

$$E[x] = \sum_{i=1}^{n-1} \sum_{j=i+1}^n \frac{2}{j-i+1} \quad \boxed{j-i=k}$$

$$j = i+1 \Rightarrow k = i+1 - i = 1$$

$$j = n \Rightarrow k = n - i$$

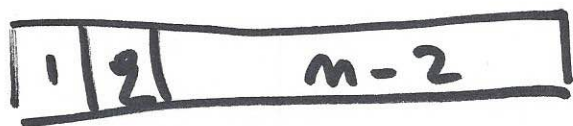
$$E[x] = \sum_{i=1}^{n-1} \sum_{k=1}^{n-i} \frac{2}{k+1} \leq \sum_{i=1}^{n-1} \sum_{k=1}^n \frac{2}{k}$$

$$\leq \sum_{k=1}^n \frac{2}{k+1} \leq \sum_{k=1}^n \frac{2}{k}$$

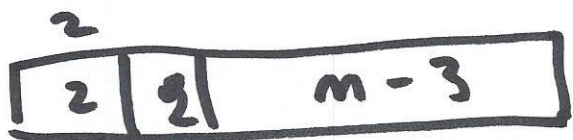
$$E[x] \leq \sum_{i=1}^{n-1} O(\lg n) = \boxed{O(n \lg n)} \quad O(\lg n)$$



$$\max(T(0), T(n-1)) \cdot \frac{1}{n}$$



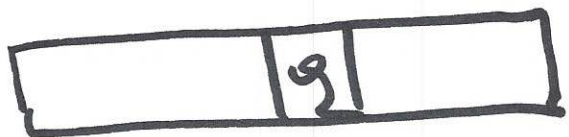
$$\max(T(1), T(n-2)) \cdot \frac{1}{n}$$



$$\max(T(2), T(n-3)) \cdot \frac{1}{n}$$

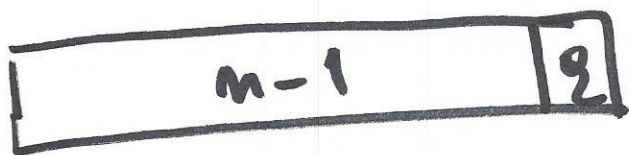
⋮

⋮



$$\max(T(\frac{n}{2}), T(\frac{n}{2})) \cdot \frac{1}{n}$$

⋮



$$\max(T(n-1), T(0)) \cdot \frac{1}{n}$$

$$\sum (\underbrace{\text{outcomes}} \times \underbrace{\text{prob}})$$