CHAPTER I

INTRODUCTION

Minimizing a network's length is one of the oldest optimization problems in mathematics and, consequently, it has been worked on by many of the leading mathematicians in history. In the mid-seventeenth century a simple problem was posed: Find the point P that minimizes the sum of the distances from P to each of three given points in the plane. Solutions to this problem were derived independently by Fermat, Torricelli and Cavalieri. They all deduced that either P is inside the triangle formed by the given points and that the angles at P formed by the lines joining P to the three points are all $120°$, or P is one of the three vertices and the angle at P formed by the lines joining P to the other two points is greater than or equal to $120°$.

In the nineteenth century a mathematician at the University of Berlin, named Jakob Steiner, studied this problem and generalized it to include an arbitrarily large set of points in the plane. This generalization created a star when P was connected to all the given points in the plane, and is a geometric approach to the 2-dimensional center of mass problem.

In 1934 Kössler and Jarník generalized the network minimization problem even further [1]: Given n points in the plane find the shortest possible connected network containing these points. This generalized problem, however, did not become popular until the book, *What is Mathematics*, by Courant and Robbins [2], appeared in 1941. Courant and Robbins linked the name Steiner with this form of the problem proposed by Kössler and Jarník, and it became known as the Steiner Minimal Tree problem. The general solution to this problem allows multiple points to be added, each of which is called a Steiner Point, creating a tree instead of a star.

Much is known about the exact solution to the Steiner Minimal Tree problem. Those who wish to learn about some of the spin-off problems are invited to read the introductory article by Bern and Graham [3], the excellent survey paper on this problem by Hwang and Richards [4], or the recent volume in The Annals of Discrete Mathematics devoted completely to Steiner Tree problems [5]. Some of the basic pieces of information about the

Steiner Minimal Tree problem that can be gleaned from these articles are: (i) the fact that all of the original n points will be of degree 1, 2, or 3, (ii) the Steiner Points are all of degree 3, (iii) any two edges meet at an angle of at least $120°$ in the Steiner Minimal Tree, and (iv) at most $n - 2$ Steiner Points will be added to the network.

This research concentrates on the Steiner Minimal Tree problem, henceforth referred to as the SMT problem. We present several algorithms for calculating Steiner Minimal Trees, including the first parallel algorithm for doing so. Several implementation issues are discussed, some new results are presented, and several ideas for future work are proposed.

In Chapter II we review the first fundamental algorithm for calculating SMT's and we present a simple, effective heuristic which achieves some quite remarkable results. In Chapter III problem decomposition for SMT's is outlined, and a new parallel algorithm for this decomposition is proposed. In Chapter IV we present and discuss the implementation of a parallel algorithm which serves as the basis for the solution to the entire problem. In Chapter V the problem of extracting the correct answer from a list of potential candidates is discussed. Results are presented in Chapter VI, with conclusions and future work following in Chapter VII.