$$2n+1 \leq 2^n \qquad \forall\, n \geq 3$$

Base case: $n = 3$: $7 \leq 8$ ✓

Inductive case:

Assume: $\boxed{2n+1 \leq 2^n}$

Prove: $2(n+1)+1 \leq 2^{n+1}$

$$2n + 2 + 1 \;\boxed{\leq}\; 2^n + 2 \leq 2^n + 2^n = 2^{n+1}$$

$$\leq 2^n$$

$n! \geq 2^{n-1} \quad \forall n \geq 1$

<u>Base case:</u> $n=1 \quad 1! \geq 2^0$ ✓

<u>Inductive case:</u>

Assume: $n! \geq 2^{n-1}$

Prove : $(n+1)! \geq 2^n$

$$\boxed{(n+1)!} = (n+1) \cdot \underbrace{n!}_{\geq 2^{n-1}} \geq \underbrace{(n+1)}_{\geq 2} \cdot 2^{n-1} \geq$$

$$\geq 2 \cdot 2^{n-1} = \boxed{2^n}$$

$$\sum_{i=1}^{n} (2i-1) = n^2 \quad \forall n \geq 1$$

<u>Base case:</u> $n = 1$ ✓

<u>Inductive case:</u>

Assume: $\sum_{i=1}^{n} (2i-1) = n^2$

Prove: $\sum_{i=1}^{n+1} (2i-1) = (n+1)^2$

$$\text{LHS} = \sum_{i=1}^{n} (2i-1) + \underbrace{2(n+1)-1}_{i=n+1} =$$

$$= n^2 + 2n + 1 = (n+1)^2$$

$$T(n) = c + \underbrace{T\left(\frac{n}{2}\right)}$$

$$= c + T\left(\frac{n}{4}\right)$$

$$= c + c + \underbrace{T\left(\frac{n}{4}\right)}$$

$$= c + T\left(\frac{n}{8}\right)$$

$$= c + c + c + T\left(\frac{n}{8}\right)$$

$$= 3c + T\left(\frac{n}{2^3}\right)$$

.... i times ...

$$= i \cdot c + T\left(\frac{n}{2^i}\right) = c \cdot \log n + T(1)$$

$$\underbrace{n = 2^k} \quad : \quad \frac{2^k}{2^i} = 1 \implies i = k$$

$$\hookrightarrow k = \log n$$

$$T(n) = \Theta(\log n)$$

$$T(n) = n + 2 \cdot \underbrace{T\left(\frac{n}{2}\right)}_{= \frac{n}{2} + 2 \cdot T\left(\frac{n}{4}\right)}$$

$$= n + 2 \cdot \left[\frac{n}{2} + 2 \cdot T\left(\frac{n}{4}\right)\right] =$$

$$= n + n + 4 \cdot \underbrace{T\left(\frac{n}{4}\right)}_{= \frac{n}{4} + 2 \cdot T\left(\frac{n}{8}\right)}$$

$$= n + n + n + 8 \cdot T\left(\frac{n}{8}\right)$$

$$= 3n + 2^3 \cdot T\left(\frac{n}{2^3}\right) = \dots \quad i \text{ times}$$

$$\dots \quad = i \cdot n + 2^i T\left(\frac{n}{2^i}\right)$$

$$n = 2^k \qquad \frac{n}{2^i} = 1 \Rightarrow \boxed{i = \log n}$$

$$T(n) = n \cdot \log n + n \cdot T(1)$$

$$= \Theta(n \log n)$$

$$T(n) = n + T(n-1)$$

$$= n + (n-1) + T(n-2)$$

$$= n + (n-1) + (n-2) + T(n-3)$$

$$+\, n-1$$

$$= n + (n-1) + (n-2) + (n-3) + \dots + 2 + T(1)$$

$$= \frac{n(n+1)}{2} - 1 + T(1)$$

$$= \theta(n^2)$$