Wael Abd-Almageed Electrical and Computer Eng. Dept. H University of New Mexico Albuquerque, NM 87131 wamageed@eece.unm.edu

M. Sami Fadali Electrical Engineering Dept. University of Nevada Reno, NV 89557 Fadali@ee.unr.edu George Bebis Computer Science Dept. University of Nevada Reno, NV 89557 bebis@cs.unr.edu

Abstract

In this paper, we introduce a new non-intrusive approach to estimating the eye position during pursuit motion of the eye. We introduce a new characterization for the pursuit eye movement. Our characterization is based on the decomposition of the pursuit eye motion into a deterministic component and a random component. We use a discrete Kalman filter to estimate the random component and calculate the deterministic component. We add the two components to obtain an estimate of the eye position. Simulation results are provided to illustrate the eye position estimation.

Key words: Kalman filter, eye tracking, gaze tracking, pusuit motion

1. Introduction

Eye tracking has long been recognized as an important task in both Computer Vision and Human-Computer Interaction (HCI.) In HCI, for example, determining the focus of attention plays an important role in responding to the user's intensions.

Two main HCI applications that require eye tracking are hands-free cursor control and object movement. In these applications, accurate tracking of eye position is required. An automatic text reader is another application that requires accurate eye position estimation.

In virtual environments, the user should be able to interact with the objects in the environment in natural and easy ways. Several studies have focused on developing interaction techniques using hands [8] with less emphasis on eyebased methods.

Current eye tracking approaches rely heavily on intrusive techniques such as measuring the reflection of some light (usually infrared) that is shone into the eye, measuring the electric potential of the skin around the eyes or applying special contact lenses that facilitate the eye tracking process [1]. User acceptance for any of these intrusive techniques has always been a problem when applying them. Other nonintrusive techniques, such as neural networks [2], have also been proposed but with less success than the intrusive ones.

^{*} This work was funded, in part, by the National Science Foundation (NSF) grant number 0088086 In this paper, we introduce a new non-intrusive approach to tracking eyes undergoing pursuit motion. We use a discrete Kalman filter to track the user's eye position. The main advantages of the proposed approach are: 1) it requires no physical contact with the user, 2) it results in high tracking accuracy, and 3) low computational complexity so that it can be implemented for real-time applications.

This paper is organized as follows. Section 2 provides a short survey of eye tracking techniques. Section 3 presents a brief overview of different types of eye movement. Section 4 is the core part of the paper in which we introduce the proposed approach. In section 5 we discuss eye motion simulation results using our estimation approach. In section 6 we provide conclusions and suggestions for future work.

2. Eye Tracking Approaches

We can broadly classify eye tracking techniques into three major categories according to the way they contact the user [1]. The first category is based on directing a beam of light (typically infrared) into the eye and then measuring the light reflected from the eye. The second category is based on measuring the potential of the skin around the eye. The last category is based on applying special type of contact lenses that facilitate eye tracking.

2.1 Measuring light reflectance techniques

Four tracking techniques use the light reflected from the eye. These techniques are:

1. Limbus tracking (Limbus is the boundary between the white sclera and the dark iris.)

2. Pupil tracking (the boundary between the pupil and the iris is usually used.)

3. Purkinje Images tracking (Purkinje Images are the different reflections from the boundaries of the lens and cornea. See Figure 1.)

4. Corneal Reflections tracking.

2.2 Electric Potential of the Skin

This class of techniques is based on the fact that there exists an electrostatic field that rotates along with the eye. By recording small differences in the skin potential around the eye, the position of the eye can be detected.

2.3 Contact Lenses



Figure 1: Purkinje Images

In this class of techniques, the user wears a special type of contact lenses that allows accurate detection of the direction of the gaze.

As expected, any intrusive technique is not easily accepted by the users and cannot be used in everyday life. For example, intrusive techniques are not suitable for HCI applications. This motivates the development of new nonintrusive techniques that achieve same level of accuracy.

3. Eye Movements

Eye movement is generally classified into four main types [3], based on what the eye is viewing at a specific moment. These four classes are described as follows.

1. Saccadic Eye Movements

Saccades are very fast jumps from one eye position to another. The velocity of saccades can be as high as 800 degrees/second [3]. This kind of eye movement is usually used while following a rapidly moving object or scene.

2. Pursuit Eye Movements

As its name implies, this type of eye movement is used while tracking (or pursuiting) a slowly moving object. It consists of two components: a slowly varying motion component plus a saccadic component. This saccadic component occurs occasionally as a correction mechanism for the eye position [3]. The saccadic component occurs when the eye current eye position is not accurate with respect to the moving object.

3. Fixation Eye Movements

During fixating on a static scene or object, the eye actually undergoes three different types of movements [3]. The first is a continuous high frequency tremor component. The frequency of this component ranges from 30 to 80 cycles per second. The second eye movement component during fixation is a slow drift of the eyeballs in a random direction. The last component is a flickering eye movement that corrects the eye position that changed by the drift component.

4. Nystagmus Eye Movement



Figure 2: Decomposition of Pursuit Eye Motion

This pattern of eye movement occurs when viewing a fast moving repetitive scene (the train window phenomenon.) It consists of pursuit motion in one direction to follow a position in the scene followed by fast motion in the opposite direction to select a new position in the scene.

4. Kalman Filter Tracker

The Kalman filter has been used extensively in Computer Vision research. Most of the applications that utilize Kalman filter focus on the problems of object tracking and structure from motion [7]. To the best of our knowledge, Kalman filter has not been used to track eye movements. In this paper, we apply the Kalman filter to track the pursuit motion of the eye.

As discussed in Section 3, the pursuit eye movement occurs when the eye is tracking a slowly moving object. Figure 2 shows that the pursuit eye movement can be decomposed into two components. One component is a simple position-velocity-acceleration motion and the other one is the saccadic component. This saccadic component could be thought of as a random component that corrects the position of the eye.

The saccadic component has a random time of occurrence, a random duration and a random amplitude. The duration of these saccadic movements typically range from 30 to 120 milliseconds, while their velocity can be as large as 800 degrees/second. This means that, the acceleration of the saccadic component increases rapidly and decreases rapidly causing a fast move and an accurate stop at the required position. We, therefore, model the acceleration of this motion as a Gauss-Markov process [6]. The autocorrelation function of this process is shown in Figure 3 and is given by the equation

$$R(\tau) = \sigma^2 \exp(-\beta|\tau|) \tag{1}$$



Figure 3: Autocorrelation Function of Gauss-Markov Process

The power spectral density of Gauss-Markov process is

$$S_n(s) = \frac{2\sigma^2\beta}{-s^2+\beta^2}$$
(2)

Because the standard Kalman filter formulation requires the noise to be Gaussian white noise, we use a shaping filter. The spectral decomposition for the Gauss-Markov process is

$$S(s) = \frac{\sqrt{2\sigma^2\beta}}{s+\beta} \frac{\sqrt{2\sigma^2\beta}}{-s+\beta}$$
(3)

We characterize the pursuit eye movement by the following state-space equations

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$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ n(t) \end{bmatrix}$$
(4)

$$z = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$
(5)

where x_1 , x_2 and n(t) represents the position, velocity and Gauss-Markov random acceleration component respectively.

We augment the state vector \mathbf{x} with an additional state variable to account for the Gauss-Markov process as follows

$$\begin{bmatrix} \dot{x}_{1} \\ \dot{x}_{2} \\ \dot{x}_{3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\beta \end{bmatrix} \begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$
(6)

$$z = \left[\sqrt{2\sigma^2 \beta} \ 0 \ 0 \right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$
(7)

where u(t) is a unity Gaussian white noise. In the discrete form, the model is

 $x_k = \phi_k + w_k \tag{8}$

$$z_k = H_k x_k + v_k \tag{9}$$

where $\phi = \exp(F\Delta t)$, *w* is the process Gaussian white noise and *v* is the measurement Gaussian white noise.

To determine the covariance matrix Q of the process noise w for the model of equation (8) and (9), we use the Van-Loan procedure [5]. We now have the discrete statespace system model and the noise characterization needed to design a discrete Kalman filter.

5. Results and Discussion

The objective of the proposed technique is to develop a tracker for the pursuit motion of the eye. According to our formulation of the tracking problem, the input to the Kalman filter can be considered as a zero-mean Gaussian white noise.

To test the performance of the proposed tracker, we have simulated the actual (true) position of the eye by generating slowly varying sequences of numbers that represent the actual eye position (e.g. large wavelength sinusoidal signal.) The generated sequences were then corrupted by zeromean Gaussian white noise. The composite signal represents the measured (corrupted) position of the eye. The composite signal is fed into the filter to estimate the original signal, the correct eye position.

Figures 4a-7a show four simulations to the input signal to the filter, the measured eye position. Figures 4b-7b show the filtering results of the four simulation experiments. The figures on the left row show the input signal to the filter, which is composed of the true eye position corrupted by zero-mean Gaussian white noise. The figures on the right row show the filtering results superimposed on the original signal. The filtered signals represent the correct eye position. Table 1 shows the error levels of the corrupting noise and the noise variance in each case of the four experiments. We have implemented the proposed approach using MAT-LAB. The time needed to process 1800 samples was 0.3 seconds (i.e. 0.17 ms/sample) on a 1GHz P-III machine running MS-Windows 2000 operating system. This time is the time required to filter one sample of the corrupted signal to obtain the original one. This time does not include the time required to measure the eye position from the eye image.

Our results show a significant tracking error reduction with the use of the Kalman filter compared to directly using the measurement of the eye position. This error reduction is the main advantage of the proposed approach at a small cost of additional computational time to filter the measurements.

The simulation results shown are for pursuit motion in one direction. However, to implement the tracker in the 2D space (spatial dimensions of images), two independent filters must be used, one filter for each spatial dimension. The two filters can be independently designed assuming that the motion in one direction is statistically independent of the motion in the other direction.

	Noise Variance	Noise-to-Signal %	Estimation Error- to-Signal %	Error Reduction
Experiment 1	1	7.381%	1.619%	78.061%
Experiment 2	4	14.377%	2.637%	81.658%
Experiment 3	1	12.294%	3.22%	73.792%
Experiment 4	4	25.491%	5.439%	78.6631%

Table 1: Experimental Results



Figure 4.a: Experiment 1: Test Sequence Simulating Measured Eye Position



Figure 5.a: Experiment 2: Test Sequence Simulating Measured Eye Position



Figure 6.a: Experiment 3: Test Sequence Simulating Measured Eye Position



Figure 4.b: Experiment 1: Filtering Results Superimposed on True Eye Position



Figure 5.b: Experiment 2: Filtering Results Superimposed on True Eye Position



Figure 6.b: Experiment 3: Filtering Results Superimposed on True Eye Position



Figure 7.a: Experiment 4: Test Sequence Simulating Measured Eye Position

6. Conclusions and Future Work

In this paper, we propose a new non-intrusive approach for tracking the position for an eye undergoing pusuit movement. Our approach is based on the physiological fact that the pursuit motion can be decomposed into a regular position-velocity-acceleration component and a random component that occurs occasionally.

We presented some simulation results based on computer generated simulation data. Our simulation results show that the Kalman filter can be used effectively to estimate the true eye position during pursuit motion. The three main advantages of the proposed approach are: (1) this is a non-intrusive approach which will not be rejected by the users compared to intrusive techniques, (2) the proposed approach significantly reduces the measurement error of the eye position, and (3) the short time required to process the measurements represents a low computational overhead. This processing time does not include the time needed to process the image to measure the eye position. The low computational complexity of the algorithm arises from the scarcity of the model matrices, which simplifies the matrix multiplications and inversions when running the Kalman loop. This low computational complexity characteristic of the proposed approach makes it suitable for real-time applications.

The next step in our research is to test the tracker performance using real image data. To develop a feature extractor that measures the eye position reliably, we need to have a head-mounted camera to eliminate the effect of head and body movements. Hence, we can obtain image sequences for the eye without the superimposed head and body movements. An alternative approach, that does not require the use of the costly camera, is to augment the current model to compensate for the head and body motion. This would allow us to implement the proposed approach without the use of a costly head-mounted camera.



Figure 7.b: Experiment 4: Filtering Results Superimposed on True Eye Position

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