

**CS 477/677 Analysis of Algorithms**

**Spring 2020**

**Homework 5**

**Due date: March 10, 2020**

**1. (U & G-required) [30 points]**

(a) [15 points] Suppose you have a weighted coin in which heads comes up with probability  $\frac{3}{4}$  and tails with probability  $\frac{1}{4}$ . If you flip heads you win \$2, but if you flip tails, you lose \$1. What is the expected value of a coin flip?

(b) [15 points] In a box, I have 20 marbles: 2 red, 3 yellow, 4 blue, 5 green and 6 black. I select one marble at random from the urn, and I win money based on the following rule: red (\$10), yellow (\$5), blue (\$2), green (\$1), black (\$0). What is the expected value of my winnings?

**2. (U & G-required) [20 points]** Why do we want the loop index  $i$  in line 2 of BUILD-MAX-HEAP to decrease from  $\lfloor A.length/2 \rfloor$  to 1 rather than increase from 1 to  $\lfloor A.length/2 \rfloor$ ?

**3. (U & G-required) [30 points]**

(a) [15 points] Exercise 13.1-2 (page 311).

(b) [15 points] Exercise 13.1-3 (page 311).

**4. (U & G-required) [20 points]**

Show that the longest simple path from a node  $x$  in a red-black tree to a descendant leaf has length at most twice that of the shortest simple path from node  $x$  to a descendant leaf.

**5. (G-required) [20 points]**

Suppose you have a bag with 12 slips of paper in it. Some of the slips have a 2 on them and the rest have a 7 on them. If the expected value of the number shown on a slip randomly drawn from the bag is 3.25, then how many slips have a 2?

**Extra credit:**

**6. [20 points]** Show that if all the outcomes of an event are equally likely, then the expected value of the event is simply the average of the values of all of the outcomes.