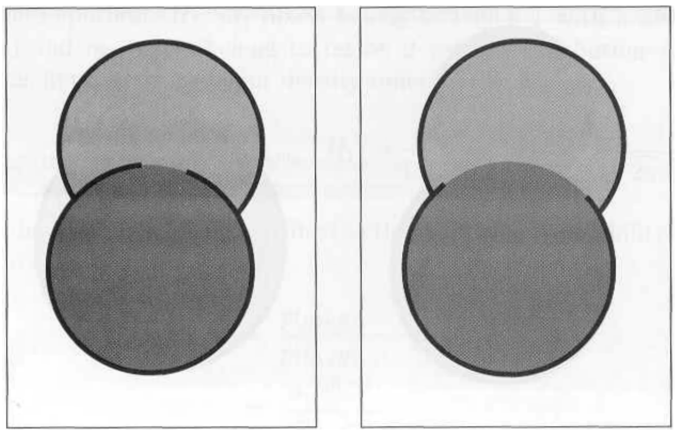


Approach 2

Merge adjacent regions R_1 and R_2 if

$$\frac{W}{S} > T_3$$

where W is the length of the weak part of the boundary, and S is the common boundary between R_1 and R_2.



Region Splitting

(Jain et al., section 3.4.3)

- Region splitting operations add missing boundaries by splitting regions that contain parts of different objects.

- Splitting schemes begin with a partition satisfying condition (5), for example, the whole image.

$$(5)\ P(R_i \cup R_j)\text{=False}$$

- Then, they proceed to satisfy condition (4) by gradually splitting image regions.

$$(4)\ P(R_i) = \text{True}$$

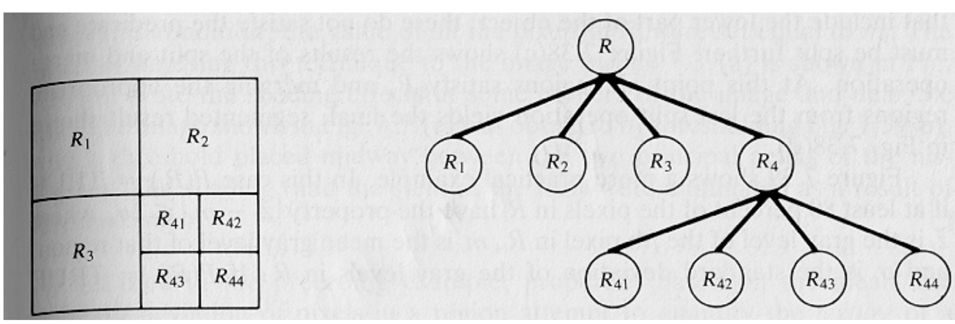
- Two main difficulties in implementing this approach:

  * Deciding when to split a region (e.g., use variance, surface fitting).

  * Deciding how to split a region.

Regular Decomposition

(1) If $P(R)$=False, split R into four quadrants
(2) If P is false on any quadrant, subsplit



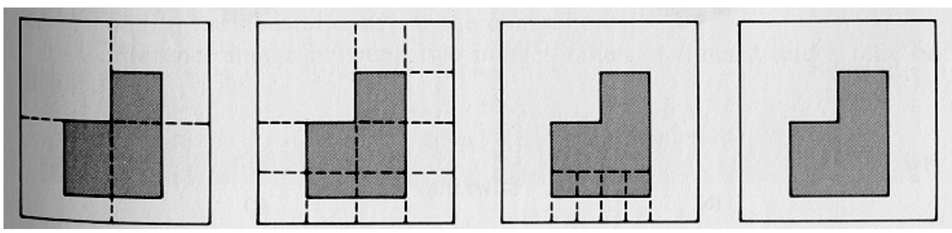
Region splitting and merging

(Jain et al., section 3.4.4)

- Splitting or merging might not produce good results when applied separately.

- Better results can be obtained by interleaving merge and split operations.

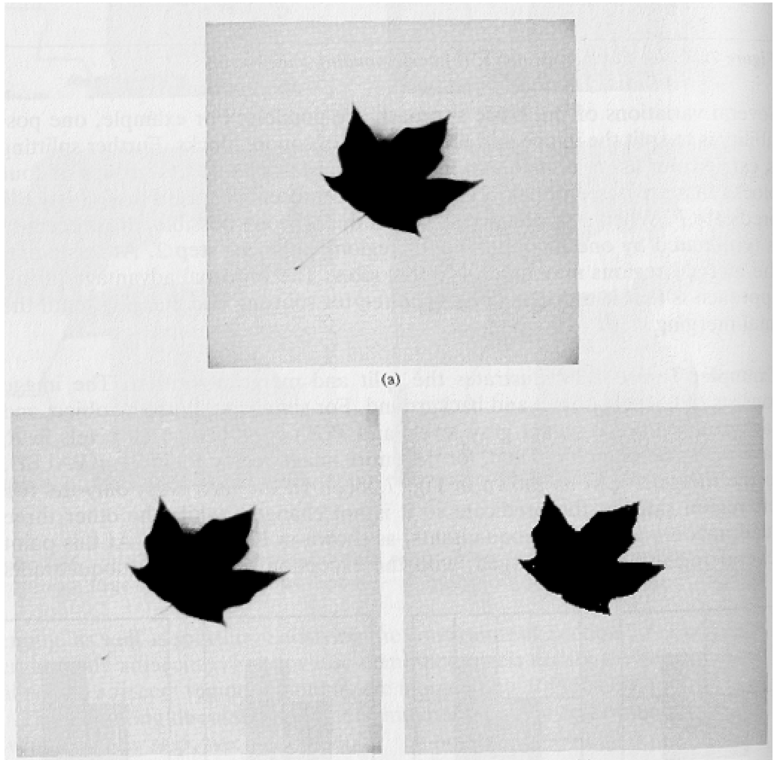
- This strategy takes a partition that possibly satisfies neither condition (4) or (5) with the goal of producing a segmentation that satisfies both conditions.



(1) Split into four disjointed quadrants any region R_i where $P(R_i)$=False

(2) Merge any adjacent regions R_j and R_k for which $P(R_j \cup R_k)$=True;

(3) Stop when no further merging or splitting is possible



split and merge thresholding

$$P(R_i) = True \text{ if}$$

$$|z_i - m_i| \leq 2\sigma_i \text{ for 80\% of the pixels in } R_i$$

$$(m_i, \sigma_i \text{ are the mean and standard deviation of pixels in } R_i)$$