# The reconstruction problem

Both intrinsic and extrinsic parameters are known: we can solve the reconstruction problem unambiguously by triangulation.

Only the intrinsic parameters are known: we can solve the reconstruction problem but only up to an unknown scaling factor.

Neither the extrinsic nor the intrinsic parameters are available: we can solve the reconstruction problem but only up to an unknown, global projective transformation.

# Reconstruction by triangulation

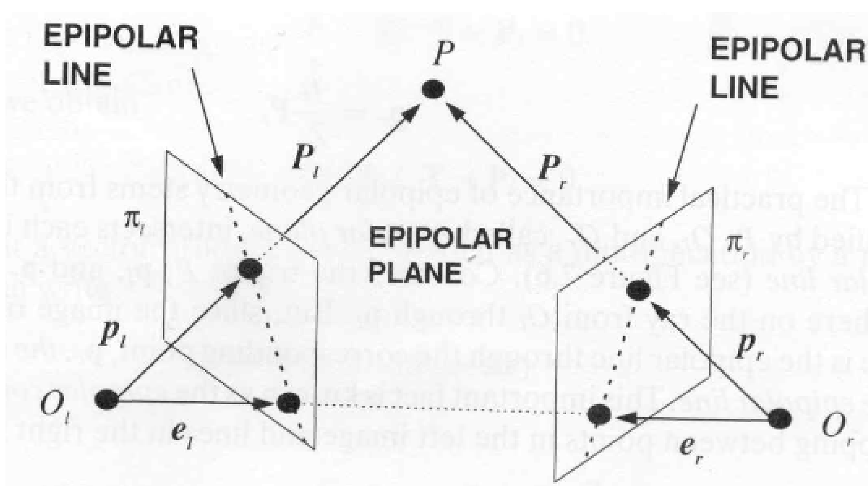
*Assumptions and problem statement*

Both the extrinsic and intrinsic camera parameters are known.

Compute the location of the 3D points from their projections $p_l$ and $p_r$

• **What is the solution?**

- The point $P$ lies at the intersection of the two rays from $O_l$ through $p_l$ and from $O_r$ through $p_r$.

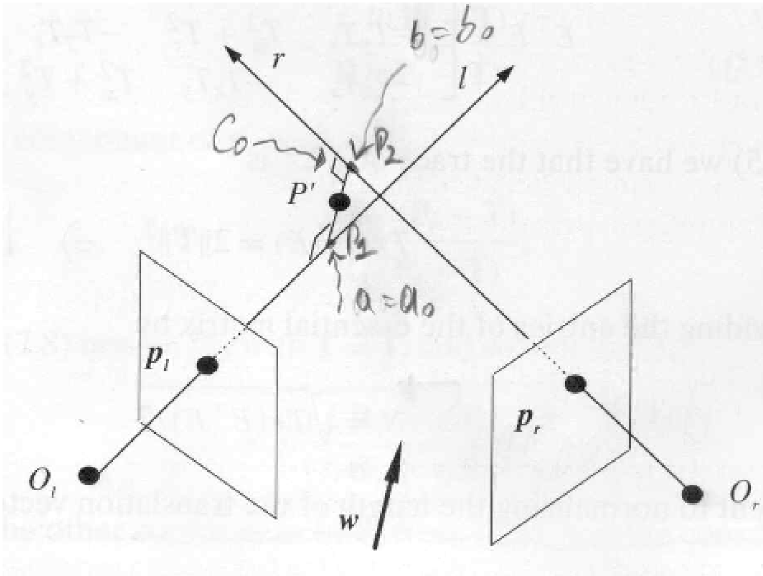


## • Practical difficulties

- The two rays will not intersect exactly in space because of errors in the location of the corresponding features.

- Find the point that is closest to both rays.

- This is the midpoint $P'$ of line segment being perpendicular to both rays.



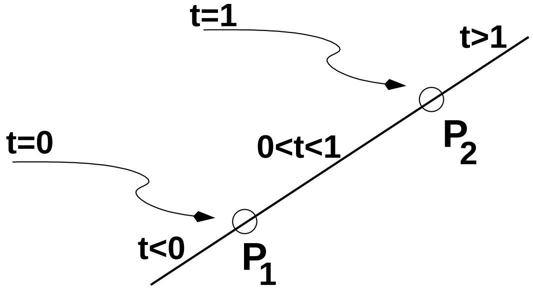

## • Parametric line representation (review)

- The parametric respresentation of the line passing through $P_1$ and $P_2$ is given by:

$$P(t) = P_1 + t(P_2 - P_1) \quad \text{or}$$

$$x(t) = x_1 + t(x_2 - x_1) \quad \text{and} \quad y(t) = y_1 + t(y_2 - y_1)$$

- The direction of the line is given by the vector $t(P_2 - P_1)$



• **Computing** $P'$

- Parametric representation of the line passing through $O_l$ and $p_l$:

$$O_l + a(p_l - O_l) = ap_l$$

- Parametric representation of the line passing through $O_r$ and $p_r$:

$$O_r^L + b(p_r^L - O_r^L) = T + bR^T p_r$$

$$\text{since } O_r^L = R^T O_r^R + T = T \text{ and } p_r^L = R^T p_r^R + T$$

$$(p_r^R = p_r, \quad , p_l^L = p_l)$$

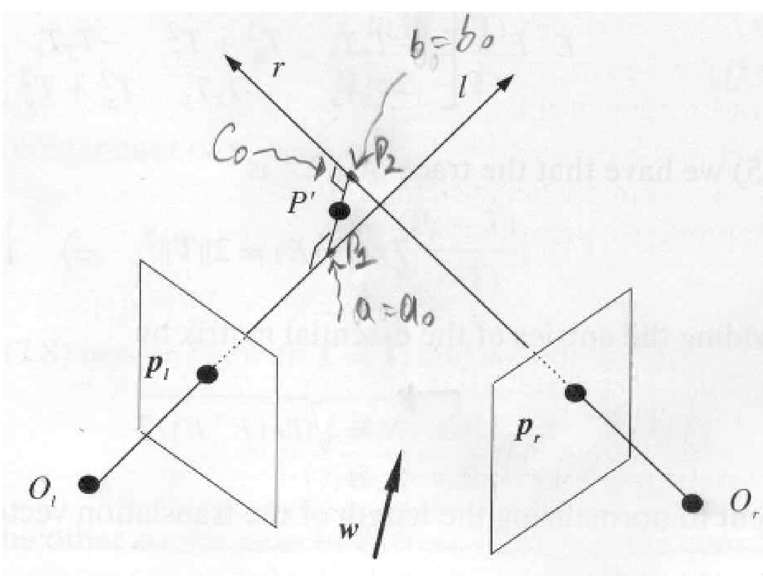
- Suppose the endpoints $P_1$ and $P_2$ are given by

$$P_1 = a_0 p_l \quad \text{and} \quad P_2 = T + b_0 R^T p_r$$

- The parametric equation of the line passing through $P_1$ and $P_2$ is given by:

$$P_1 + c(P_2 - P_1)$$

- The desired point $P'$ (midpoint) is computed for $c = 1/2$



• **Computing $a_0$ and $b_0$**

- Consider the vector $w$ orthogonal to both $l$ and $r$ is given by:

$$w = (p_l - O_l) \text{ x } (p_r^L - O_r^L) = p_l \ x \ R^T p_r$$

$$(\text{since } p_r^R = R(p_r^L - O_r^L))$$

- The line $s$ going through $P_1$ with direction $w$ is given by:

$$a_0 p_l + cw = a_0 p_l + c(p_l \ x \ R^T p_r)$$

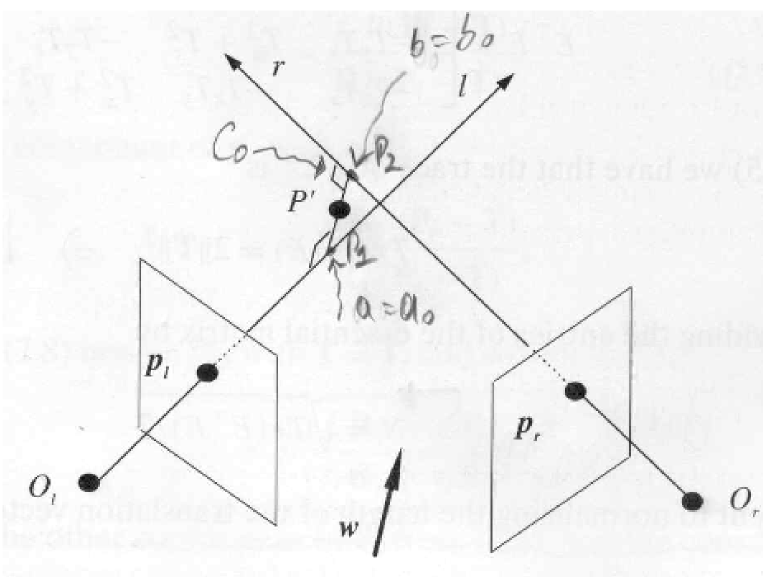
- The lines $s$ and $r$ intersect at $P_2$:

$$a_0 p_l + c_0(p_l \ x \ R^T p_r) = T + b_0 R^T p_r$$

$$(\text{assume } s \text{ passes through } P_2 \text{ for } c = c_0)$$

- We can obtain $a_0$ and $b_0$ by solving the following system of equations:

$$a_0 p_l - b_0 R^T p_r + c_0(p_l \ x \ R^T p_r) = T$$



# Reconstruction up to a scale factor

*Assumptions and problem statement*

| |
|---|
| Only the intrinsic camera parameters are known.<br>We have established $n \geq 8$ correspondences to compute $E$<br>Compute the location of the 3D points from their projections $p_l$ and $p_r$. |

## Comments:

We cannot recover the true scale of the viewed scene since we do not know the baselile T (recall: $Z = f \frac{T}{d}$).

Reconstruction is unique only up to an unknown scaling factor.

This factor can be determined if we know the distance between two points in the scene.

## Estimate $E$

- We can estimate $E$ using the 8-point algorithm.

- The solution is unique up to an unknown scale factor.

## Recover T

$$E^T E = S^T R^T R S = S^T S = \begin{bmatrix} T_y^2 + T_z^2 & -T_x T_y & -T_x T_z \\ -T_y T_x & T_z^2 + T_x^2 & -T_y T_z \\ -T_z T_x & -T_z T_y & T_x^2 + T_y^2 \end{bmatrix}$$

note: $Tr(E^T E) = 2||T||^2$ or $||T|| = \sqrt{Tr(E^T E)/2}$

To simplify the recovery of $T$, let's divide $E$ by $||T||$:

$$\hat{E}^T \hat{E} = \begin{bmatrix} 1 - \hat{T}_x^2 & -\hat{T}_x \hat{T}_y & -\hat{T}_x \hat{T}_z \\ -\hat{T}_y \hat{T}_x & 1 - \hat{T}_y^2 & -\hat{T}_y \hat{T}_z \\ -\hat{T}_z \hat{T}_x & -\hat{T}_z \hat{T}_y & 1 - \hat{T}_z^2 \end{bmatrix} \quad \text{where} \quad \hat{T} = T/||T||$$

(we can compute $\hat{T}$ much easier from $\hat{E}^T \hat{E}$)

## Recover R

It can be shown that

$$R_i = w_i + w_j \; x \; w_k \text{ where } w_i = \hat{E}_i \; x \; \hat{T}$$

## Ambiguity in $(\hat{T}, R)$

There is a twofold ambiguity in the sign of $\hat{E}$ and $\hat{T}$

There will be four different estimates for $(\hat{T}, R)$

## 3D reconstruction

Let's compute the $z$ coordinate of each point in the *left camera* frame:

$$p_l = f_l \frac{P_l}{Z_l} \quad \text{and} \quad p_r = f_r \frac{P_r}{Z_r} = f_r \frac{R(P_l - \hat{T})}{R_3^T(P_l - \hat{T})} \quad (\text{since } P_r = R(P_l - T))$$

The first component $x_r$ of $p_r$ is:

$$x_r = f_r \frac{R_1^T(P_l - \hat{T})}{R_3^T(P_l - \hat{T})}$$

Substiting $P_l = \dfrac{p_l Z_l}{f_l}$ into the above equation we get:

$$Z_l = f_l \frac{(f_r R_1 - x_r R_3)^T \hat{T}}{(f_r R_1 - x_r R_3)^T p_l}$$

Compute $X_l$ and $Y_l$ from $P_l = \dfrac{p_l Z_l}{f_l}$

Compute $(X_r, Y_r, Z_r)$ from $P_r = R(P_l - T)$

*Algorithm*

Input: a set of corresponding points $p_l$ and $p_r$

1. Estimate $E$

2. Recover $\hat{T}$

3. Recover $R$

4. Reconstruct $Z_l$ and $Z_r$

5. If the signs of $Z_l$ and $Z_r$ are:

    5.1 both negative for some point, change the sign of $\hat{T}$ and goto step 4

    5.2 one negative, one positive, for some point, change the sign of each entry of $\hat{E}$ and goto step 3.

    5.3 both positive for all points, exit.

*Comment:* the algorithm should not go through more than 4 iterations (why?)

# Reconstruction up to a projective transformation

- We will not discuss this case (more complicated)

- Look in the book if interested (pp. 166-170)