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Designing Auction Mechanisms for Dynamic Spectrum Access

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Abstract With the increasing demands for radio spectrum, techniques are being explored that would allow dynamic access of spectrum bands that are under-utilized. In this regard, a new paradigm called dynamic spectrum access is being investigated where wireless service providers (WSPs) would dynamically seek more spectrum from the under-utilized licensed bands when and where they need without interfering with the primary users. Currently, there is little understanding on how such a dynamic allocation will operate so as to make the system feasible under economic terms. In this paper, we consider the dynamic spectrum allocation process where multiple WSPs (bidders) compete to acquire necessary spectrum band from a common pool of spectrum. We use auction theory to analyze the allocation process when the demand from WSPs exceeds the available spectrum. We investigate various auction mechanisms under different spectrum allocation constraints to find WSPs' bidding strategies and revenue generated by spectrum owner. We show that sequential bidding of bands provides better result than the concurrent bidding when WSPs are constrained to at most single unit allocation. On the other hand, when the bidders request for multiple units, (i.e., they are not restricted by allocation constraints) synchronous auction mechanism proves to be beneficial than asynchronous auctions.

Keywords auctions · dynamic spectrum access (DSA) · cognitive radio · winner determination · knapsack

1 Introduction

Privatization of the telecommunications industry coupled with technological advancements and economic liberalization has stimulated competition among wireless service providers (WSPs) and driven down the prices. In addition, the transformation from second generation (2G) mobile telephony to third generation (3G) technologies has also boosted this competition to a great extent resulting in numerous WSPs in one geographic region.

In most countries, the competitive behavior among WSPs was initiated by spectrum auctions held in 2000 and 2001 [4]. These auctions were conducted either by the government or under the supervision of a regulatory body. These regulatory bodies set the rules and regulation which govern the access and use of spectrum. Though the auctions were very successful in some countries (e.g., United Kingdom, Germany), they were open to criticism in others (e.g., Austria, Switzerland, Netherlands) [4]. Through the Federal Communications Commission (FCC), spectrum was auctioned in the United States – the results of which are hotly debated. For example, 824–849 MHz, 1.85–1.91 GHz,

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1.930–1.99 GHz are reserved for licensed cellular and PCS services and require a valid FCC license, whereas 902–928 MHz, 2.40–2.50 GHz, 5.725–5.825 GHz are free-for-all unlicensed bands (http://www.ntia.doc.gov/osmhome/osmhome.html). These spectrum allocations are usually long–term and any changes are made under the strict guidance of FCC.

Recent studies have shown that the spectrum usage is both space and time dependent, and therefore static allocation of spectrum often leads to low spectrum utilization (http://www.sharedspectrum.com/inc/content/measurements/nsf/NYC_report.pdf). In static spectrum allocation, a large part of the radio bands are allocated to the military, government and public safety systems. However, the utilization of these bands are significantly low. One may argue that spectrum allocated to cellular and PCS network operators are highly utilized. But in reality, spectrum utilization even in these networks vary over time and space and undergo under–utilization. Often times, the usage of spectrum in certain networks is lower than anticipated, while there might be a crisis in others. Static allocation of spectrum fails to address the issue of spectrum access even if the service providers (with statically allocated spectrum) are willing to pay for extra amount of spectrum for a short period of time if there is a demand from the users they support.

In order to break away from the inflexibility and inefficiencies of static allocation, a new paradigm called *Dynamic Spectrum Access* (DSA) is being investigated [2]. In DSA, spectrum bands would be allocated and de-allocated dynamically from coordinated access band (CAB) [3]. Examples of such bands include the public safety bands (764–776 MHz, 794–806 MHz) and unused broadcast UHF TV channels (450–470 MHz, 470–512 MHz, 512–698 MHz, 698–806 MHz). Note that, these bands will be dynamically leased to WSPs on a short–term basis in addition to the statically allocated spectrum that all WSPs already have.

Allocating spectrum dynamically among competing WSPs raises one important question 'how would the optimal allocation be achieved'. With the exact value of spectrum unknown to both the seller and the buyers (WSPs), the use of *auctions* is a rational choice since the spectrum bands in the CAB is less than the usual aggregate demand from the WSPs. For every geographic region, auction can be conducted in a periodic manner taking the service providers in that region into account. The service providers are the bidders and the spectrum owner is the seller in this auction. From here onwards, we use the terms service providers and bidders interchangeably. Spectrum would be allocated and de–allocated every auction period, which is also known as the lease duration. At the end of the lease period, all WSPs would release their bands and fresh auction will be again initiated. The service providers buy spectrum from auctioneer and sells the spectrum in form of services to the end users to make additional profits. The demands for spectrum and the revenue generated from the end users become the driving factors for the WSPs to participate in the auction. As WSPs compete for a part of the available spectrum and are willing to pay a price for *that* part only, the kind of auction model needed must be more efficient than the traditional auctions. Moreover, as the bidding behavior is different for different auction mechanisms, it is obvious that the outcome of these auctions will be dependent on specific auction type and need to be studied separately.

In this paper, we investigate spectrum auction mechanisms when multiple units of spectrum are available and the demand from WSPs exceeds the available spectrum. We study various auction mechanisms under different allocation constraints to find WSPs' bidding strategies and revenue generated by the auctioneer. First, we investigate the special case where WSPs (bidders) are granted at most one spectrum chunk from the pool of spectrum chunks in each allocation period. We study both sequential and concurrent auctions, i.e., when bands are auctioned one after another and when all the bids for all the bands are submitted simultaneously. Substitutable and non–substitutable – both types of bands are considered and analyzed in this regard. The novelty of this research is that we focus on calculating the optimal bid (bidder's reservation price) and the revenue generated by auctioneer under the auction setting of *single unit grant* from *multiple unit auction pool*. We show that sequential auction provides better result than concurrent auction. As the more general case, we also consider the spectrum allocation where bidders are not constrained to single unit of spectrum. We devise a "Dynamic spectrum allocator knapsack auction" mechanism with the help of sealed bid, second price auction strategies that is used to determine the winning set of WSPs and dynamically allocate and de-allocate spectrum to the winning set of WSPs. The synchronous and asynchronous allocation policies are investigated and compared in terms of average spectrum allocated, average revenue generated, and probability of winning.

The rest of the paper is organized as follows. In Section 2, we discuss the relevant works that deal with spectrum auctions. Basics of auctions are presented in Section 3. In Section 4, we propose the auction model for single unit grant and analyze sequential and concurrent auctions. In Section 5, auction design for multiple unit grant is proposed and analyzed. Simulation model and results are presented in Section 6. Conclusions are drawn in the last section.

2 Related work

Auction theory has been used to determine the value of a commodity that has an undetermined or variable price. A large number of Internet auction sites have been set up to process both consumer–oriented and business–oriented transactions. Currently, most auction sites (e.g., eBay, http://www.ebay.com/) support a basic bidding strategy through a proxy service for a single-unit auction where ascending bidding continues till a winner emerges. In this type of auction, there is only one item for auction and all the bidders bid for that only item. In such single unit auction, Vickrey proved that "English" and "Dutch" type auctions yield the same expected revenue under the assumptions of risk neutral participants and privately known value drawn from a common distribution [16].

With the emergence of spectrum markets [1, 17], single unit auction models are no longer valid. Multi–unit auctions have been used to investigate pricing policies of network resources (e.g., transmission rate, bandwidth or link capacity) in [5–8, 12, 15] and references therein. The key issue addressed in [5] concerns how the available bandwidth within the network should be shared between competing streams of elastic traffic; the stability and fairness of a class of rate control algorithms are also investigated. The implications of flat pricing and congestion pricing for capacity expansion are studied in [6]. A bandwidth pricing mechanism based on second-price auctions that solves congestion problems in communication networks has been proposed in [7, 8]. A decentralized auction-based approach to price edge-allocated bandwidth in a differentiated services Internet is presented in [12]. Most works done so far on auctions are extensions of Vickrey auction [16] with somewhat strong assumptions. First, the auctions are designed in such a way such that the bidders with higher bids are always favored, e.g., in any classical auction. But favoring higher bidders does not always necessarily maximize the revenue. Moreover, FCC's intention is not only to maximize but also to be fair to the market, where bidders have varying demands. Bidders in these auctions may either look for *single unit* or *bundle of units* from the available pool of multiple units of resources. Second, a major part of the literature assumes the objects in multi-object auctions to have a common value, which may not be true for spectrum auctions. This is because revenue generated from the same spectrum band through the services for end–users may be different for different WSPs due to many factors such as their locations, interference from others etc. Considering such constraints, a spectrum architecture called DIMSUMnet was proposed in [2]. In [9, 10], the authors introduced a DSA scheme in which a spectrum manager periodically auctions short-term spectrum licenses. The spectrum is sold at a unit price, and the assumptions underneath is that a large number of spectrum buyers are present and none has enough power to influence the market clearing price. Spectrum auctioning mechanisms under heterogeneous wireless access networks have been investigated in [11].

3 Auction design and classifications

Good auction design is important for any type of successful auction and often depends on the item being sold. For example, the auctions held in Ebay (http://www.ebay.com/) are typically used to sell an art object or a valuable item. In contrast to Ebay auctions, spectrum auction is similar to the multi–unit auction, where multiple units are up for auction. Multiple bidders present their bids for a part of the spectrum band, where sum of all these requests exceed the total spectrum band capacity thus causing the auction to take place. Moreover, unlike classic single unit auction, multiple winners evolve in this auction model constituting a winner set. The determination of winner set often depends on the auction design strategy taken by the spectrum owner.

There are three important issues behind any auction design. They are (i) attracting bidders (enticing bidders by increasing their probability of winning), (ii) preventing collusion thus preventing bidders to control the auction and (iii) maximizing auctioneer's revenue [13]. It is not at all intended that only big companies with high spectrum demand should acquire entire spectrum. The goal is to increase competition and bring fresh new ideas and services. As a result it is necessary to make the small companies, who also have a demand of spectrum, interested in taking part in the auction.

For spectrum auctions, we assume that there are multiple service providers who are willing to buy more spectrum for short lease periods to serve more end–users and to make more profit. The WSPs determine their spectrum requirements and the price (bids) they are willing to pay. Spectrum is then allocated dynamically by the spectrum owner depending on these bids and the requested amounts of spectrum based on some *winner determination* strategy.

To determine which WSPs must get the requested spectrum, the auctioneer must answer couple of questions. First, what is the objective of the spectrum allocator (FCC for reference)? Apart from maximizing the revenue, the spectrum owner must also be fair in leasing out the unused spectrum bands for the purpose of self-coexistence. The second question that follows is what would be the pricing and market mechanisms? In such auction models, multiple spectrum chunks are available simultaneously and service providers are interested in one or multiple units of these chunks. In Fig. 1, we present the broad classification of auctions based on the spectrum allocation constraints and the part shown within the circle is the focus of this research. Thus unlike classic single unit auction, multiple winners evolve in this auction model constituting a winner set. The determination of winner set often depends on the auction design strategy taken by the spectrum owner. The spectrum band could be substitutable or non-substitutable. By substitutable band we mean a bidder will not care about *which* band(s) he gets as long as all the bandwidths are equal. (We ignore the physical characteristics of signals when they operate at different frequencies.) Non-substitutable bands are the ones with different bandwidths and thus different valuations.

Within multiple units available for spectrum auction category (refer to Fig. 1), we consider both the spectrum allocation mechanisms, (i) bidders are granted at most one spectrum unit from the available pool, (ii) bidders are not constrained and thus are granted multiple units. The use of constrained auction mechanism, where bidders are granted at most one single unit is justified by the newly proposed IEEE 802.22 wireless network spectrum sharing model where IEEE 802.22 devices share the spectrum in the sub-900 MHz. The available spectrum is limited and thus spectrum owner needs to ensure availability of some free spectrum chunks for the incoming requests. For the more generalized scenario, where bidders are granted multiple spectrum bands without any allocation constraint, we only analyze the non-substitutable bands. This is because substitutable bands under multiple units grant is isomorphous to non-substitutable bands under the single unit grant from multiple units for the bidders.

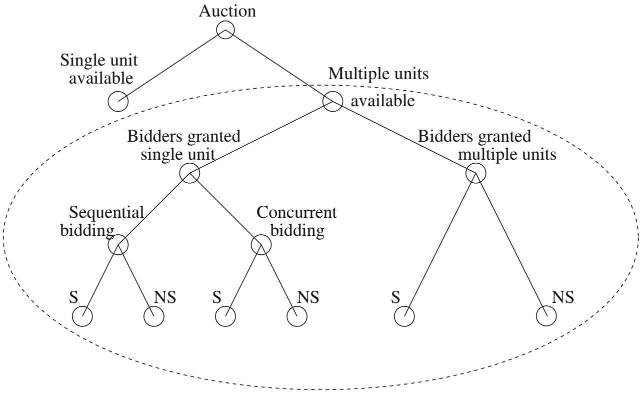


Figure 1 Auction classifications (*S* substitutable, *NS* non-substitutable)

4 Auction design for single unit grant

Let $\mathcal{S} = \{s_1, s_2, \cdots, s_m\}$ be a vector of m substitutable spectrum bands and $\mathcal{N} = \{\mathcal{N}_1, \mathcal{N}_2, \cdots, \mathcal{N}_n\}$ be the n bidders engaged in the auction. For proper auction setting, we assume $n > m$. Without loss of generality, we assume the WSPs to be greedy, i.e., they always try to maximize profit. Let $\mathcal{B} = \{b_1, b_2, \cdots, b_n\}$ denote the n-bid vector from the bidders submitted to the auctioneer where b_i is the bid from the ith bidder. After the auction is completed, winners obtain the lease of the bands for a certain period. The service providers then use the total allocated band (the static band already allocated plus dynamic band won) to provide service to the end users. We follow the sealed-bid auction policy to prevent collusion. We assume all the bidders in the auction to be rational such that losing bidders in any auction round will increase their bids by certain amount in the next round if their bids were less than the true valuation of the bands. Similarly, winning bidder(s) will decrease their bids by certain amount in the next round to increase their payoff(s) till a steady state is reached. At the end of each auction round, the auctioneer only broadcasts the information of minimum bid submitted in that round. Note that, the justification behind not broadcasting any other information (e.g., maximum bid) and only broadcasting minimum submitted bid information in the proposed model is that bidders are only allowed to know the lower bound of the bids. Knowing the lower bound will encourage only the potential bidders (bidders with reservation price higher than or equal to the broadcasted bid information) to participate in the next auction rounds.

The WSPs use the acquired spectrum to provide services to the end users. The revenue generated from the end users gives an indication of the true valuation price of the band. Providers use this valuation price profile to govern their future bidding strategy for forthcoming auction periods. To complicate matters in real-world scenario, the revenue generated even from one particular spectrum band can be different for different service providers depending on company policy and pricing for the end-users. Note that, this assumption does not contradict the definition of substitutable band. (With the substitutable band assumption, one single provider sees no difference between any two bands but two providers can have different revenues from same band.) As a result, the common valuation price for a spectrum band will also be different for different service providers. We present this true valuation price of a substitutable band as a vector, $V = \{V_1, V_2, \cdots, V_n\}$ for n bidders. Later, in Section 4.4, we will reform the valuation price vector for the non-substitutable bands. With the valuation price and bids from a bidder formally defined, payoff of a bidder is given by,

$$\begin{cases} V_i - b_i & \text{if } i\text{th bidder wins} \\ 0 & \text{if } i\text{th bidder loses} \end{cases} \tag{1}$$

We analyze and compare the sequential and concurrent bid mechanisms under the above mentioned auction setting. Auctions among the WSPs occur periodically with the periods being the *DSA* period. In the sequential mechanism, spectrum bands are auctioned one after another in one DSA period and each winning bidder gets at most one spectrum band, i.e., winning bidder is not allowed to participate for the remaining auction rounds in that DSA period. Thus each DSA period consists of m auction rounds with decreasing number of bidders. In contrast to the sequential bid, in concurrent bidding, each DSA period consists of only one auction round. All the bidders submit their bids concurrently at the beginning of each DSA period. When one DSA period expires, all the bands are returned to the auctioneer and the process repeats for both sequential and concurrent bids.

4.1 Sequential auction for substitutable bands

In sequential auction, m spectrum bands are auctioned one after another. First, n bidders submit their sealed bids for band s_1 and the winner is determined. Winner of s_1 does not participate for the rest of the auction in that DSA period. Remaining $(n-1)$ bidders then bid for spectrum band s_2 and so on till all the spectrum bands are auctioned. Let us analyze the properties of sequential auction.

4.1.1 Probability of winning

We assume a time instance when the auction for k spectrum bands are over and k winners have emerged. As a result, there are $(n-k)$ bidders participating for $(m-k)$ spectrum bands. We assume that bids from all the bidders are uniformly distributed. The probability density function of bid submissions in sequential auction mechanism can be given by,

$$f(b) = \frac{1}{V_{max} - b_{min}} \tag{2}$$

where, V_{max} is the maximum valuation possible of a spectrum band and b_{min} is the minimum bid of all the bids submitted by the existing bidders.

Now, let us assume that bidder i submits a bid b_i at the beginning of $(k+1)$th band auction. All the other $(n-k-1)$ bidders also submit their corresponding bids for the $(k+1)$th band. Bidder i will win the $(k+1)$th band if and only if all the $(n-k-1)$ bidders' bids are less than b_i. Let us first find the probability that any other bid b_j, $(j \in (n-k-1)$ bidders) is less than b_i. The probability that any bid $b_j < b_i$, such that, $j \neq i$; $j, i \in (n-k)$ bidders, can be given by

$$P(b_j < b_i \mid j \neq i;\ j \in (n-k-1) \text{ bidders}) = \int_{b_{min}}^{b_i} f(b)\,db \tag{3}$$

Substituting $f(b)$ and integrating, we obtain,

$$P(b_j < b_i \mid j \neq i;\ j \in (n\text{–}k\text{–}1) \text{ bidders}) = \frac{b_i - b_{min}}{V_{max} - b_{min}} \tag{4}$$

If bidder i is to win the $(k+1)$th band, we need to calculate the probability that all the $(n-k-1)$ bidders' bids are lower than the bid b_i. Thus probability of bidder i winning the $(k+1)$th band can be given by,

$$P(\forall\, b_j < b_i \mid j \neq i; \forall j \in (n-k-1) \text{ bidders}) = \prod^{n-k-1} P(b_j < b_i \mid j \neq i;\ j \in (n-k-1) \text{ bidders}) \tag{5}$$

Using Eq. 4 in Eq. 5, we obtain the probability of a bidder winning the $(k+1)$th auction round as

$$P_{seq}(i^{th}\ bidder\ winning) = \left(\frac{b_i - b_{min}}{V_{max} - b_{min}}\right)^{(n-k-1)} \tag{6}$$

4.1.2 Optimal bid analysis

We define *optimal bid* of ith bidder as the bid that wins a band and maximizes the payoff for ith bidder. In other words, optimal bid denotes the reservation bid of a bidder, exceeding which, the bidder is in the risk of obtaining low payoff. If on the other hand, the bid submitted is less than the optimal bid, probability of winning also decreases.

The ith bidder's expected payoff is given by,

$$E_i = (V_i - b_i) \times P(ith\ bidder\ winning) \tag{7}$$

Substituting $P_{seq}(i^{th}\ bidder\ winning)$ from Eq. 6 into Eq. 7, we obtain,

$$E_i = (V_i - b_i)\left(\frac{b_i - b_{min}}{V_{max} - b_{min}}\right)^{(n-k-1)} \tag{8}$$

Let us evaluate bid b_i^* that will maximize E_i. To maximize E_i, we equate the first derivative of E_i to 0, i.e.,

$$\frac{\partial E_i}{\partial b_i} = \frac{(V_i - b_i)(n-k-1)(b_i - b_{min})^{(n-k-2)}}{(V_{max} - b_{min})^{(n-k-1)}} - \frac{(b_i - b_{min})^{(n-k-1)}}{(V_{max} - b_{min})^{(n-k-1)}} = 0 \tag{9}$$

We obtain the optimal bid for ith bidder in $(k+1)$th auction round as

$$b^*_{i_{seq}} = \frac{(n-k-1)V_i + b_{min}}{(n-k)} \tag{10}$$

In our auction formulation, as all the bidders are rational, the natural inclination of the losing bidders would be to increase their bids (if the bids are less than the bidders' true valuation prices). As the auction progresses, b_{min} will be non-decreasing. Thus in the steady state, with increase in auction rounds, $b_{min} \rightarrow V_{min}$, where V_{min} is the minimum true valuation price of the bands.

4.2 Concurrent auction for substitutable bands

In concurrent auction, m spectrum bands are auctioned concurrently where all the n bidders submit their bids together at the beginning of a DSA period. As all the bands are substitutable, each bidder submits just one bid. Each of the highest m bidders win a spectrum band. Let us analyze the properties of concurrent auction here.

4.2.1 Probability of winning

In concurrent auction setting, a bidder's choice would be to be among the highest m bidders and to maximize the payoff profit. The probability of winning would then boil down to the probability of generating a bid such that all the bids from $(n-m)$ losing bidders are below this bid.

The probability of bidder i winning a band in concurrent auction can be given by,

$$P_{con}(i^{th}\ bidder\ winning) = \prod^{n-m} P(b_j < b_i \mid j \neq i;\ j \in (n-m)\ \text{bidders}) \tag{11}$$

As a greedy bidder, the aim of the bidder is not only to win but also to maximize the profit. In other words, the aim is to win with the lowest possible bid.

Simplifying and expanding Eq. 11, we obtain the probability of a bidder winning in concurrent auction with maximized profit as

$$P_{con}(i^{th}\ bidder\ winning) = \left(\frac{b_i - b_{min}}{V_{max} - b_{min}}\right)^{(n-m)} \tag{12}$$

4.2.2 Optimal bid analysis

The expected payoff is given by

$$E_i = (V_i - b_i) \times P_{con}(ith\ bidder\ winning) \tag{13}$$

Substituting $P_{con}(i^{th}\ bidder\ winning)$ from Eq. 12 into Eq. 13, we obtain,

$$E_i = (V_i - b_i)\left(\frac{b_i - b_{min}}{V_{max} - b_{min}}\right)^{(n-m)} \tag{14}$$

To maximize E_i, we take the first derivative of E_i and equate to 0,

$$\frac{\partial E_i}{\partial b_i} = \frac{(V_i - b_i)(n-m)(b_i - b_{min})^{(n-m-1)}}{(V_{max} - b_{min})^{(n-m)}} - \frac{(b_i - b_{min})^{(n-m)}}{(V_{max} - b_{min})^{(n-m)}} = 0 \tag{15}$$

Solving Eq. 15, we obtain the optimal bid for ith bidder in concurrent auction as

$$b^*_{i_{con}} = \frac{(n-m)V_i + b_{min}}{(n-m+1)} \tag{16}$$

This bid is optimal in the sense that this is the minimum bid to maximize the probability of winning a spectrum band and thus maximizes the expected payoff. Next, we present a comparison between optimal bids for both sequential and concurrent auction to study the dominant strategies for bidders.

4.3 Dominant strategy—sequential and concurrent auction

The optimal bids for sequential and concurrent auctions are given in Eqs. 10 and 16 respectively. Let us consider their difference as

$$b_{diff} = b^*_{i_{seq}} - b^*_{i_{con}} \tag{17}$$

We consider two cases. First, under the transient state and second, when steady state has been reached. We define steady state as the state when all the bidders eventually settle down to their corresponding fixed bids and after that bidders will have no extra payoff in unilaterally changing their bids. Transient state is the learning phase where bidders have not reached the steady state and are willing to experiment with their bids. Under the transient state, we again consider two possibilities. One at the beginning of the allocation period (even before the first band auction in sequential setting: all m bands remaining) and the other after k spectrum bands auctions are over.

Transient state – No bands auctioned so far: The difference in optimal bids $b^*_{i_{seq}}$ and $b^*_{i_{con}}$ is

$$b_{diff} = \frac{(n-1)V_i + b_{min}}{n} - \frac{(n-m)V_i + b_{min}}{(n-m+1)} \tag{18}$$

Simplifying we obtain,

$$b_{diff} = \frac{(m-1)(V_i - b_{min})}{n(n-m+1)} \tag{19}$$

We know that for a bidder to win a spectrum band, the following conditions must be true.

$$V_i \geq b^*_{i_{seq}} > b_{min} \text{ and } V_i \geq b^*_{i_{con}} > b_{min} \tag{20}$$

From conditions presented in Eq. 20 and for $m > 1$, we can conclude that b_{diff} in Eq. 19 is a positive quantity ($b_{diff} > 0$). This establishes the fact that optimal bid (reservation price of the bidder) to win in sequential auction setting is more than that in concurrent auction. It is also clear from Eq. 19 that with increase in the number of available bands, m, while keeping n fixed, b_{diff} increases, i.e., the difference between reservation prices in sequential auction and concurrent auction increases. Thus increasing available spectrum bands for auction, which should have been an incentive for the auctioneer, does not benefit auctioneer in real world scenario in concurrent auction setting.

Transient state – k bands auctioned so far: All bidders participating in $(k+1)$th auction round have the chance to iterate their bids thus increasing the minimum bid. Note that, compared to concurrent auction, in sequential auction, bidders get the opportunity to revisit their bids $(m-1)$ times more in each DSA period. Then in concurrent auction, as the bidders have less number of chances to resubmit their bidding strategies, it is clear that minimum bid submitted in concurrent auction would be less than the minimum bid submitted in sequential auction.

After k spectrum bands auctions are over let the minimum bids in sequential and concurrent auctions be b_{min_1} and b_{min_2} respectively; such that $b_{min_2} \leq b_{min_1}$. Substituting values of $b^*_{i_{seq}}$ and $b^*_{i_{con}}$ in Eq. 17, we get the difference in optimal bids between sequential and concurrent auction as,

$$b_{diff} = \frac{(n-k-1)V_i + b_{min_1}}{(n-k)} - \frac{(n-m)V_i + b_{min_2}}{(n-m+1)} \tag{21}$$

Simplifying Eq. 21, we obtain,

$$b_{diff} = \frac{(m-k-1)(V_i - b_{min_1})}{(n-k)(n-m+1)} + \frac{(n-k)(b_{min_1} - b_{min_2})}{(n-k)(n-m+1)} \tag{22}$$

As all the terms in Eq. 22 are positive, it can be concluded that optimal bids in sequential auction setting is more than that in concurrent auction setting. Thus, from the auctioneer's perspective, it is more beneficial to follow the sequential bidding mechanism for substitutable bands.

Steady state reached: In this case, we assume that the auction has been run for sufficient large number of times to reach the steady state both for sequential and concurrent mechanisms. As we mentioned previously, auctioneer broadcasts the minimum bid submitted so the history of minimum bids are known to all the bidders. Thus as we assume the auction model to achieve the steady state, minimum bid submitted both for sequential and concurrent mechanism would be the same.

Then the difference in optimal bids between sequential and concurrent auction is given as,

$$b_{diff} = \frac{(m-k-1)(V_i - b_{min})}{(n-k)(n-m+1)} \tag{23}$$

As all the terms in Eq. 23 are positive, it can be concluded again that optimal bids in sequential auction setting is more than that in concurrent auction setting.

4.4 Concurrent and sequential auctions for non-substitutable bands

In this section, we present the concurrent and sequential auction models for m non-substitutable bands. For every bidder, the value of each of these m bands is different. We assume that bidders have complete information about the valuation and rankings of the bands. Under the complete information scenario, n bidders submit bids concurrently at the beginning of the allocation period.

Let the true valuation price be in the form of a vector of vectors,

$$\mathbf{V} = \{\{\mathbf{V}_1\}, \{\mathbf{V}_2\}, \cdots, \{\mathbf{V}_n\}\} \tag{24}$$

where $\{\mathbf{V}_i\}$ is the valuation price vector of ith bidder for all m spectrum bands, i.e.,

$$\mathbf{V}_i = \{V_{i1}, V_{i2}, \cdots, V_{im}\} \tag{25}$$

Let the reservation price of ith bidder for all m spectrum bands be

$$R_i = \{r_{i1}, r_{i2}, \cdots, r_{im}\} \tag{26}$$

With all the values for bands known, it is obvious that a bidder i will choose to submit bid for *that* spectrum band which will maximize his payoff profit,

$$U_i = V_{ij} - r_{ij}; \qquad j \in m \tag{27}$$

The dominant strategy of bidder i in concurrent auction would be to choose the band which will provide him the maximum payoff profit U_i. Thus it may happen that jth band provides the maximum payoff profit for l bidders which will result in l bidders competing for jth band excluding all other bands from the spectrum band list. Moreover, in concurrent auction the losing bidders do not have chance to revisit their bid strategy even if there might be less valuation bands unoccupied by any bidder. This problem does not happen if the auction is sequential as bidders get chances to revisit their bid strategies. We compare concurrent and sequential auction revenue generation from the auctioneer's perspective.

4.4.1 Concurrent auction

Before we calculate the aggregate revenue for the auctioneer, let us first analyze only one band j. If $l > 1$ bidders aim for this band j, then the revenue Rev_{lj} generated from this band would be the maximum bid submitted from all these l bidders. If only 1 bidder aims for the band j, the revenue generated will be the bid submitted by the sole bidder. If no bidder aims at the band j, the revenue generated will be zero from band j.

Then the total revenue generated from all the n bidders and m bands in the concurrent auction setting can be expressed in the following recursive way

$$Rev_{con}[n, m] = Rev_{l1} + Rev_{con}[n - l, m - 1] \tag{28}$$

where $Rev_{con}[n, m]$ is the total revenue generated from n bidders and m bands and l can take values from 0 to n. The disadvantage in such a concurrent setting is that $(n - l)$ may be 0 even if some of the bands are still left unoccupied. Thus all the bands are not sold out in auction even if $n > m$ and thus auctioneer does not get full benefit of all the bands.

4.4.2 Sequential auction

Similarly, we formulate the revenue generated from the sequential auction. The total revenue generated can be presented as a recursive expression

$$Rev_{seq}[n, m] = Rev_{l1} + Rev_{seq}[n - 1, m - 1] \tag{29}$$

where l can take values from 0 to n. We find that as the bands are sequentially auctioned, all the bands are sold out thus providing better revenue possibility than concurrent auction.

5 Auction design for multiple unit grant

So far, we analyzed the scenario where only a single band would be assigned to a service provider from the multiple bands available. In this section, we relax this constraint and investigate the more generalized case where service providers can win multiple spectrum bands available from the common spectrum pool. We propose our auction model and formulate the conflict among the service providers and spectrum broker under such multiple units grant.

To ensure successful auction design, we consider three important issues on which the success of the auction depends. They are (i) maximizing auctioneer's revenue, (ii) attracting bidders by increasing their probability of winning, and (iii) preventing collusion so that bidders can not control the auction. It is not at all intended that only big companies with high spectrum demand should acquire these additional spectrum bands. The goal is to increase competition and bring fresh new ideas and services. As a result it is necessary to make the small companies, who also have a demand of spectrum, interested to take part in the auction. This way, revenue can be maximized and maximum use of the available spectrum from the CAB can be made.

The situation described above maps directly to the 0-1 knapsack problem, where the aim is to fill the sack as much as possible maximizing the valuations of the sack. Here, we compare the spectrum bands present in CAB as the total capacity of the sack and the bids presented by service providers as the valuations for the spectrum amount they request. We propose this auction procedure as *"Dynamic Spectrum Allocator Knapsack Auction"*.

We formulate the above mentioned knapsack auction as follows. Let us consider that there are n bidders looking for the additional amount of spectrum from the CAB. All the bidders submit their demand through sealed bids. We follow sealed bid auction strategy because sealed bid auction has shown to perform well in all-at-a-time auction bidding and has a tendency to prevent collusion. Note that, each service provider has knowledge about its own bidding quantity and bidding price but do not have any idea about any other service providers' bidding quantity and price. We assume that the spectrum band available in CAB is W. Now, if the spectrum requests submitted by some or all of the service providers exceed the spectrum available in CAB then the auction is held to solve the conflict among these providers.

Let, $i = 1, 2, \cdots, n$ denote the bidders (service providers). We denote the strategy taken by service provider i as q_i, where q_i captures the demand tuple of this ith service provider and is given by

$$q_i = \{w_i, x_i\} \tag{30}$$

where, w_i and x_i denote the amount of spectrum and bidding price for that spectrum respectively.

Auction is best suited when the total demand is more than the supply, i.e.,

$$\sum_{i=1}^{n} w_i > W \tag{31}$$

Our goal is to solve the dynamic spectrum allocation problem in such a way so that earned revenue is maximized from the spectrum owner's point of view, by choosing a bundle of bidders, subject to condition such that total amount of spectrum allocated does not exceed W. Thus the allocation policy of the spectrum owner would be,

$$maximize_i \sum_i x_i \tag{32}$$

subject to the condition,

$$\sum_i w_i \le W \tag{33}$$

5.1 Synchronous and asynchronous auctions

Spectrum allocation with the help of proposed sealed bid knapsack auction can be done either synchronously or asynchronously [14]. In synchronous auction, bids from all the bidders are taken simultaneously and allocation/de-allocation of spectrum from and to the CAB are done only at fixed intervals. On the other hand, in asynchronous auction, bids are submitted by bidders asynchronously and allocation/de-allocation of spectrum from and to the CAB are not done at fixed intervals.

Asynchronous auction: As the name suggests, this auction procedure of spectrum is asynchronous among the service providers as shown in Fig. 2. Whenever a service provider comes up with a request for spectrum from the CAB, the spectrum owner checks to see if that request can be serviced from the available pool of CAB. If the requested amount of spectrum is available, spectrum owner assigns this chunk to the service provider for the requested time and declines if the spectrum requested is not available. Similarly, if more than one service provider come up with requests for spectrum from the CAB, the spectrum owner checks to see if all the requests can be serviced from the available pool of CAB. If they can be serviced, the spectrum is assigned but if all the requests can not be granted, then auction is initiated. We denote the strategy taken by service provider i as q_i^a. q_i^a captures the demand tuple of this ith service provider in asynchronous allocation mode and is given by

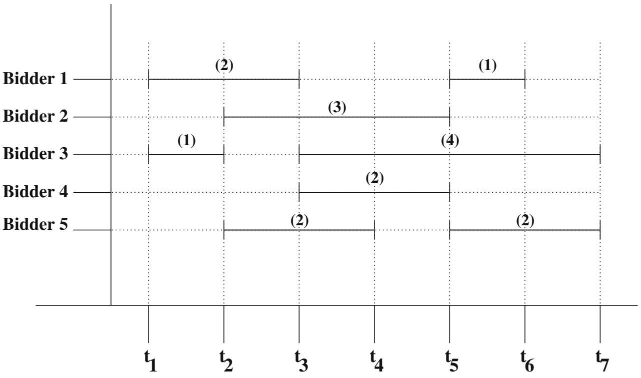


Figure 2 Asynchronous allocation in different intervals of time

$$q_i^a = \{w_i, x_i, T_i\} \tag{34}$$

where, w_i and x_i denote the amount of spectrum and bidding price for that spectrum respectively. T_i is the duration for which the spectrum amount is requested. The numbers inside the parenthesis in the Fig. 2 denote the duration T_i of the spectrum lease allocated to the corresponding bidders. As the decision about whether to allocate or not to allocate spectrum to a service provider is taken instantly in this allocation procedure by looking at the available pool only this allocation procedure is not very effective and may not maximize the earned revenue. It may happen that a service provider *B* is willing to pay a higher price than a service provider *A* who paid a lower price for the same demand, but unfortunately *B's* request came up after *A's* request. In this allocation procedure, as the spectrum owner does not have any idea about the future, *A's* request will be processed and *B's* will be declined (assuming that the available pool does not change at the time of *B's* arrival). Thus revenue could not be maximized through this allocation procedure.

Synchronous auction: In synchronous auction, spectrum bands are allocated and de-allocated at fixed intervals as shown in Fig. 3. All the service providers with a demand present their requests to the spectrum owner and the price they are willing to pay. Spectrum owner takes all the requests, processes them using some strategy and then allocates the spectrum bands to the providers at the same time for the same lease period. When the lease period expires, all the allocated spectrum chunks are returned to the common pool for future use. For example, lease periods for all the bidders are indicated as 1 in the Fig. 3.

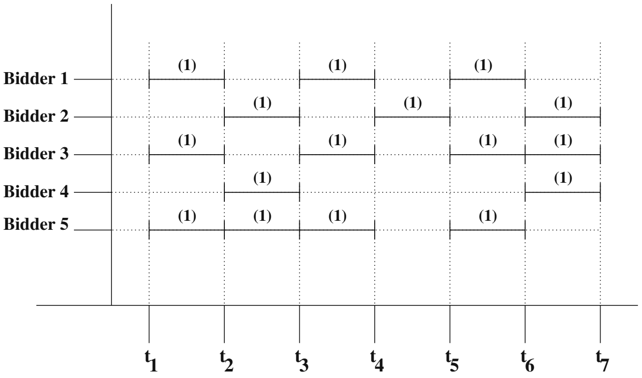


Figure 3 Synchronous allocation of spectrum in fixed intervals

5.2 Performance Comparison

We analyze and compare the synchronous and asynchronous strategies with the help of knapsack auction. Below, we present two lemmas to show the performance comparison between synchronous allocation coupled with knapsack auction and asynchronous allocation of spectrum.

Lemma 1 *Average Revenue generated in asynchronous allocation through knapsack auction procedure can not be better than average revenue generated in synchronous allocation for a given set of biddings.*

Proof We assume that there are n bidders competing for W amount of spectrum. In asynchronous allocation mode, the bid strategies taken by ith service provider is given by tuple q_i^a, while in synchronous mode, the tuples are represented by, q_i.

We prove the above proposition with the help of counter-example. We arbitrarily decide two time intervals, t_j and t_{j+1} for the asynchronous mode allocation. We assume that first deallocation(s) of spectrum and new allocation(s) are happening at time t_{j+1} after time t_j. Moreover, we assume that the asynchronous allocation at time t_j is maximal and provide the maximum revenue. Let, m be the number of bidders who were granted spectrum at time t_j. Then, the maximum revenue generated at time t_j can be given by,

$$\sum_{i}^{m} x_i \tag{35}$$

Now, we assume l of m bidders de-allocate at time t_{j+1} and rest $(m-l)$ bidders continue to use their spectrum. Then the revenue generated by these $(m-l)$ bidders is given by,

$$\sum_{i}^{m-l} x_i \tag{36}$$

Moreover, the $(n-m)$ bidders, who were not granted spectrum at time t_j, will also compete for the rest of the spectrum,

$$W - \sum_{i}^{m-l} w_i \tag{37}$$

Now, we need to find, whether the revenue generated in this asynchronous mode at time t_{j+1} can exceed the synchronous mode revenue at the same time by the same set of bidders. For simplicity, we assume that the bidders do not change their bidding requests in time intervals t_j and t_{j+1}.

By the property of 0-1 knapsack auction, we know that the revenue generated by a subset (we denote this subset by Q) of $n-l$ bidders will be a local maxima, if only the revenue obtained from all the $(n-l)$ bidders are considered simultaneously, i.e., synchronous allocation of spectrum to $(n-l)$ interested bidders (note that l is the set of bidders de-allocating their spectrum at time t_{j+1} and are not taking part in auction at time t_{j+1}). But on the other hand, in the asynchronous mode, $(m-l)$ bidders are already present and thus knapsack auction is conducted among $(n-m)$ bidders for the spectrum $W - \sum_i^{m-l} w_i$. Then, it can be easily said from the property of 0-1 knapsack auction that, this asynchronous mode will generate the same local maxima as the synchronous mode, if and only if all $(m-l)$ bidders (who are already present from the previous time interval) fall under the optimal subset Q. If any of the bidders out of $(m-l)$ bidders do not fall under the optimal subset Q, then it is certain that asynchronous mode allocation will not be able to maximize the revenue for that given set of biddings. □

Let us provide a simple example to clarify the proof. *An illustrative example:* Let us consider that 5 bidders who compete for a total capacity of 14 and the bid tuples generated by them at time interval t_j are $(6, 10, 2)$, $(5, 9, 3)$, $(7, 14, 1)$, $(2, 8, 2)$ and $(3, 9, 3)$ respectively. The first number of the tuple denotes spectrum amount requested, while the second and third numbers denote the price willing to pay for that spectrum request and time duration for which the spectrum request is done respectively. As we can see from the above tuples that bidder 3's request has duration 1, that means, bidder 3 will de-allocate first at time t_{j+1}.

We execute both asynchronous and synchronous knapsack auction. In asynchronous mode, the revenue generated at time t_j is 31 with the optimal subset of bidders given by bidders 2, 3, and 4. Now at time t_{j+1}, bidder 3 exits, while bidders 2 and 4 continue. The remaining spectrum left in the CAB is 7 for which the bidders 1 and 5 compete. Then the revenue generated at time t_{j+1} is 27 and the bidders granted are 1, 2, and 4.

On the other hand, in synchronous allocation, each of the providers are allocated and de-allocated at fixed time intervals. Then with the same set of bid requests of spectrum amount and price, it is seen that maximum possible revenue generated at time t_{j+1} out of the bidders $1, 2, 4$ and 5 (as bidder 3 is not interested to take part in auction at time t_{j+1}) is 28, while the optimal subset of bidders is given by $Q = \{1, 2, 5\}$. This clearly shows that asynchronous auction may not provide the maxima all the time depending on the bidders de-allocating and requesting.

Lemma 2 *Asynchronous allocation through knapsack auction procedure is sub-optimal while synchronous allocation is optimal.*

Proof We define a process as optimal that always provides a local maxima for a given set of values, while a sub-optimal process may or may not achieve that local maxima with the same set of values. With the help of this definition and the proof provided in Lemma 1, we can similarly prove Lemma 2. □

5.3 Bidders' strategies

In knapsack auction, we investigate bidders' strategies for both first and second price bidding. In first price auction, bidder(s) with the winning bid(s) pays their winning bid(s). In contrast, in second price auction, bidder(s) with the winning bid(s) do not pay their winning bid but pay some other lower winning bid according to the strategy fixed by the auctioneer.

For investigating the bidders' strategy, we consider a particular bidder j. Let each bidder i submit the demand tuple q_i. Then the optimal allocation of spectrum to the bidders is done by the auctioneer taking all the demand tuples into consideration. We denote this optimal spectrum allocation as M, where M incorporates all the demand tuples q_i and is subject to conditions presented in Eqs. 32 and 33. Moreover, we assume that the jth bidder's request falls among the optimal allocation M, i.e., jth bidder has been granted the spectrum. Then the revenue generated by auctioneer is given by,

$$\sum_{i \in M} x_i \tag{38}$$

where, all the bids of bidders present in the optimal allocation M, are summed.

In contrast, let us assume a case where jth bidder does not exist at all and the auction is held among the rest of the bidders. Let the optimal allocation be denoted by M^* and is again subject to conditions presented in Eqs. 32 and 33. Then the revenue generated by auctioneer in this case is given by,

$$\sum_{i \neq j, i \in M^*} x_i \tag{39}$$

Then the minimum winning price charged to jth bidder can be given by,

$$a_j = \sum_{i \neq j, i \in M^*} x_i - \sum_{i \neq j, i \in M} x_i \tag{40}$$

It is clear from the above equation that bidder j's request is granted if

$$x_j > a_j, \tag{41}$$

bidder j's request is not granted if

$$x_j < a_j \tag{42}$$

and bidder j is indifferent between winning and losing if

$$x_j = a_j \tag{43}$$

With these insights, we try to find the bidders' strategies in first price and second price bidding under the knapsack auction model.

Lemma 3 *In second price bidding, the dominant strategy of the bidder is to bid their reservation price.*

Proof Before proving this lemma, let us explain the reservation price or true evaluation price of the bidder. When a service provider (bidder) buys spectrum from the spectrum broker, the service provider needs to sell that spectrum in form of some service to the end users who are willing to pay for that service. The revenue generated from the end users for that amount of spectrum can be the true evaluation price or reservation price for that service provider (bidder).

Let us assume jth service provider (bidder) has the demand tuple $q_j = \{w_j, x_j\}$ and its reservation price for that amount of spectrum requested be r_j. As per Eq. 40, jth bidder's request will be granted and hence be in the optimal allocation M, only if the bid generated by jth bidder is more than a_j. Then according to the second price bidding policy, jth bidder will pay the second price which is a_j. The expected payoff obtained by jth bidder is given by,

$$E_j = r_j - a_j \tag{44}$$

We proceed to show that jth bidder's true bid is its reservation price r_j as claimed in the lemma using counter proof approach.

We assume that jth bidder does not bid its true evaluation of the spectrum requested, i.e., $x_j \neq r_j$. Two cases might arise depending on the relative values of x_j and r_j.

Case 1 Bid is less than the reservation price, i.e., $x_j < r_j$.

- $r_j > x_j > a_j$: bidder j falls inside the optimal allocation M and its request is granted. The expected payoff obtained by jth bidder is still given by $(r_j - a_j)$.
- $r_j > a_j > x_j$: bidder j loses and its request is not granted. Accordingly, the expected payoff is 0.
- $a_j > r_j > x_j$: bidder j still loses and the expected payoff is again given by zero.

Case 2 Bid is more than the reservation price, i.e., $x_j > r_j$.

- $x_j > r_j > a_j$: bidder j falls inside the optimal allocation M and its request is granted. The expected payoff obtained by jth bidder is still given by $(r_j - a_j)$.
- $x_j > a_j > r_j$: though bidder j wins but the expected payoff becomes negative in this case. The expected payoff obtained by jth bidder is now given by $(r_j - a_j) < 0$. Bidder j definitely will not be interested in this scenario.
- $a_j > x_j > r_j$: bidder j loses and the expected payoff is again 0.

Thus it is clear that if bidder j wins, then the maximum expected payoff this bidder can obtain is given by $E_j = r_j - a_j$ and bidding any other price above or below its reservation price r_j will not increase the payoff. Thus the dominant strategy of the bidders in second price bidding is to bid their reservation prices. □

Lemma 4 *In first price bidding, the bid is upper bounded by the reservation price.*

Proof In contrast to the Lemma 3, in first price bidding, the expected payoff obtained by jth bidder can be given by,

$$E_j = r_j - x_j \tag{45}$$

as the price paid by the bidder is the same as the bid. Then, to increase the expected payoff, i.e., to keep $E_j > 0$, x_j must be less than r_j.

At the same time, for winning, bid x_j must be greater than a_j, as specified in Eq. 40. Thus dominant strategy for the bidders in first price auction is $r_j > x_j > a_j$. □

6 Simulation model and results

To prove the effectiveness of the proposed auction models and the bidding strategies, we conducted simulation experiments. We divided the experiments into broad categories. Auctions with the allocation constraint of at most one spectrum band grant are discussed in Subsection 6.1. In Subsection 6.2, we present the results of the auction model where bidders are granted multiple spectrum bands.

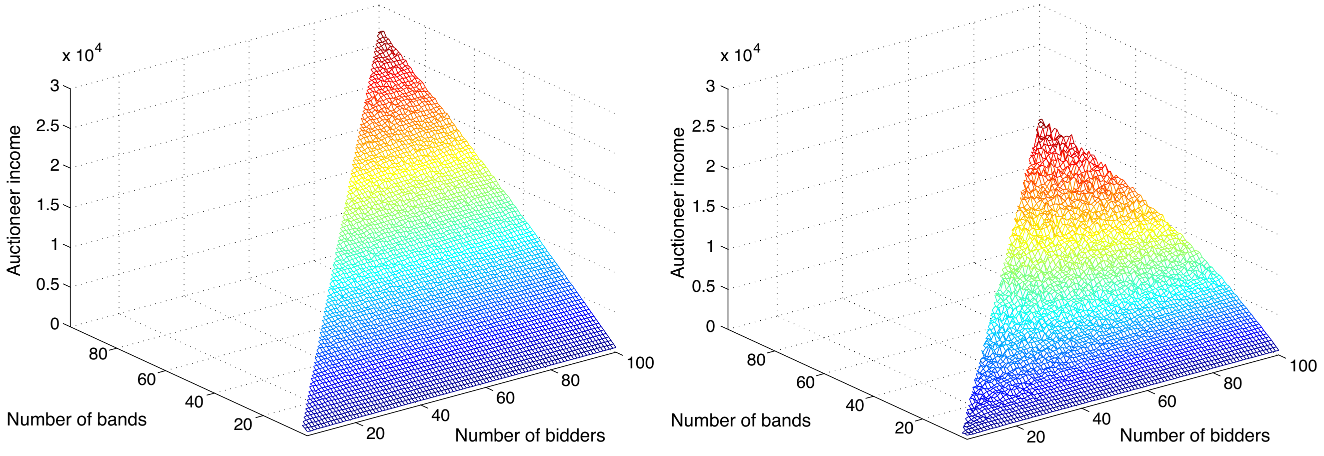
6.1 Results for single unit grant

For single unit grant, we present a comparison between sequential and concurrent bidding for both substitutable and non-substitutable bands. We assume the number of bands to be less than the number of bidders for the auction to take place.

6.1.1 Substitutable spectrum bands

The parameters for this auction setting are as follows. We assume all the spectrum bands are of equal value to all the bidders. Note that throughout this simulation model, we use the notation *unit* instead of any particular currency. The *reservation price* for each bidder is assumed to follow a uniform distribution with minimum and maximum as 250 and 300 units respectively. Moreover, the bids presented by the bidders are also assumed to follow a uniform distribution between 100 and 300 units.

In Fig. 4a and b, we compare the auctioneer's revenue for both sequential and concurrent bidding with varying number of bidders and spectrum bands. As discussed earlier, the revenue generated in the sequential auction setting is more than that in the concurrent one. In fact, with increase in number of bands and bidders,



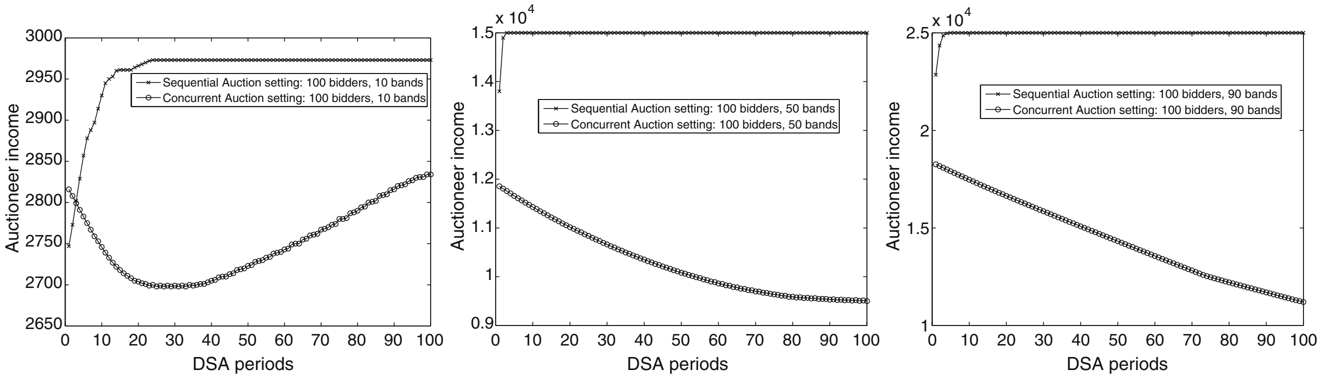
(a)

(b)

Figure 4 **a** Revenue in sequential auction, and **b** revenue in concurrent auction; with number of bidders and substitutable spectrum bands

revenue generated in sequential setting is almost 200% more than the revenue in concurrent setting, thus proving that sequential auction to be more beneficial from the auctioneer's perspective.

In Fig. 5a, b and c, we present the revenue generated by auctioneer in both sequential and concurrent biddings with increase in DSA periods. We assume that the bidders use auction histories of previous rounds to submit their bids in future rounds. Thus a winning bidder in one DSA period will try to submit a lower bid in next DSA period to increase his surplus profit whereas a losing bidder will increase his bid provided the previous bid was less than his reservation price. For all three results, we fixed the number of bidders as $n = 100$ but varied the number of bands as $m = 10$, $m = 50$ and $m = 90$. We find that the *difference* in the revenue generated between sequential setting and concurrent setting increases with number of bands (note the y-axis scale value in Fig. 5a, b and c). Thus sequential auction provides more revenue than the concurrent auction. Moreover, we find that with increasing number of bands, sequential auction reaches steady state much faster than the concurrent auction. This happens due to the fact that as more and more number of bands are available in the common pool for the auction ($m \rightarrow n$), greedy bidders will get more incentive bidding less than their true valuation prices as was proved earlier. This of course will not happen in the sequential auction. Thus sequential auction is clearly a better choice for auctioneer to generate higher revenue.

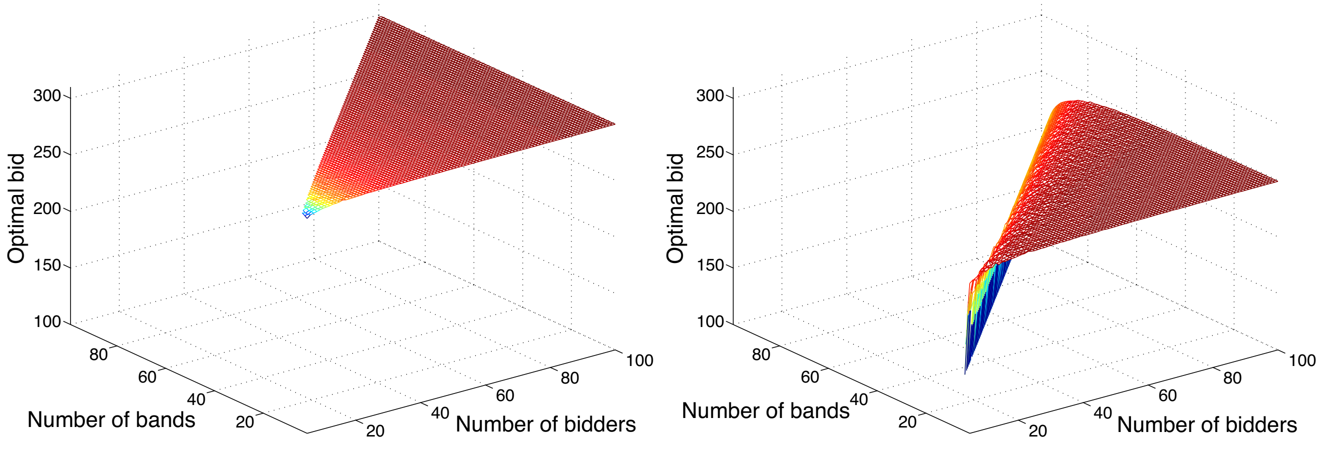


(a)

(b)

(c)

Figure 5 Auctioneer's revenue with substitutable bands: **a** 100 bidders and 10 spectrum bands; **b** 100 bidders and 50 spectrum bands; **c** 100 bidders and 90 spectrum bands



(a) (b)

Figure 6 Substitutable bands: optimal bid for a specific bidder for **a** sequential auction, and **b** concurrent auction

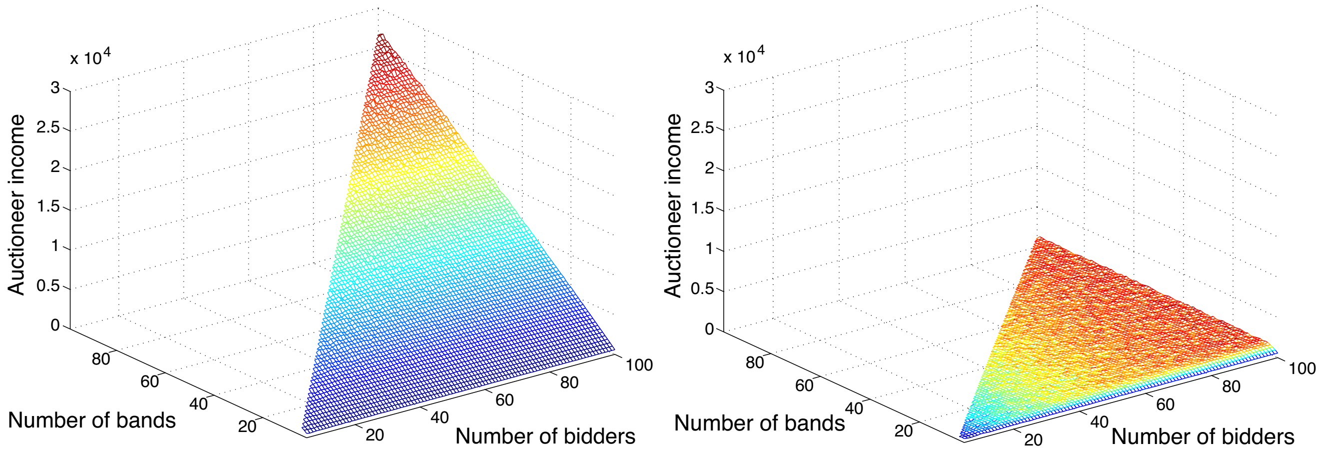
Next, we present the optimal bid for a specific bidder to win a spectrum band for both sequential and concurrent bidding in Fig. 6a and b. It can be observed that the optimal bid for the concurrent auction is less than the optimal bid for the sequential auction and even decreasing with $m \to n$. Thus in concurrent auction setting, auctioneer will not receive any incentive increasing the number of bands in the common pool thus reducing the whole purpose of dynamic spectrum allocation.

6.1.2 Non-substitutable spectrum bands

For non-substitutable bands, the bands are not of equal value. We assume the band's true value follow a uniform distribution with minimum and maximum being 450 and 500 units respectively. We follow the same distribution of bids as mentioned in the previous subsection.

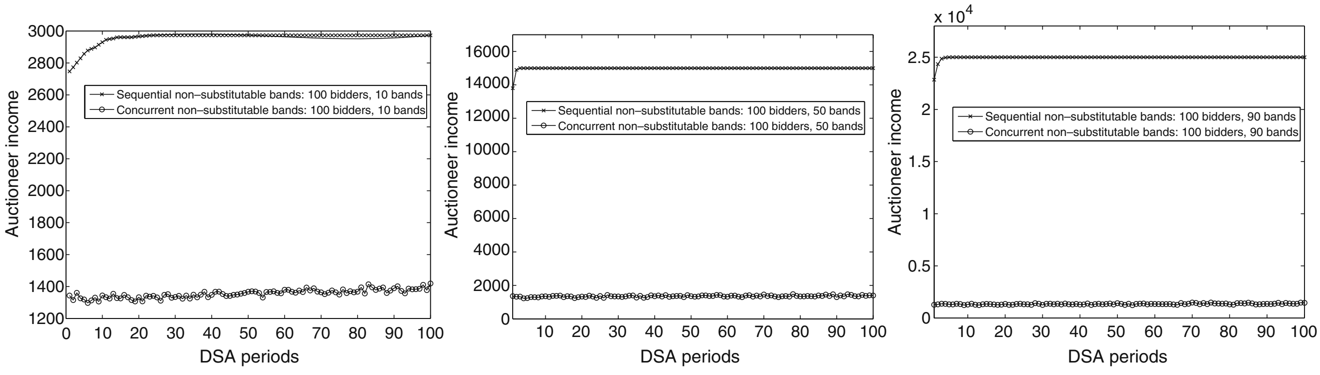
In Fig. 7a and b, we present the revenue with varying number of bidders and bands. It is clear that sequential auction provides better revenue for the auctioneer than the concurrent setting for non-substitutable bands.

In Fig. 8a, b and c, we present the revenue generated by auctioneer in both sequential and concurrent biddings with increase in auction rounds. Similar to the previous case, we assume that the bidders use auction histories of previous rounds to submit their bids in future rounds. We find that the *difference* in the revenue generated between sequential setting and concurrent



(a) (b)

Figure 7 **a** Revenue in sequential auction, and **b** Revenue in concurrent auction; with number of bidders and non-substitutable spectrum bands



(a) (b) (c)

Figure 8 Auctioneer's revenue with non-substitutable spectrum bands: **a** 100 bidders and 10 spectrum bands; **b** 100 bidders and 50 spectrum bands; **c** 100 bidders and 90 spectrum bands

setting under non-substitutable bands is even more than that of the substitutable bands of previous case (note the y-axis scale changes in Fig. 8a, b and c). Thus sequential auction setting is clearly a better choice for auctioneer to generate better revenue for both types of bands.

6.2 Results for multiple unit grant

We simulate the dynamic spectrum allocator knapsack auction model and show how the synchronous allocation outperforms the asynchronous allocation when bidders are granted multiple non-substitutable spectrum bands.

6.2.1 Spectrum auctioning methodology and parameters

The main factors that we consider for comparing the performance of the proposed synchronous knapsack sealed-bid auction with the asynchronous auction are the revenue generated by spectrum owner, total spectrum usage, and probability of winning for bidders. We consider the following for the simulation model.

- *Bid tuple:* The bid tuple q_i generated by bidder i in synchronous auction consists of amount of spectrum requested, w_i and the price the bidder is willing to pay, x_i. In asynchronous auction, the duration is also advertised in addition to the above two. Each bidder has a reservation or evaluation price for the amount of spectrum requested and the bid is governed by this reservation price.
- *Bidders' strategies:* We follow second price sealed-bid mechanism. We could have chosen the first price bidding policy; the only reason for choosing second price policy is that it has more properties than first price in terms of uncertainty [16]. After each round of auction, the only information bidders know is whether their request is granted or

Table 1 Simulation parameters

Parameter	Value
Total amount of spectrum	125
Minimum amount of spectrum that can be requested	11
Maximum amount of spectrum that can be requested	50
Minimum bid for per unit of spectrum	25
Minimum time requested for spectrum leasing in asynchronous allocation	1
Maximum time requested for spectrum leasing in asynchronous allocation	5
Fixed time for spectrum leasing in synchronous allocation	1

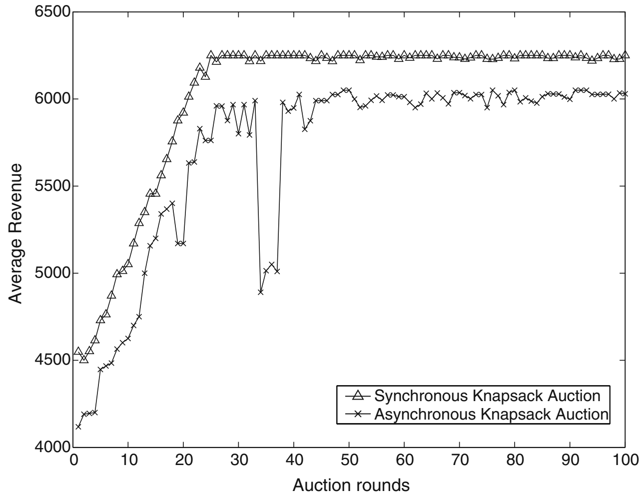


Figure 9 Revenue generated with auction rounds

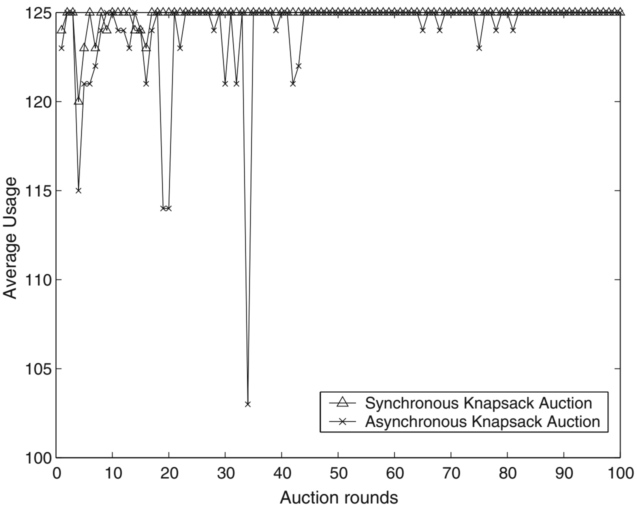


Figure 10 Spectrum usage with auction rounds

not. We assume that all the bidders are present for all the auction rounds; bidders take feedback from previous rounds and generate the bid tuple for next round.

- *Auctioneer's strategies:* Spectrum owner tries to maximize the revenue generated from the bidders.

For better insight into the results, we compare the proposed synchronous sealed bid knapsack auction with the asynchronous sealed bid knapsack auction under the second price bidding policy, i.e., bidder(s) with the winning bid(s) do not pay their winning bid but pay the second winning bid. Simulation parameters are shown in Table 1.

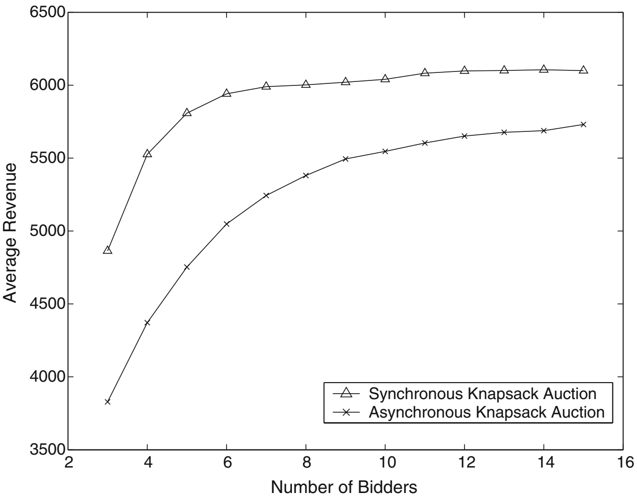


Figure 11 Average revenue generated with number of service providers

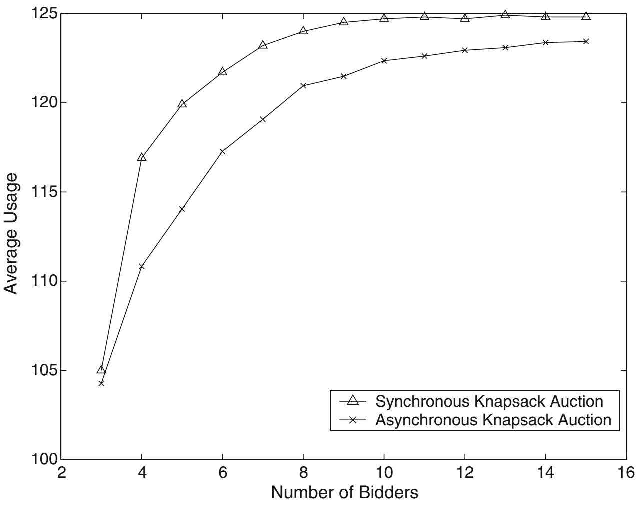


Figure 12 Average spectrum usage with number of service providers

6.3 Simulation results

Figures 9 and 10 compare revenue and spectrum usage for both the strategies (synchronous and asynchronous) with increase in auction rounds. The number of bidders considered in this simulation is 15. Note that, both revenue and usage are low at the beginning and subsequently increases with rounds. When auction starts, bidders always act skeptical, thus initial bids are always much lower than their true potential bids. With the increase in auction rounds, bidders get an idea of the bids of other bidders and thus try to increase or decrease their bids accordingly.

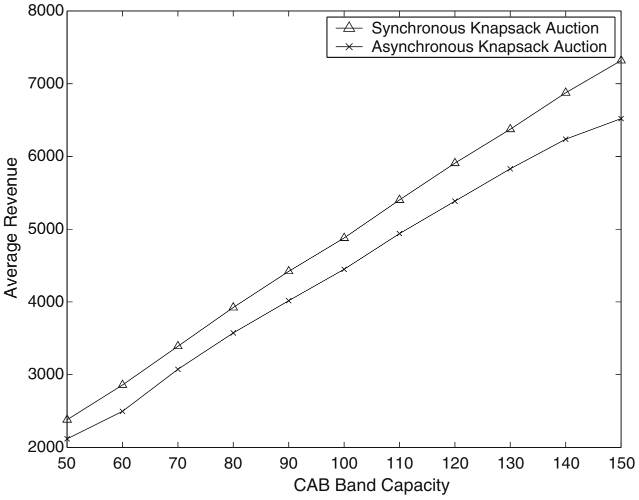


Figure 13 Average revenue generated with increase in CAB

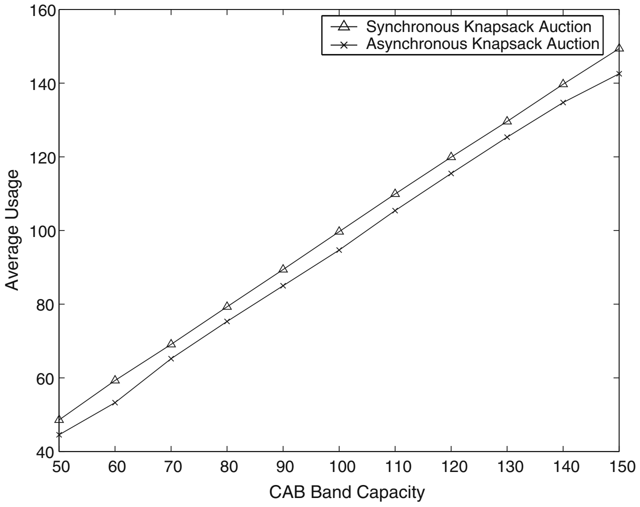


Figure 14 Average spectrum usage with increase in CAB

Figures 11 and 12 show the average revenue and spectrum usage with varying number of bidders for both the auction strategies. We observe that the proposed synchronous knapsack auction generates approximately 10% more revenue compared to the asynchronous knapsack auction and also reaches steady state faster. The average spectrum usage is also more with the synchronous allocation policy.

Figures 13 and 14 show the average revenue and spectrum usage with increase in capacity in CAB for both the auction strategies. It is clear that with increase in CAB, synchronous strategy provides more revenue and makes optimal use of CAB than the asynchronous strategy and thus provides more incentive for the spectrum owner.

In Fig. 15, we look at the auction model from the bidders' perspective. Higher revenue requires more number of bidders. We compare the two strategies in terms of the probabilities to win a bid. We observe that the proposed synchronous auction strategy has a significantly higher probability of winning compared to asynchronous auction strategy. This implies that bidders will be encouraged to take part in the synchronous knapsack auction; thus increasing the competition among the providers and increasing the chance to generate more revenue.

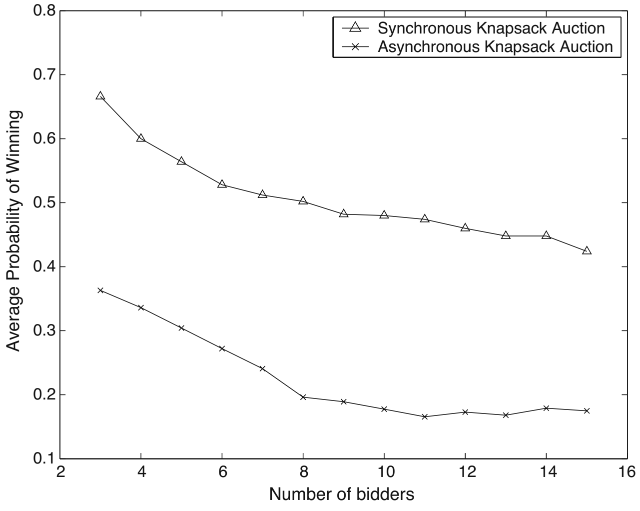


Figure 15 Average probability of winning spectrum with number of bidders

7 Conclusions

In this research, we investigate possible auction mechanisms for dynamic spectrum allocation. We first focus on the scenario where there are multiple spectrum bands in the common pool of auction but each bidder is allocated at most one spectrum band. Through analysis and simulation we show that the popular conception of concurrent auction does not prove beneficial in this case. In this regard, we considered two metrics: revenue generated by auctioneer and optimal bid of the bidders for comparison of sequential and concurrent auctions. We have shown that sequential auction proves to be the better choice for DSA auctions with spectrum allocation constraint.

On the other hand, we also studied the scenario without any allocation constraint. We proposed an auction mechanism for DSA that is based on the well known knapsack problem. Both synchronous and asynchronous auction strategies are studied and compared in this context. Through simulations it was found that it is in the best interest of both bidders and spectrum owner to adopt the synchronous auction. We showed how the optimal usage of spectrum band is achieved and the revenue is maximized for the spectrum owner. The proposed mechanism yields higher probability of winning for the service providers and thus encourages the providers to participate in the bidding process.

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