CS 477/677 Analysis of Algorithms

Homework 1

Due February 4, 2020

1. (U & G-required) [30 points] Arrange the following list of functions in ascending order of growth rate. That is, if function g(n) immediately follows function f(n) in your list, then f(n) should be O(g(n)).

$f_1(n) = (n-2)!$

$f_2(n) = 5\lg(n+100)^{10}$

$f_3(n) = 2^{2n}$

$f_4(n) = 0.001n^4 + 3n^3 + 1$

$f_5(n) = \ln^2 n$

$f_6(n) = \sqrt[3]{n}$

$f_7(n) = 3^n$

2. (U & G-required) [30 points] Using mathematical induction, show that the following relations are true for every n ≥1:

a) $\sum_{i=1}^{n}(-1)^{i+1} i^2 = \frac{(-1)^{n+1} n(n+1)}{2}$

b) $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$

3. (U & G-required) [20 points] Find the order of growth of the following sums:

a) $\sum_{i=1}^{n}(2i-1)$

b) $\sum_{i=1}^{n} i(i+1)$

4. (U & G-Required) [20 points] For each of the following functions, indicate the class Θ(g(n)) the function belongs to. Use the simplest g(n) possible in your answers.

a) $(n^3 + 3)^{20}$

b) $\sqrt[3]{5n^6 + 2}$

c) $4n^2 lg \frac{(n+1)^2}{2} + n^3$

d) $\frac{8^{n+1}}{2^n}$

5. (G-Required) [20 points] Assume you have functions f and g such that f(n) is O(g(n)). For each of the following statements, decide whether you think it is true or false and give a proof or a counter-example.

a) $2^{f(n)}$ is $O(2^{g(n)})$

b) $f(n)^2$ is $O(g(n)^2)$

6. [Extra credit - 20 points] Assume you have five algorithms with the running times listed below (these are the exact running times). How much slower do each of these algorithms get when you (a) double the input size, or (b) increase the input size by 1?

i) n^2

ii) n^3

iii) $100n^2$

iv) nlgn

v) 2^n