# Virtual Bidder Group Auction Mechanism for Dynamic Spectrum Access

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***Abstract*—In order to fully utilize spectrum, auction-based dynamic spectrum access has become a promising approach which allows unlicensed wireless users to lease unused bands from spectrum license holders. Traditionally, the property of spectrum goes to a unique winner with the highest bid after the auction, and the bidders with low bids would be probably not served. This also may result in spectrum resource wasting when the assigned band is larger than the original request. In fact, because the spectrum is divisible goods, it can be shared among a group of users. In this paper, we propose a novel virtual bidder group (VBG) mechanism in spectrum auction which allows multiple winners to obtain the spectrum item simultaneously. Furthermore, a heuristic phase-optimization algorithm is proposed to reduce the crucial exponential computing time issue of band allocation optimization in double-side bandwidth auction scenario with multiple auctioneers. Simulation results prove that the VBG scheme could significantly improve the realized system data rate and the bandwidth utility, which indicates higher revenue for the spectrum license holder; meanwhile, the proposed phase-optimization algorithm exhibits relative low complexity and could provide a near-optimal performance.**

***Keywords-virtual bidder group (VBG) mechanism; multi-winner; spectrum auction; phase-optimization algorithm***

## I. INTRODUCTION

As the demand for wireless spectrum has been growing rapidly with the deployment of new wireless applications and devices in the recent years, academic and industrial fields have begun to consider more flexible and comprehensive uses of available spectrum. With the development of cognitive radio technologies [1], dynamic spectrum access becomes a promising approach, which allows unlicensed users (secondary users) to lease the licensed bands from spectrum holders (primary operators) for communication purpose, while the primary operators also try to rent out their available spectrum for benefit gain.

Due to the relationship between primary operator and secondary user in spectrum-leasing, the well developed Auction Theory [2], one of the most important models in game theory, can be applied to formulate and analyze the dynamic bandwidth allocation interaction between them [3]. There are existing works to study the auction-based dynamic spectrum access mechanisms. In [4], the author characterized the bandwidth competition among coexisting streaming overlays as a decentralized collection of dynamic auction games, based on which he developed a strategy to achieve efficient multioverlay streaming. Based on the auction method, an algorithm for uplink resource allocation in OFDMA networks presented in [5] was proved to offer near-optimal performance through both theoretical analysis and simulation studies. In [6], the authors proposed two auction mechanisms to deal with the resource allocation for multi-relay asynchronous cooperative networks. An approach comparing the generated revenue between sequential auction and concurrent auction is proposed in [7], and the former mechanism is proved to be better.

Although the schemes in the existing work have enhanced spectrum allocation efficiency, there still exist some critical drawbacks. Firstly, in most current auction-based spectrum allocation, one licensed band (or a package of multiple bands) is sold to a unique bidder. Potentially, the spectrum would not be full-utilized if the quantity of the spectrum goods is greater than the requirement of the winner bidder. Secondly, as the auctioneer prefers to sell the goods to the one with the highest bid, the bidders with relative low bids might not be served for a long time. However, since the spectrum resource is divisible [8], awarding one band to multiple users would be a better choice. Therefore, we present a virtual bidder group (VBG) mechanism in this paper. Several bidders form a virtual group taking part in the auction, and the spectrum item can be shared among the winning group members simultaneously, with interference constraint. In this paper, we adopt the Vickrey' second-price sealed-bid auction, which could obtain the maximal total *social utility* as well as elicit all the participants to report in their true valuation of the goods [9]. However, as the efficient bandwidth allocation is determined by a mixed-integer programming (MILP) problem, computational complexity becomes major concern especially in double-side auction where multiple auctioneers coexist. Thus, we further develop a heuristic phase-optimization algorithm to reduce the exponential calculation time with a near-optimal performance.

The rest of this paper is organized as follows. In Section Ⅱ, the system model and key notions in this paper is introduced. Section Ⅲ explains the proposed VBG spectrum allocation mechanism in single-side auction scenario in details. Then, we further extend VBG mechanism to double-side auction in Section Ⅳ, and develop a heuristic phase-optimization algorithm to reach calculation efficiency. We give the simulation result and analysis in Section Ⅴ. Section Ⅵ concludes the work in this paper.

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## II. AUCTION MODELS

We first consider a cognitive radio network where $N$ secondary users try to lease licensed spectrum from a primary operator to transmit data, then we extend it to a multi-auctioneers case in Section IV, where $M$ primary operators coexist in a network. We model the above scenarios as single-side and double-side auctions respectively, where the auctioneer is the primary operator and the bidder is the secondary user. We assume each primary operator hold one available spectrum item unit for renting, and the secondary user could obtain no more than one item. The property of a spectrum item keeps unchanged during a leasing period. As illustrated in Fig. 1, a leasing period is divided into auction duration and occupation duration separately. In the auction duration, the system launches a new round spectrum auction following a specific allocation and charging rules [2]. Then, depending on the allocation result, the winning bidders transmit their data using the leased band in occupation duration.

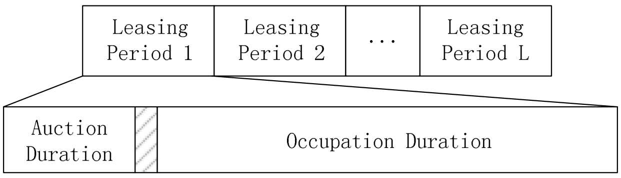


Figure 1. Leasing period construction

At the beginning of auction duration, auctioneer receives sealed bids $\vec{b} = [b_1, b_2, ..., b_N]$ from bidders simultaneously, where $b_i$ is the bid from bidder $i$. According to the bids and the sellers' available band, the auctioneer decides the resource allocation $\vec{x} = [x_1, x_2, ..., x_N]$ and the price $\vec{p} = [p_1, p_2, ..., p_N]$ charging for the bidders, where $x_i$, $p_i$ are the amount of spectral band rent to bidder $i$ and its corresponding payment respectively. We can define the set of winners as $Win \subseteq \{1, 2, ...N\}$, $i \in Win$ if and only if $x_i > 0$. The social utility obtained by the auctioneer in a leasing period is $\sum_{i=1}^{N} p_i = \sum_{i \in Win} p_i$.

In order to define the valuation of an item for a bidder, we consider the utility function for a spectrum item unit as:

$$u_i(R, r) = \frac{1}{1 + \exp(-gR_i - hr_i)}, \quad (1)$$

where $R_i[bps]$ is the generated data rate in bidder $i$, $r_i[bps/Hz]$ stands for the spectrum efficiency determined by user's instantaneous channel condition, according to the relative modulation and coding scheme. $g$ and $h$ are parameters that determine the exact shape of Eq. (1). Note that the above expression produces a higher user utility with a larger amount of data to transmit and a better channel condition.

## III. SINGLE-SIDE SPECTRUM AUCTION

In this section, we first review the traditional single-side auction with Vickrey's second-price sealed-bid mechanism, and analyze its weakness when applied to spectrum auction; then, we propose our novel virtual bidder group (VBG) mechanism to cover its weakness.

### A. Vickrey's second- price sealed-bid mechanism

In Vickrey's second-price sealed-bid auction, every bidder submits their bids for the item. Assume that the bidders' bids of the object are all different and all positive; number the bidders 1 through $N$ in such the following way:

$$b_1 > b_2 > ... > b_N > 0. \quad (2)$$

The winner of this round is the bidder with the highest bid $b_1$. According to the second price mechanism, instead of paying its offered bid, the winner pays the second highest bid among all the bids $p_1 = b_2$ [10]. Charging bidders the *opportunity cost* of the items they win guarantees the *social utility* to be maximal; and reporting one's true valuations for the item is the player's optimal choice, because there is no incentive for him to bid higher or lower than his true valuation for a item. Therefore, the bids they submitted are equal to the utility for this item $(b_1, ..., b_N) = (u_1, \cdots u_N)$.

### B. Weakness in former spectrum auction

In most of the former spectrum auction mechanisms, spectrum resource is treated as indivisible good, whose property is unique after the auction. Thus, the spectrum item unit goes to a single winner in a spectrum leasing period. We set an example to illustrate the weakness for single-winner spectrum auction by applying traditional Vickrey's second-price mechanism.

An example is given in Table I. There are three for-lease spectrum item units in the operator, with different bandwidth specified in the second column. In each leasing period, secondary users submit their frequency bandwidth requirement combined with their bid to operator. Depending on the item's spectral capacity and the second-price mechanism, the allocation and charging result of each period is given in the last column. There are several critical drawbacks in this traditional mechanism. First of all, the spectrum item unit may not be fully utilized. As illustrated, though the spectrum in first period goes to user 1 with the largest bandwidth requirement of $70KHz$, the left $10KHz$ spectrum resource is not used, which could be considered as wasted to some extent. To give a precise evaluation of bandwidth usage, we define the bandwidth utility as:

$$\sum_{i \in Win} w_i \Big/ \sum_{k=1}^{K} W_k, \quad (3)$$

where $W_k$ is the total spectral bandwidth in period $k$. The total utility achieved after three periods of auction is $0.72$, which is obtained from $(70 + 60 + 50)/(80 + 100 + 70)$. Secondly, the bidders with low bids would probably wait for a long time

TABLE I. TRADITIONAL SEQUENTIAL SECOND-PRICE AUCTION

| Lease Period | Band-width (KHz) | Bandwidth Requirement and Bids | | | | Result |
|---|---|---|---|---|---|---|
| | | User 1 | User 2 | User 3 | User 4 | |
| 1 | 80 | 70,350 | 30,150 | 50,250 | 60,300 | [1,300] |
| 2 | 100 | -- | 30,150 | 50,250 | 60,300 | [4,250] |
| 3 | 70 | -- | 30,150 | 50,250 | -- | [3,150] |

before being granted the spectrum unit, as he has to wait all the bidders with higher bids served until he win a spectrum unit.

### *C. Enhanced spectrum auction with VBG mechanism*

In our proposed virtual bidder group (VBG) mechanism, at the auction duration of a leasing period, each secondary user has an estimate of its data rate depending on its historic records and the relative utility for the item: $\{(\vec{R},\vec{u})\}=\{(R_1,u_1),...(R_N,u_N)\}$ which are user's private information. The bidders are no longer necessary to be individual secondary users, but a group of bidders satisfying the constraint condition that their accumulated bandwidth requirement is less or equal to the total bandwidth of the item: $\sum_{i\in H} w_i \le W$ in a specific leasing period, where $H$ stands for the temporary virtual group, the members of which are chosen by operator following the bandwidth constraint described above. Therefore, with exquisite spectrum gap design, the members in the group would not deteriorate the performance of each other.

In VBG mechanism, the auctioneer holds the same purpose as in Vickrey's second-price auction to achieve the maximal social utility $\overline{U^*}\left(\overline{x^*}\right)$, with an *efficient* allocation of goods $\overline{x^*}=[x_1^*,x_2^*,...x_N^*]$. Thus, the *efficient* allocation in VBG is determined by solving the following equation:

$$\begin{aligned}\overline{U^*}\left(\overline{x^*}\right)&=\max\left(\overline{U}\left(\overline{x}\right)\right), \qquad (4)\\ &=\max\left(\sum_{i=1}^{N}p_i(x_i)\right)=\max\left(\sum_{i\in Win}^{N}p_i(w_i)\right)\\ &=\max\left(\sum_{i\in Win}p_i(R_i/r_i)\right),\quad s.t.\sum_{i\in Win}R_i/r_i\le W\end{aligned}$$

where $w_i$ is user $i$ 's spectrum band request calculated by $R_i/r_i$. In this paper, the allocated bandwidth $x_i$ to winning bidder $i$ is equal to its request, i.e. if the operator's available spectrum capacity is less than the bidder's required one, this spectrum item is considered to be useless to that bidder. Clearly, the above linear optimization problem is non-convex since it is in form of *mixed-integer programming* (MILP) problem, which is NP-hard in general [11]. Its constraint conditions reflect the individual bandwidth requirement relationship, which can be solved by traditional exhaustive research algorithm.

After the virtual winning bidder group obtains the spectrum unit, the members in this group divide the unit at the second level. As the total bandwidth demands from secondary users may less than the bandwidth of the item, we divide the surplus band $W^{diff}=W-\sum_{i\in H}w_i$ into $|H|$ (number of bidders in the winning virtual group) different parts $\left[\overline{w}^{diff}\right]=\left[w_1^{diff},...w_{|H|}^{diff}\right]$, where $w_i^{diff}=W^{diff}\frac{w_i}{W}$. Therefore, the final bandwidth a winning bidder obtained can be expressed as: $\tilde{w}_i=w_i+w_i^{diff}$, where $\sum_{i\in H}\tilde{w}_i=W$.

Now, the only unsolved problem is splitting the payment among the secondary users within the winning virtual group. The charging scheme in VBG is a little different from the one in Vickrey's second-price auction. As the bidder could be a virtual bidder group, the price charged to this virtual bidder is $P_H=\overline{U^*_{N/\{H\}}}\left(\overline{w^*}\right)-\overline{U^*_{N/\{H\}}}\left(\overline{w^{-W_H}}^*\right)$, which can be interpreted as the damage this virtual bidder caused to other bidders. Then, at the second level, $P_H$ is afforded among the winning group members in a fair manner: $p_{H,i}=P_H\frac{\tilde{w}_i}{W}$.

## IV. DOUBLE-SIDE SPECTRUM AUCTION

### *A. Bandwidth allocation optimization in double auction*

In this section, we extend our VBG mechanism to a double-side spectrum auction, as it is more practical to consider $M$ primary operators coexist in the same cognitive radio network. In double-side auction, not only the bidders compete to obtain available spectrum, but the auctioneers also try to rent out their band item for monetary gain.

In the double auction, the system also aims to maximize the social utility with efficient allocation, which can be similarly determined by resolving the following optimization problem:

$$\begin{aligned}\overline{U^*}\left(\overline{x^*}\right)&=\max\sum_{j=1}^{M}\sum_{i=1}^{N}p_{i,j}(x_{i,j}). \qquad (5)\\ &=\max\sum_{j=1}^{M}\sum_{i\in Win}p_{i,j}(w_{i,j})=\max\sum_{j=1}^{M}\sum_{i\in Win}p_{i,j}(R_i/r_{i,j})\\ &s.t.\sum_{i\in Win}R_i/r_{i,j}\le w_j,\forall j\in\{1,...,N\}\\ &s.t.\sum_{j=1}^{M}c_{i,j}\le 1,\forall i\in\{Win\}, where\begin{cases}c_{i,j}=0 & if\,x_{i,j}=0\\ c_{i,j}=1 & if\,x_{i,j}>0\end{cases}\end{aligned}$$

### *B. Optimum solution using greedy algorithm*

Eq. (5) could also be solved by exhaustive algorithm. However, the computational complexity reaches $O\left(2^{MN}\right)$, which rises exponentially with the increasing number of primary operators and secondary users. In order to diminish the complexity, we can apply greedy algorithm to divide the global optimization problem into optimal steps, which can bring computation simplicity.

The idea is to simply serve the user one by one, that is, we allocate user 1 with bandwidth $w_{1,j}$, which could generate the maximal social utility from a specific operator $j$. And we eliminate user $1$ from auction in this leasing period and refresh the remaining bandwidth in operator $j$ : $W_j - w_{1,j}$ . Then, serve the second user in the same way to allocate it with proper spectrum, and so on so forth.

Although the greedy algorithm could significantly reduce the complexity to $O(MN)$ , it can only obtain optimal allocation locally, which may not lead to global optimum.

*C. Heuristic solution using phase-optimization algorithm*

Next, we introduce a novel heuristic phase-optimization algorithm, which combines the advantage of greedy algorithm and exhaustive algorithm, to provide a near-optimal bandwidth allocation with low computational complexity.

With the similar idea in greedy algorithm to divide the global optimum issue into optimal structures, we split optimization process into $G$ phases, during each of which $N/G$ users are granted with spectrum item. After that, the served $N/G$ users are eliminated from potential bidder set. Therefore, we could obtain $C_{N'}^{N'/G}$ different kinds of combinations in every phase, based on which we search for the optimal bandwidth allocation among all combinations with exhaustive algorithm. Here, $N'$ is the instantaneous number of potential bidder. The detailed phase-optimization algorithm is described as follows:

**Step1**: find the $C_{N'}^{N'/G}$ combinations among $N'$ users, each with the number of $N/G$ users;

**Step2**: in phase $r$ , calculate the maximal potential social utility obtained in all $C_{N'}^{N'/G}$ combinations $\left\{\tilde{p}_{r,1},...,\tilde{p}_{r,s},...,\tilde{p}_{r,C_{N'}^{N'/G}}\right\}$ using exhaustive algorithm;

**Step3**: search the maximal item $\tilde{p}_{r,\max} = \max\left\{\tilde{p}_{r,1},...,\tilde{p}_{r,C_{N'}^{N'/G}}\right\}$ from potential social utility set;

**Step4**: update the remaining available bandwidth in all operators: $W_j - \sum_{i\in CG_{\max}} w_{i,j}, \forall j \in \{1,...,M\}$ , where $CG_{\max}$ stands for the winning combination group which produce $\tilde{p}_{r,\max}$ ;

**Step5**: eliminate the members in $CG_{\max}$ from potential bidder in this leasing period: $N'' = N' - G$ ;

**Step6**: repeat step 1 to step 5 until all operators' remaining band could not satisfy any users' requirement, or all users are granted required bandwidth.

**Proposition** Phase-optimization algorithm reduces computational complexity of bandwidth allocation optimization to no more than $G\ C_N^{N/G} \cdot O\left(2^{\frac{MN}{G}}\right)$ .

***Proof*** In phase $r$ , there are $C_{N(r)}^{N(r)/G}$ kinds of combinations of potential bidder group, each with $N/G$ users. Therefore, the complexity of exhaustive solution in finding the optimal bandwidth allocation among these $N/G$ users and $M$ operators is $O\left(2^{M\cdot\frac{N}{G}}\right)$ . In this way, the computation complexity accumulated in all phases is expressed as:

$$O\left(C_{N_{(1)}}^{N_{(1)}/G}2^{\frac{MN}{G}} + C_{N_{(2)}}^{N_{(2)}/G}2^{\frac{MN}{G}}+,...,+C_{N_{(G)}}^{N_{(G)}/G}2^{\frac{MN}{G}}\right)$$
$$= O\left[\left(C_{N_{(1)}}^{N_{(1)}/G} + C_{N_{(2)}}^{N_{(2)}/G}+,...,+C_{N_{(G)}}^{N_{(G)}/G}\right)\cdot 2^{\frac{MN}{G}}\right], \quad (6)$$

As $N_{(r)}$ is the instantaneous number of users in phase $r$ ,

$$\text{Eq. (6)} \le O\left[\left(\underbrace{C_N^{N/G} + C_N^{N/G}+,...,+C_N^{N/G}}_{G}\right)\cdot 2^{\frac{MN}{G}}\right] \quad (7)$$
$$= G\ C_N^{N/G}\cdot O\left(2^{\frac{MN}{G}}\right).$$

Phase-optimization algorithm reduces partial exponential complexity to linear parameter. This mitigates computational issue greatly. As we choose a less number of phases $G$ to divide the optimization problem, the closer complexity phase-optimization algorithm is to the global optimum solution; while a larger value $G$ denotes our algorithm equips a more similar characteristic with traditional greedy algorithm.

## V. PERFORMANCE EVALUATION

In this section, we provide numerical results corresponding to VBG mechanism and traditional second price auction, as well as the computation efficiency comparison between different algorithms in optimum solution. In the experimental setup, we consider 25 secondary users, each with generated data rate randomly selected between $50Kbps$ and $300Kbps$ . The secondary users are randomly distributed in a circle uniformly with the diameter $D = 500m$ . We adopt the same definition of spectral efficiency as described in [12] between operator and secondary user:

$$r_i = \log_2\left[1+\frac{P_S}{N_0}\left(\frac{d_i}{D/2}\right)^{-2}\right], \quad (8)$$

where $P_S$ is the signal power, $N_0$ is the AWGN variance, and $d_i$ is the distance between them. We set $P_S = 2N_0$ which guarantees a $SNR = 3dB$ , when a secondary user is at the distance of $d_i = D/2 = 250m$ from the operator.

Fig. 2 compares the achieved total social utility between traditional single-winner mechanism and our proposed VBG mechanism in single-side spectrum auction. We assume the item's bandwidth in primary operator is $2MHz$ . the total social utility generated in VBG mechanism is up to 9, which is about 8 times larger than the one achieved by single-winner mechanism. Furthermore, we illustrate the efficiency of different algorithms in solving spectrum allocation. Exhaustive algorithm generates the highest utility compared to the rest two; however, it leads to the most computation complexity. Contrarily, greedy algorithm can obtain the lowest complexity, but it brings only $2/3$ social utility compared to the former one.

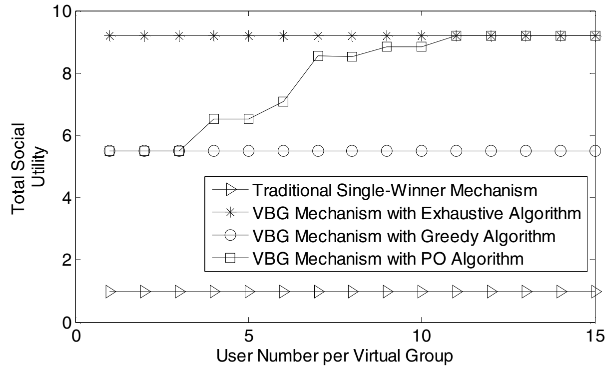


Figure 2. Total social utility comparison between single-winner mechanism and VBG mechanism, also among different algorithms in VBG mechanism.

After the number of virtual group member reaches 7, phase-optimization algorithm could perform nearly as well as exhaustive algorithm, while requires a much lower complexity.

Fig. 3 depicts comparison of system total data rate achieved by single-winner mechanism and VBG mechanism separately in double auction scenario, where multiple primary operators coexist. There are 3 primary operators offering available spectrum item in the auction. With increasing of the spectrum item bandwidth, the total data rate realized by VBG mechanism rises quickly until all secondary users are accepted, while the rate obtained by single-winner mechanism keeps unchanged. That is because only one secondary user with the highest bid could be served in single-winner mechanism. Furthermore, when the spectrum resource is scarce, phase-optimization algorithm could generate larger data rate compared with greedy algorithm, and the system bandwidth utility obtained by former algorithm is about 10% larger than the latter one, as illustrated in Fig.4.

## VI. Conclusion

In this paper, we propose a virtual bidder group (VBG) auction mechanism in dynamic spectrum access. Because the spectrum is divisible resource, multiple secondary users can obtain the same spectrum item simultaneously, which distinguishes it from traditional spectrum auction where there is only one winning bidder. As in double auction scenario, exponential calculation complexity becomes a major concern in optimization problem, we apply a heuristic phase-optimization algorithm to achieve computing simplicity while provide a near-optimal performance. Simulation results show the proposed mechanism achieves a better social utility. Meanwhile, it generates higher system data rate as well as a more full-utilized spectrum, which indicates a larger amount of revenue for spectrum license holder.

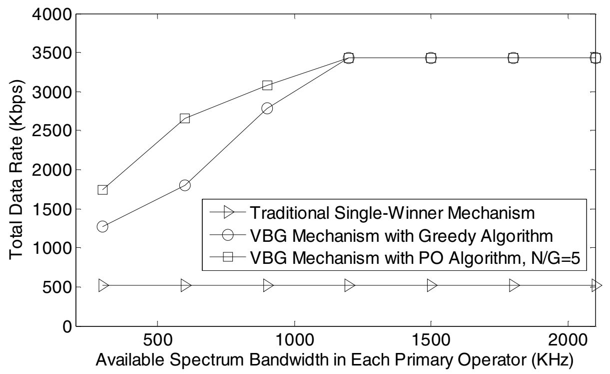


Figure 3. System total data rate comparison based on different size of spectrum item unit in each primary operator.

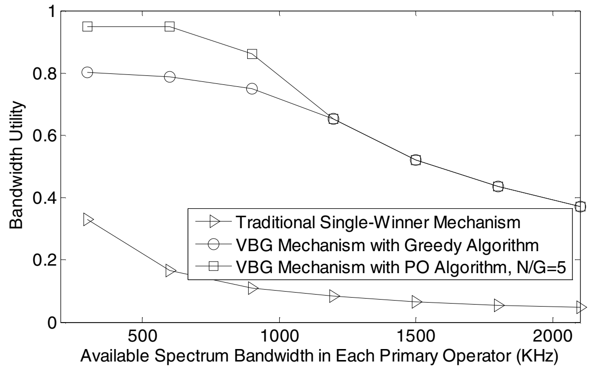


Figure 4. System bandwidth utility comparison based on different size of spectrum item unit in each Primary Operator.

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