Lecture 06: Coordination

Last update: October 3, 2018
Physical clocks

Problem
Sometimes we simply need the exact time, not just an ordering.

Solution: Universal Coordinated Time (UTC)
- Based on the number of transitions per second of the cesium 133 atom (pretty accurate).
- At present, the real time is taken as the average of some 50 cesium clocks around the world.
- Introduces a leap second from time to time to compensate that days are getting longer.

Note
UTC is broadcast through short-wave radio and satellite. Satellites can give an accuracy of about ±0.5 ms.
Clock synchronization

Precision
The goal is to keep the deviation between two clocks on any two machines within a specified bound, known as the precision $\pi$:

$$\forall t, \forall p, q : |C_p(t) - C_q(t)| \leq \pi$$

with $C_p(t)$ the computed clock time of machine $p$ at UTC time $t$.

Accuracy
In the case of accuracy, we aim to keep the clock bound to a value $\alpha$:

$$\forall t, \forall p : |C_p(t) - t| \leq \alpha$$

Synchronization
- **Internal synchronization**: keep clocks precise
- **External synchronization**: keep clocks accurate
Clock drift

Clock specifications

- A clock comes specified with its maximum clock drift rate $\rho$.
- $F(t)$ denotes oscillator frequency of the hardware clock at time $t$.
- $F$ is the clock’s ideal (constant) frequency $\Rightarrow$ living up to specifications:

$$\forall t : (1 - \rho) \leq \frac{F(t)}{F} \leq (1 + \rho)$$

Observation

By using hardware interrupts we couple a software clock ($C_p(t)$) to the hardware clock, and thus also its clock drift rate:

$$C_p(t) = \frac{1}{F} \int_0^t F(t) dt \Rightarrow \frac{dC_p(t)}{dt} = \frac{F(t)}{F}$$

$$\Rightarrow \forall t : 1 - \rho \leq \frac{dC_p(t)}{dt} \leq 1 + \rho$$

Fast, perfect, slow clocks

- Fast clock: $\frac{dC_p(t)}{dt} > 1$
- Perfect clock: $\frac{dC_p(t)}{dt} = 1$
- Slow clock: $\frac{dC_p(t)}{dt} < 1$
Detecting and adjusting incorrect times

Getting the current time from a time server

Computing the relative offset $\theta$ and delay $\delta$

Assumption: $\delta T_{\text{req}} = T_2 - T_1 \approx T_4 - T_3 = \delta T_{\text{res}}$

$$\theta = T_3 + \frac{(T_2 - T_1) + (T_4 - T_3)}{2} - T_4 = \frac{((T_2 - T_1) + (T_3 - T_4))}{2}$$

$$\delta = \frac{((T_4 - T_1) - (T_3 - T_2))}{2}$$
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$$\delta = \frac{((T_4 - T_1) - (T_3 - T_2))}{2}$$

Network Time Protocol

Collect eight $(\theta, \delta)$ pairs and choose $\theta$ for which associated delay $\delta$ was minimal.
Keeping time without UTC

Principle
Let the time server scan all machines periodically, calculate an average, and inform each machine how it should adjust its time relative to its present time.

Using a time server

The Berkeley algorithm
The Happened-before relationship

**Issue**

What usually matters is not that all processes agree on exactly what time it is, but that they agree on the order in which events occur. Requires a notion of ordering.
The Happened-before relationship

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What usually matters is not that all processes agree on exactly what time it is, but that they agree on the order in which events occur. Requires a notion of ordering.

The happened-before relation
- If $a$ and $b$ are two events in the same process, and $a$ comes before $b$, then $a \rightarrow b$.
- If $a$ is the sending of a message, and $b$ is the receipt of that message, then $a \rightarrow b$.
- If $a \rightarrow b$ and $b \rightarrow c$, then $a \rightarrow c$.

Note
This introduces a partial ordering of events in a system with concurrently operating processes.
Logical clocks

Problem

How do we maintain a global view on the system’s behavior that is consistent with the happened-before relation?

Attach a timestamp $C(e)$ to each event $e$, satisfying the following properties:

1. If $a$ and $b$ are two events in the same process, and $a \rightarrow b$, then we demand that $C(a) < C(b)$.
2. If $a$ corresponds to sending a message $m$, and $b$ to the receipt of that message, then also $C(a) < C(b)$.

Problem

How to attach a timestamp to an event when there’s no global clock

⇒ maintain a consistent set of logical clocks, one per process.
Logical clocks

Problem

How do we maintain a global view on the system’s behavior that is consistent with the happened-before relation?

Attach a timestamp $C(e)$ to each event $e$, satisfying the following properties:

**P1** If $a$ and $b$ are two events in the same process, and $a \rightarrow b$, then we demand that $C(a) < C(b)$.

**P2** If $a$ corresponds to sending a message $m$, and $b$ to the receipt of that message, then also $C(a) < C(b)$. 
Logical clocks

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Problem

How to attach a timestamp to an event when there’s no global clock ⇒ maintain a consistent set of logical clocks, one per process.
Logical clocks: solution

Each process $P_i$ maintains a local counter $C_i$ and adjusts this counter

1. For each new event that takes place within $P_i$, $C_i$ is incremented by 1.
2. Each time a message $m$ is sent by process $P_i$, the message receives a timestamp $ts(m) = C_i$.
3. Whenever a message $m$ is received by a process $P_j$, $P_j$ adjusts its local counter $C_j$ to $\max\{C_j, ts(m)\}$; then executes step 1 before passing $m$ to the application.

Notes

- Property P1 is satisfied by (1); Property P2 by (2) and (3).
- It can still occur that two events happen at the same time. Avoid this by breaking ties through process IDs.
Logical clocks: example

Consider three processes with **event counters** operating at different rates.

- **Process P_1**:
  - **Events**: 0, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60
  - **Clocks**: 0, 1, 2, 3, 4

- **Process P_2**:
  - **Events**: 0, 6, 12, 18, 24, 30, 36, 42, 48, 56, 64
  - **Clocks**: 0, 1, 2, 3, 4

- **Process P_3**:
  - **Events**: 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100
  - **Clocks**: 0, 1, 2, 3, 4

**Example**:
- **P_1** adjusts its clock to 6.
- **P_2** adjusts its clock to 16.
- **P_3** adjusts its clock to 61.

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**Diagram**

- **Arrow m_1** from P_1 to P_2.
- **Arrow m_2** from P_2 to P_3.
- **Arrow m_3** from P_3 to P_1.
- **Arrow m_4** from P_1 to P_3.

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**Legend**

- **P_1**: First process.
- **P_2**: Second process.
- **P_3**: Third process.
- **m_1, m_2, m_3, m_4**: Time events.
Logical clocks: where implemented

Adjustments implemented in middleware

1. Application layer
   - Application sends message
   - Adjust local clock and timestamp message

2. Middleware layer
   - Middleware sends message
   - Adjust local clock
   - Message is received

3. Network layer
   - Message is delivered to application
Example: Total-ordered multicast

Concurrent updates on a replicated database are seen in the same order everywhere

- $P_1$ adds $100$ to an account (initial value: $1000$)
- $P_2$ increments account by 1%
- There are two replicas

Result

In absence of proper synchronization:
replica #1 ← $1111$, while replica #2 ← $1110$. 
Solution

- Process $P_i$ sends **timestamped message** $m_i$ to all others. The message itself is put in a local queue $queue_i$.
- Any incoming message at $P_j$ is queued in $queue_j$, **according to its timestamp**, and **acknowledged** to every other process.

Note: We are assuming that communication is reliable and FIFO ordered.
Example: Total-ordered multicast

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$P_j$ passes a message $m_i$ to its application if:

1. $m_i$ is at the head of $queue_j$
2. for each process $P_k$, there is a message $m_k$ in $queue_j$ with a larger timestamp.
Example: Total-ordered multicast

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We are assuming that communication is reliable and FIFO ordered.
Vector clocks

Observation
Lamport’s clocks do not guarantee that if $C(a) < C(b)$ that $a$ causally preceded $b$.

Concurrent message transmission using logical clocks

Observation
Event $a$: $m_1$ is received at $T = 16$; Event $b$: $m_2$ is sent at $T = 20$. 
Vector clocks

Observation

Lamport’s clocks do not guarantee that if $C(a) < C(b)$ that $a$ \underline{causally preceded} $b$.

Concurrent message transmission using logical clocks

- Event $a$: $m_1$ is received at $T = 16$
- Event $b$: $m_2$ is sent at $T = 20$

Note

We \underline{cannot} conclude that $a$ causally precedes $b$. 
Causal dependency

Definition

We say that $b$ may causally depend on $a$ if $ts(a) < ts(b)$, with:

- for all $k$, $ts(a)[k] \leq ts(b)[k]$ and
- there exists at least one index $k'$ for which $ts(a)[k'] < ts(b)[k']$

Precedence vs. dependency

- We say that $a$ causally precedes $b$.
- $b$ may causally depend on $a$, as there may be information from $a$ that is propagated into $b$. 
Capturing causality

Solution: each \( P_i \) maintains a vector \( VC_i \)

- \( VC_i[i] \) is the local logical clock at process \( P_i \).
- If \( VC_i[j] = k \) then \( P_i \) knows that \( k \) events have occurred at \( P_j \).

Maintaining vector clocks

1. Before executing an event \( P_i \) executes \( VC_i[i] \leftarrow VC_i[i] + 1 \).
2. When process \( P_i \) sends a message \( m \) to \( P_j \), it sets \( m \)’s (vector) timestamp \( ts(m) \) equal to \( VC_i \) after having executed step 1.
3. Upon the receipt of a message \( m \), process \( P_j \) sets \( VC_j[k] \leftarrow \max\{ VC_j[k], ts(m)[k] \} \) for each \( k \), after which it executes step 1 and then delivers the message to the application.
Vector clocks: Example

Capturing potential causality when exchanging messages

Analysis

<table>
<thead>
<tr>
<th>Situation</th>
<th>$ts(m_2)$</th>
<th>$ts(m_4)$</th>
<th>$ts(m_2) \overset{&lt;}{\prec} ts(m_4)$</th>
<th>$ts(m_2) \overset{&gt;}{\prec} ts(m_4)$</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>(2, 1, 0)</td>
<td>(4, 3, 0)</td>
<td>Yes</td>
<td>No</td>
<td>$m_2$ may causally precede $m_4$</td>
</tr>
<tr>
<td>(b)</td>
<td>(4, 1, 0)</td>
<td>(2, 3, 0)</td>
<td>No</td>
<td>No</td>
<td>$m_2$ and $m_4$ may conflict</td>
</tr>
</tbody>
</table>
Observation

We can now ensure that a message is delivered only if all causally preceding messages have already been delivered.

Adjustment

$P_i$ increments $VC_i[i]$ only when sending a message, and $P_j$ “adjusts” $VC_j$ when receiving a message (i.e., effectively does not change $VC_j[j]$).
Causally ordered multicasting

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Adjustment

$P_i$ increments $VC_i[i]$ only when sending a message, and $P_j$ “adjusts” $VC_j$ when receiving a message (i.e., effectively does not change $VC_j[j]$).

$P_j$ postpones delivery of $m$ until:

1. $ts(m)[i] = VC_j[i] + 1$
2. $ts(m)[k] \leq VC_j[k]$ for all $k \neq i$
Causally ordered multicasting

Enforcing causal communication
Mutual exclusion

Problem
A number of processes in a distributed system want exclusive access to some resource.

Basic solutions
Permission-based: A process wanting to enter its critical section, or access a resource, needs permission from other processes.

Token-based: A token is passed between processes. The one who has the token may proceed in its critical section, or pass it on when not interested.
Permission-based, centralized

Simply use a coordinator

(a) Process $P_1$ asks the coordinator for permission to access a shared resource. Permission is granted.

(b) Process $P_2$ then asks permission to access the same resource. The coordinator does not reply.

(c) When $P_1$ releases the resource, it tells the coordinator, which then replies to $P_2$. 
The same as Lamport except that acknowledgments are not sent

Return a response to a request only when:

- The receiving process has no interest in the shared resource; or
- The receiving process is waiting for the resource, but has lower priority (known through comparison of timestamps).

In all other cases, reply is deferred, implying some more local administration.
Mutual exclusion Ricart & Agrawala

Example with three processes

(a) Two processes want to access a shared resource at the same moment.
(b) \(P_0\) has the lowest timestamp, so it wins.
(c) When process \(P_0\) is done, it sends an OK also, so \(P_2\) can now go ahead.
Mutual exclusion: Token ring algorithm

Essence

Organize processes in a logical ring, and let a token be passed between them. The one that holds the token is allowed to enter the critical region (if it wants to).

An overlay network constructed as a logical ring with a circulating token
Decentralized mutual exclusion

**Principle**
Assume every resource is replicated $N$ times, with each replica having its own coordinator $\Rightarrow$ access requires a majority vote from $m > N/2$ coordinators. A coordinator always responds immediately to a request.

**Assumption**
When a coordinator crashes, it will recover quickly, but will have forgotten about permissions it had granted.
Decentralized mutual exclusion

How robust is this system?

- Let $p = \Delta t / T$ be the probability that a coordinator resets during a time interval $\Delta t$, while having a lifetime of $T$. 

- The probability $P[k]$ that $k$ out of $m$ coordinators reset during the same interval is $P[k] = \left(\begin{array}{c} m \\ k \end{array}\right) p^k (1-p)^{m-k}$.

- If the number of coordinators that reset is more than the majority of non-faulty coordinators, i.e., $m - f \leq N/2$, or $f \geq m - N/2$, then the correctness is violated.
Decentralized mutual exclusion

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  $$P[k] = \binom{m}{k} p^k (1 - p)^{m-k}$$

- $f$ coordinators reset $\Rightarrow$ correctness is violated when there is only a minority of nonfaulty coordinators: when $m - f \leq N/2$, or, $f \geq m - N/2$. 
Decentralized mutual exclusion

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- The probability of a violation is $\sum_{k=m-N/2}^{N} P[k]$. 
Decentralized mutual exclusion

Violation probabilities for various parameter values

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<th>N</th>
<th>m</th>
<th>p</th>
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<tr>
<td>8</td>
<td>5</td>
<td>3 sec/hour</td>
<td>$&lt; 10^{-15}$</td>
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<td>24</td>
<td>3 sec/hour</td>
<td>$&lt; 10^{-73}$</td>
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### Decentralized mutual exclusion

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So....

What can we conclude?
Mutual exclusion: comparison

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Messages per entry/exit</th>
<th>Delay before entry (in message times)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centralized</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Distributed</td>
<td>$2 \cdot (N - 1)$</td>
<td>$2 \cdot (N - 1)$</td>
</tr>
<tr>
<td>Token ring</td>
<td>$1, \ldots, \infty$</td>
<td>$0, \ldots, N - 1$</td>
</tr>
<tr>
<td>Decentralized</td>
<td>$2 \cdot m \cdot k + m, k = 1, 2, \ldots$</td>
<td>$2 \cdot m \cdot k$</td>
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Distributed vs. Decentralized
- Exclusively composite
- Replicas
Election algorithms

Principle
An algorithm requires that some process acts as a coordinator. The question is how to select this special process dynamically.

Note
In many systems the coordinator is chosen by hand (e.g. file servers). This leads to centralized solutions ⇒ single point of failure.
Election algorithms

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Teasers
1. If a coordinator is chosen dynamically, to what extent can we speak about a centralized or distributed solution?
2. Is a fully distributed solution, i.e. one without a coordinator, always more robust than any centralized/coordinated solution?
Basic assumptions

- All processes have unique id’s
- All processes know id’s of all processes in the system (but not if they are up or down)
- Election means identifying the process with the highest id that is up
Election by bullying

Principle

Consider $N$ processes $\{P_0, \ldots, P_{N-1}\}$ and let $id(P_k) = k$. When a process $P_k$ notices that the coordinator is no longer responding to requests, it initiates an election:

1. $P_k$ sends an *ELECTION* message to all processes with higher identifiers: $P_{k+1}, P_{k+2}, \ldots, P_{N-1}$.
2. If no one responds, $P_k$ wins the election and becomes coordinator.
3. If one of the higher-ups answers, it takes over and $P_k$'s job is done.
Election by bullying

The bully election algorithm

Diagram showing the bully election algorithm with nodes labeled 0 to 7 and arrows indicating the election process.
Election in a ring

Principle

Process priority is obtained by organizing processes into a (logical) ring. Process with the highest priority should be elected as coordinator.

- Any process can start an election by sending an election message to its successor. If a successor is down, the message is passed on to the next successor.

- If a message is passed on, the sender adds itself to the list. When it gets back to the initiator, everyone had a chance to make its presence known.

- The initiator sends a coordinator message around the ring containing a list of all living processes. The one with the highest priority is elected as coordinator.
Election in a ring

Election algorithm using a ring

- The solid line shows the election messages initiated by $P_6$
- The dashed one the messages by $P_3$
Positioning nodes

Issue

In large-scale distributed systems in which nodes are dispersed across a wide-area network, we often need to take some notion of proximity or distance into account ⇒ it starts with determining a (relative) location of a node.
Observation

A node $P$ needs $d + 1$ landmarks to compute its own position in a $d$-dimensional space. Consider two-dimensional case.

Computing a position in 2D

$P$ needs to solve three equations in two unknowns $(x_P, y_P)$:

$$d_i = \sqrt{(x_i - x_P)^2 + (y_i - y_P)^2}$$
Global positioning system

Assuming that the clocks of the satellites are accurate and synchronized

- It takes a while before a signal reaches the receiver
- The receiver’s clock is definitely out of sync with the satellite

Basics

Observation

4 satellites $\Rightarrow$ 4 equations in 4 unknowns (with $\Delta r$ as one of them)
Global positioning system

Assuming that the clocks of the satellites are accurate and synchronized

- It takes a while before a signal reaches the receiver
- The receiver’s clock is definitely out of sync with the satellite

Basics

- $\Delta_r$: unknown deviation of the receiver’s clock.

Observation

4 satellites $\Rightarrow$ 4 equations in 4 unknowns (with $\Delta_r$ as one of them)
Global positioning system

Assuming that the clocks of the satellites are accurate and synchronized

- It takes a while before a signal reaches the receiver
- The receiver’s clock is definitely out of sync with the satellite

Basics

- $\Delta_r$: unknown deviation of the receiver’s clock.
- $x_r, y_r, z_r$: unknown coordinates of the receiver.

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- Measured distance to satellite $i$: $c \times \Delta_i$ ($c$ is speed of light)

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Basics

- $\Delta_r$: unknown deviation of the receiver’s clock.
- $x_r, y_r, z_r$: unknown coordinates of the receiver.
- $T_i$: timestamp on a message from satellite $i$
- $\Delta_i = (T_{\text{now}} - T_i) + \Delta_r$: measured delay of the message sent by satellite $i$.
- Measured distance to satellite $i$: $c \times \Delta_i$ (c is speed of light)
- Real distance: $d_i = c\Delta_i - c\Delta_r = \sqrt{(x_i - x_r)^2 + (y_i - y_r)^2 + (z_i - z_r)^2}$

Observation

4 satellites $\Rightarrow$ 4 equations in 4 unknowns (with $\Delta_r$ as one of them)
WiFi-based location services

Basic idea

- Assume we have a database of known access points (APs) with coordinates
- Assume we can estimate distance to an AP
- Then: with 3 detected access points, we can compute a position.

Car driving: locating access points

- Use a WiFi-enabled device along with a GPS receiver, and move through an area while recording observed access points.
- Compute the centroid: assume an access point $AP$ has been detected at $N$ different locations $\{\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_N\}$, with known GPS location.
- Compute location of $AP$ as $\vec{x}_{AP} = \frac{\sum_{i=1}^{N} \vec{x}_i}{N}$.

Problems

- Limited accuracy of each GPS detection point $\vec{x}_i$
- An access point has a nonuniform transmission range
- Number of sampled detection points $N$ may be too low.
Computing position

Problems

- Measured latencies to landmarks fluctuate
- Computed distances will not even be consistent

Inconsistent distances in 1D space

Solution: minimize errors

- Use $N$ special landmark nodes $L_1, \ldots, L_N$.
- Landmarks measure their pairwise latencies $\tilde{d}(L_i, L_j)$
- A central node computes the coordinates for each landmark, minimizing:

\[
\sum_{i=1}^{N} \sum_{j=i+1}^{N} \left( \frac{\tilde{d}(L_i, L_j) - \hat{d}(L_i, L_j)}{\tilde{d}(L_i, L_j)} \right)^2
\]

where $\hat{d}(L_i, L_j)$ is distance after nodes $L_i$ and $L_j$ have been positioned.
Computing position

Choosing the dimension $m$

The hidden parameter is the dimension $m$ with $N > m$. A node $P$ measures its distance to each of the $N$ landmarks and computes its coordinates by minimizing

$$\sum_{i=1}^{N} \left( \frac{\tilde{d}(L_i, P) - \hat{d}(L_i, P)}{\tilde{d}(L_i, P)} \right)^2$$

Observation

Practice shows that $m$ can be as small as 6 or 7 to achieve latency estimations within a factor 2 of the actual value.