Power Control Game in Multi-Terminal Covert Timing Channels

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Abstract—We present a game theoretic power control of overlay/overt communications to maximize the goodput (effective throughput of error-free bits) of multi-terminal covert timing channels. Most approaches in the literature on covert timing channels discuss capacities of the timing channels but do not study how the overlay communication can be controlled to maximize the goodput of covert timing channels. We study the factors of the overlay communication that affect the goodput of each timing channel in a multi-terminal covert timing network. We show that the goodput of the covert timing channel can be enhanced by increasing the rate of overlay transmission and by game theoretic power control of overlay communication. We finally extend the game theoretic power control to maximize the goodput of each covert timing channel in a multi-terminal covert timing network by maximizing the asymptotic spectral efficiency of the overlay communication.

Index Terms — Covert timing channels, multi-terminal, goodput, power control

I. INTRODUCTION

We present a game theoretic power control of overlay/overt communication to maximize the goodput (effective throughput of error-free bits) of multi-terminal covert-timing channels. Covert communication [1] initially started off as parasitic communication and later developed into an effective means of hiding communication as an underlay beneath another application. Covert channels can be applied to increase the capacity of channels beyond the service rates, e.g., as shown in [2] or to hide data from attackers, e.g., as discussed in [3]. One popular means of hiding communication is the use of covert timing channels [4], wherein, a transmitter not only transmits overlay packets to the receiver as a “normal communication”, but also transmits an underlay covert information by delaying the overlay packets by an amount \( t_0 \) corresponding to a zero bit and by an amount \( t_1 \) corresponding to a one bit. Alternatively, the transmitter and receiver can agree on a threshold, \( T \) and packets can be delayed by a random time larger than \( T \) to convey a zero and a random time less than \( T \) to convey a one.

The notion of covert timing channels has been popular as a means to enhance capacity [2]-[9] (and the references there in). Anantharam and Verdu [2] use the arrival times in queues to convey covert information and thereby increase the capacity of queues beyond the rate of service. This is further extended in [5] to include results for non-exponential service distributions and in [6] to determine bounds on the sum-timing capacity of queues with multiple flows, each with exponential service distribution. In [7], Wagner and Anantharam consider the exponential service timing channel (ESTC) and compute the zero reliability rate and propose a distance metric to achieve bounds on the probability of error. Wang and Lee [8] extend the capacity estimation in [4] including the overheads incurred to overcome the effects due to loss of synchronization. In [9], Berk et al. present statistical detection of covert timing channels by modeling it as a binary asymmetric channel.

Most studies in the literature discuss means to enhance the covert timing channel capacity. The capacity provides the maximum rate at which data can be transmitted to achieve as low bit error rates as desired. However, a parameter of interest to the transmit-receive pairs in covert timing networks is also the actual goodput, i.e., the effective rate of transmission of error-free bits. The analysis of goodput provides a more practical perspective of the kind of underlay communication that can be covertly transmitted. The goodput is typically a decreasing function of the bit error rate (BER) at the receiver. The BER occurs due to the wireless channel characteristics (e.g., multipath fading)\(^1\). The BER, in turn, is a decreasing function of the difference between the time delay corresponding to a zero, \( t_0 \) and that corresponding to a one, \( t_1 \), i.e., \( \Delta t = |t_1 - t_0| \). Larger \( \Delta t \) results in a lower BER, i.e., a larger goodput. The optimal value of \( |t_1 - t_0| \) is also expected to depend on the characteristics of the overlay communication. Hence, it is important to study how different parameters of the overlay communication affect the covert timing channel goodput.

Eavesdroppers in covert timing networks can potentially detect covert communication by measuring anomalies in the packet delays [11]\(^2\). Therefore, a larger value of \( |t_1 - t_0| \) enables a passive attacker or an eavesdropper to easily detect the covert communication as the anomalies in the packet delays are more obvious. One potential means to mitigate detection of covert communications is by using auxiliary communications [13], [14]. Here, other transmitters and receivers of covert communication transmit packets so that a malicious attacker cannot detect the covert communications. To illustrate this, we demonstrated experimental results [13], using radio prototypes.

\(^1\)Note that in communication systems, error control codes [10] can be used to reduce the BER. However, covert timing operations are low rate communications [2] and error control codes further degrade the throughput due to the overheads. Therefore, error control codes are not considered in the discussion of the BER in covert timing channels.

\(^2\)Detection can also be made by using the entropy of the overlay message (e.g., [12]) but is very complex.
based on a software abstraction layer implemented over the IEEE 802.11a/b/g stack supported by Atheros chip sets. The findings in our experiments in [13] are described below.

We conducted experiments with standard FTP communication with no underlay covert timing data and then conducted experiments with FTP communications with underlay covert timing data. Fig. 1(a) [13] presents the packet count distribution (the histogram of the number of packets received after different time intervals at the receiver) measured when normal FTP traffic is transmitted. Fig. 1(b) presents the packet count distribution measured when underlay covert timing data was transmitted along with FTP traffic. It is observed that the presence of covert timing data results in two distinct peaks in the packet count distribution, which an eavesdropper can easily exploit to detect the presence of covert communication. We then perform experiments with 6 transmit-receive pairs with covert timing data as an underlay to normal FTP traffic. For each transmit-receive pair, the communication from the other transmit-receive pairs act as auxiliary communication. Fig. 1(c) [13] presents the packet count distribution for this scenario. It is observed that the packet count distribution looks similar to the one in the scenario with normal FTP traffic with no underlay covert timing data. Thus, auxiliary communication assisted in camouflaging the covert communication. We also performed a detailed theoretical analysis in [13] and [14] to illustrate that the probability of detecting covert communication reduces as the amount of camouflaging resources (number of auxiliary communications) increases.

Although auxiliary communications (i.e., the covert communication between the other transmit-receive pairs in a multi-terminal covert timing network) assist in camouflaging covert communications, each transmitter also causes interference to the other receivers in the network. This degrades the quality of the overlay communication. It is therefore important to know how the overlay communication can be affected when multiple simultaneous covert communications take place then devise means to enhance the goodput (effective throughput of error-free bits) of the covert communication. Covert timing channels not only require large goodput, but also should be protected from being detected by an eavesdropper. Note however, that protecting the covert communication from being detected by a malicious eavesdropper or attacker is already achieved by deploying auxiliary communications, as shown in [13] and [14]. Therefore, we focus only on the goodput here.

To explore this in detail, we attempt to answer the following questions in this paper: (1) What are the characteristics of the overlay communication that affect the goodput of covert communication? (2) What are the additional factors that affect the performance of multi-terminal covert timing networks? We first model the inter-arrival time of packets perceived by the receiver due to the propagation characteristics of the wireless medium, as a Gaussian process. We perform experiments with covert communication as an underlay to FTP packets, to validate this model. A hypothesis test based analysis is presented based on the Gaussian process model of the inter-arrival times of packets, to study the relation between the characteristics of the overlay application and the goodput of single-terminal covert timing networks. We then apply the results obtained for the single-terminal networks to multi-terminal networks. Multi-terminal covert timing networks suffer from multiple access interference (MAI) in addition to errors due to wireless propagation characteristics. This affects the performance of the overlay communication, which, in turn, affects the performance of the covert communication. We present a game theoretic power control to counter the MAI and study the goodput of multi-terminal covert timing networks with game theoretic power control. We finally extend the game theoretic power control to perform an asymptotic analysis of the spectral efficiency of the overlay communications in multi-terminal covert timing networks which is used to provide a rate control to maximize the covert timing channel goodput. Our results indicate that covert timing operations are more effective when performed as underlay to applications with high-rate and/or smaller packet sizes.

The rest of the paper is organized as follows. In Section II, we describe the experiments performed on a covert timing test bed, which we then use to model the inter-arrival times of packets at the receiver. Section III describes the system model. In Section IV, we present the analysis to maximize the goodput of a single-terminal and multi-terminal covert timing network and some numerical results. The asymptotic spectral efficiency of overlay applications in multi-terminal covert timing networks are also presented. Conclusions are drawn in Section V.

II. MODELING INTER-ARRIVAL TIMES

We conduct experiments in which a transmitter transmits overlay FTP packets to a receiver and also transmits covert information by delaying the FTP packets by 35 ms and 65 ms corresponding to a ‘0’ bit and ‘1’ bit, respectively. For the experimental setup, we use our covert network test bed prototype proposed and described in [13], [14], i.e., radio prototypes based on a software abstraction layer implemented over the IEEE 802.11a/b/g stack supported by Atheros chip sets. We count the number of packets delayed by various amounts of time and use the statistical frequencies of these packet counts to compute the mean and the variance of the inter-arrival times corresponding to a ‘1’ bit. We fit a Gaussian probability density function (pdf) to theoretically characterize the distribution of the inter-arrival times and compare the theoretical Gaussian fit with the data obtained from the experiments.

Fig. 2 presents the comparison of the theoretical Gaussian pdf fit with that of the data obtained from experiments. It is observed that the packet count distribution follows closely, a normal distribution with mean 65 ms corresponding to packets conveying a covert ‘1’ bit. We use this result to make the proposition that when the transmitter and receiver negotiate an inter-packet arrival time of \( t_0 \) corresponding to a covert ‘0’ bit, the inter-packet arrival time at the receiver is a random variable which is normally distributed with mean \( t_0 \) and variance \( \sigma_0^2 \).

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3The values 35 ms and 65 ms were just chosen for an illustrative experimental purpose so that the packet time distribution for the covert ‘0’ bit and that for the covert ‘1’ bit can be clearly measured. As such, any two values can be used in the experiments for the time intervals corresponding to the covert ‘0’ and ‘1’ bits.

4Similar results were observed for a covert ‘0’ bit.
Similarly, when a covert ‘1’ bit is transmitted with a negotiated inter-packet delay, $t_1$, the arrival time is normally distributed with mean $t_1$ and variance $\sigma_1^2$. It may be possible that $\sigma_0^2 = \sigma_1^2$, but we present the generalized analysis which is valid both for $\sigma_0^2 \neq \sigma_1^2$ as well as $\sigma_0^2 = \sigma_1^2$.

III. SYSTEM MODEL

Consider a multi-terminal network with bandwidth $W$ Hz, with $M$ transmitters/sources and $M$ receivers/destinations. For each transmit-receive pair, the transmission from other transmitters act as auxiliary communication that camouflage the covert communication. Transmitter $i$ transmits overlay packets to receiver $i$ at rate, $r_i$ and underlay covert information to receiver $i$ with time intervals, $t_0^{(i)}$ and $t_1^{(i)}$, corresponding to the covert bits ‘0’ and ‘1’, respectively. Transmitter $i$ transmits at power $P_i$. Our objective is to obtain the optimal values of $t_0^{(i)}$, $t_1^{(i)}$ and $P_i$ which maximizes the effective goodput of the timing channel between the $i^{th}$ transmit-receive pair. It is also of interest to determine the asymptotic spectral efficiency of the system, i.e., when $M$ and $W$ become very large and use this to determine the optimal rate of transmission of the overlay application and thus, maximize the effective goodput of the timing channels between each transmit-receive pair.

In order to determine $t_0^{(i)}$ and $t_1^{(i)}$ for each $i$, we first study a single transmit-receive pair, where the transmitter transmits overlay packets of length $L_p$ at rate, $r$ and delay the packets by a time interval $t_0$ and $t_1$ corresponding to bits ‘0’ and ‘1’, respectively. Due to the wireless propagation characteristics, the receiver receives the packet after a random time interval $\tau$. Based on $\tau$, the receiver performs a likelihood ratio based hypothesis test to make a decision on the transmitted bit to be a 0 or a 1. The decision making mechanism could lead to bit-errors in the covert timing channel. It is desired to design $|t_1 - t_0|$ so that the BER is minimum. Later, it is desired to apply these results in conjunction with game theoretic power control to maximize the goodput of each transmit-receive pair in the $M$–terminal covert timing network.

IV. PERFORMANCE ANALYSIS

A. Single-Terminal Analysis

The transmitter transmits packets of length $L_p$ corresponding to an overlay application at a rate, $r$. Hence, the packet time, $t_p$, is given by $t_p = L_p/r$. To transmit a covert ‘0’ bit, the transmitter is required to transmit a packet for a time $t_p$, wait for a time, $t_0$ and then transmit another packet. Similarly, to transmit a covert ‘1’ bit, the transmitter is required to transmit a packet for a time $t_p$, wait for a time, $t_1$ and then transmit another packet. Therefore, the transmitter requires on an average, a time $N t_p + b_0 t_0 + b_1 t_1$ to transmit $N - 1$ covert bits with $b_0$ covert ‘0’ bits and $b_1$ covert ‘1’ bits. The average rate of transmission for the covert communication, $r_{cov}$, can then be written as

$$ r_{cov} = \frac{N - 1}{N t_p + E[b_1] t_1 + E[b_0] t_0}. $$

(1)

Let $\pi_0$ be the probability that a covert ‘0’ bit is transmitted and $\pi_1 = 1 - \pi_0$ be the probability of transmitting a covert ‘1’ bit. Then $E[b_0] = (N - 1) \pi_0$ $E[b_1] = (N - 1) \pi_1$ and hence, for large, $N$, $r_{cov}$ can be written as

$$ r_{cov} = \frac{1}{t_p + \pi_0 t_0 + \pi_1 t_1}. $$

(2)

and for $\pi_0 = \pi_1 = 0.5$,

$$ r_{cov} = \frac{2}{2t_p + t_0 + t_1}. $$

(3)

The receiver can wrongly decode the covert bits (due to the wireless channel characteristics) and suffer a BER, $p_e$. Hence, the effective goodput of the timing channel, $r_e$, is the actual
transmission rate \( r_{cov} \) weighted by the loss due to BER, can be approximated as
\[
r_e = r_{cov}(1 - 2p_c).
\] (4)

In the above, the factor of 2 occurs due to the following reason. If the channel errors are very large causing BER of more than 0.5 (which can be estimated from large values of \( \sigma_1 \) and \( \sigma_0 \)), then receivers can simply invert the bits in the transmitted covert message. Thus, the performance of a channel with BER, \( p_c \), is same as that of the channel with BER, \( 1 - p_c \). The worst performance occurs when \( p_c = 0.5 \) because inverting the bits serve no purpose for the receiver and the receiver can make a decision independent of the transmitted bits. Thus, the effective rate of covert transmission is zero because the receiver does not recover covert bits, but the decision on these bits are independent of what was transmitted by the transmitter. Thus, when \( p_c = 0 \), \( r_e = r_{cov} \) and when \( p_c = 0.5 \), \( r_e = 0 \).

The BER, \( p_c \), is evaluated as follows. Let \( p(\tau|0) \) be the probability density function (pdf) of \( \tau \) when a 0 is transmitted and \( p(\tau|1) \) be the pdf of \( \tau \) when a 1 is transmitted. Based on the discussion in Section II, \( \tau \sim N(\mu, \sigma^2) \) when a covert ‘0’ bit is transmitted and \( \tau \sim N(\mu, \sigma^2) \) when a covert ‘1’ bit is transmitted. Therefore,
\[
p(\tau|0) = \frac{1}{\sqrt{2\pi}\sigma_0} \exp\left\{ -\frac{(\tau - \mu_0)^2}{2\sigma_0^2} \right\},
\] (5)
\[
p(\tau|1) = \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left\{ -\frac{(\tau - \mu_1)^2}{2\sigma_1^2} \right\}. \tag{6}
\]

The optimum likelihood ratio test based decision is \( \Lambda(\tau) \overset{1}{\geq} \Lambda_0 \), where \( \Lambda_0 \) is a pre-specified threshold and the likelihood ratio, \( \lambda(\tau) \), is defined as \( \lambda(\tau) = p(\tau|1)/p(\tau|0) \). The value of \( \Lambda_0 \) that minimizes the average error is unity [15]. From (5) and (6), the optimum decision for \( \Lambda_0 = 1 \) can be written as
\[
q(\tau) = a\tau^2 + b\tau + c \geq 0,
\] (7)
where \( a = \frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2} \), \( b = -2 \left( \frac{\mu_0}{\sigma_0^2} - \frac{\mu_1}{\sigma_1^2} \right) \) and \( c \approx \frac{\mu_1^2}{\sigma_1^2} - \frac{\mu_0^2}{\sigma_0^2} \). Let \( p_e(0|1) \) be the BER when a bit 1 is transmitted by the transmitter and decoded as a 0 by the receiver. Similarly, let \( p_e(1|0) \) denote the BER due to wrongly decoding a 0 as a 1. The expressions for \( p_e(0|1) \) and \( p_e(1|0) \) for the decision rule in (7) can be written as
\[
\begin{align*}
p_c(0|1) &= Q\left( \frac{|t_1 - t_0|}{\sigma_1 + \sigma_0} \right) - Q\left( \frac{|t_1 - t_0|}{\sigma_1 + \sigma_0} \right), \\
p_c(1|0) &= Q\left( \frac{|t_1 - t_0|}{\sigma_1 + \sigma_0} \right) + Q\left( \frac{|t_1 - t_0|}{\sigma_1 + \sigma_0} \right),
\end{align*}
\] (8)
(9)
where \( Q(\gamma) = \int_{\gamma}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left\{ -\frac{y^2}{2} \right\} dy \). Let \( \pi_0 \) and \( \pi_1 \) be the probabilities of transmitting a 0 and a 1, respectively. Then,
\[
p_e = p_e(0|1)\pi_1 + p_e(1|0)\pi_0.
\]
(10)

where \( \alpha = |t_1 - t_0| \). From (3), (4) and (10),
\[
r_e = \frac{2}{2p_e + 2t_0 + \alpha} \left[ 1 - 2Q \left( \frac{\alpha}{\sigma_1 + \sigma_0} \right) \right]. \tag{11}
\]

The effective goodput of the timing channel can be maximized by solving the following optimization problem, \( \max_{\alpha \geq 0} r_e \).

The value of \( \alpha \) that maximizes \( r_e \), \( \alpha^* \), can then be obtained by equating \( \frac{d\alpha}{dr} \) to zero, i.e., by solving
\[
\sqrt{\frac{2}{\pi}} \frac{2t_p + 2t_0 + \alpha^*}{\sigma_1 + \sigma_0} e^{-\frac{1}{2} \left( \frac{\alpha^*}{\sigma_1 + \sigma_0} \right)^2} = 1 - 2Q \left( \frac{\alpha^*}{\sigma_1 + \sigma_0} \right). \tag{12}
\]

The maximum value of \( r_e, r_{eff} \), can then be obtained as
\[
r_{eff} = r_e(\alpha^*) = \frac{2\sqrt{2}}{\sqrt{\pi(\sigma_1 + \sigma_0)} e^{-\frac{1}{2} \left( \frac{\alpha^*}{\sigma_1 + \sigma_0} \right)^2}}. \tag{13}
\]

It is noted that although (13) seems to suggest that making \( \alpha^* = 0 \) maximizes \( r_{eff} \), it is not so, because \( \alpha^* \) cannot take any arbitrary value, but only that obtained by solving (12).

From (13), the covert timing channel goodput depends on \( \alpha^* \), which, in turn, depends on \( t_p \) from (12). As a numerical example, consider a single-terminal covert timing network with \( s = 1 \mu s, \sigma_1 = 10 \mu s \) and \( \sigma_0 = 5 \mu s \). Fig. 3 presents the covert timing channel goodput with respect to the overlay packet size, \( t_p \). We also numerically evaluate the maximum effective timing channel goodput when the factor of 2 is not included in (4). Fig. 4 depicts the percentage difference as a

\[\text{We also numerically evaluate the effective timing channel goodput, when the factor of 2 is not included. The difference between timing channel goodputs in the two cases is negligible as will be observed from Fig. 4.}\]

\[\text{The expression in (11) assumes } t_1 > t_0. \text{ The analysis would be exactly same if } t_0 > t_1 \text{ and results can be obtained by replacing } t_0 \text{ by } t_1.\]
function of \( t_p \). It is observed that the percentage difference is very negligible (less than 0.7%, i.e., less than 25 bps for an actual goodput of 3.5 Kbps). It is observed that \( r_{eff} \) decreases as \( t_p \) increases. This is because, from (12), \( \alpha^* \) decreases as \( t_p \) decreases and from (13), \( r_{eff} \) increases as \( \alpha^* \) decreases. Therefore, a smaller \( t_p \) results in larger goodput for the covert timing channel. It is therefore inferred that the covert timing channel goodput not only depends on the channel conditions (i.e., \( \sigma_1, \sigma_0 \)) but also depends on the characteristics of the overlay application (i.e., \( t_p \)).

Since \( t_p = \frac{L_p}{r} \), \( t_p \) can be decreased by decreasing \( L_p \) or increasing \( r \). Therefore, covert timing operations in single terminal networks are more effective under overlay applications with larger rate (e.g., video streaming) as against overlay applications with smaller rate (e.g., best effort applications). Similarly, covert timing operations are more effective under overlay applications with small packet sizes (e.g., ping) as against those with larger packet sizes (e.g., file transfer).

In a multi-terminal network with \( M \) transmitters and receivers, transmitter \( i \) transmits packets at rate \( r_i \) (leading to a packet time of \( t_0^{(i)} \)). These packets are delayed by an interval \( t_1^{(i)} \) corresponding to a covert ‘1’ bit and at an interval \( t_0^{(i)} \) corresponding to a covert ‘0’ bit. Let \( t_1^{(i)} > t_0^{(i)} \) \( \forall i \) and let \( \alpha_i = \frac{t_1^{(i)} - t_0^{(i)}}{t_0^{(i)}} \). Let \( r_i \) be the random time interval after which the receiver receives successive packets. The optimum value of \( \alpha^*_i \) can be obtained by solving (12) by replacing \( \alpha^* \) by \( \alpha^*_i \), \( t_p \) by \( t_1^{(i)} \), \( t_0 \) by \( t_0^{(i)} \), \( \sigma_1 \) by \( \sigma_1^{(i)} \) and \( \sigma_0 \) by \( \sigma_0^{(i)} \). The maximum effective goodput of the \( i \)th covert timing channel, \( r_{eff}^{(i)} \), is then given by

\[
r_{eff}^{(i)} = \frac{2\sqrt{2}}{\pi} e^{\frac{1}{\sigma_1^{(i)} + \sigma_0^{(i)}}} \left( \frac{\alpha^*_i}{\sigma_1^{(i)} + \sigma_0^{(i)}} \right)^2.
\]

As will be explained in Section IV-B, transmit power also plays an important role in the multi-terminal case.

**B. Multi-terminal Network**

Consider a multi-terminal covert timing network with \( M \) transmitters and receivers. The \( i \)th transmitter transmits with power \( P_i \). In addition to the signal transmitted by the \( i \)th transmitter, the \( i \)th receiver also receives the interference from the other transmitters. This can cause bit errors (and hence, packet errors) at the \( i \)th receiver due to multiple access interference (MAI). Although the covert information in timing channels is stored in the inter-packet delays and not in the actual packets corresponding to the overlay application, it is still essential that the packets corresponding to the overlay application be received error-free at the receiver, particularly in multi-terminal networks. This is because, when a packet is received in error, the receivers will not be able to determine the correct identities of the transmitter and the receiver of the packets. The erroneous packets have to be discarded and cannot be used to convey covert information. Thus, in a multi-terminal covert timing network, the effective rate of the overlay application between each transmit-receive pair degrades by a factor which depends on the BER caused due to MAI. This, in turn, degrades the effective covert timing channel goodput.

The BER of any timing channel due to MAI depends on the signal-to-interference-noise ratio (SINR) of that channel, which, in turn, depends on the transmit powers of all the transmitters, as will be seen from (15). It is therefore of interest to control the transmit powers so that the BER in each of the timing channels due to MAI is kept at desired levels. We make the following assumptions in the analysis of the multi-terminal covert timing network.

- The system has bandwidth \( W \) Hz.
- Each transmitter can transmit at a maximum power \( P_{max} \).
- The channel gain from transmitter \( i \) to receiver \( j \) is \( h_{ij} \).
- The channel gain matrix, \( H \), is given by \( H = [h_{ij}]_{1 \leq i \leq M, 1 \leq j \leq M} \).
- In addition to the interference from other transmitters, the channel between the \( i \)th transmit-receive pair experiences additive white Gaussian noise (AWGN) with power spectral density \( N_0 \) Watts/Hz.

The signal-to-interference noise ratio (SINR) obtained by the \( i \)th receiver, \( x_i \), is given by

\[
x_i = \frac{P_i h_{ii} G_i}{\sum_{j \neq i} P_j h_{ji} + N_0 W},
\]

where \( G_i \) is the spreading gain at receiver \( i \), given by \( G_i = \frac{W}{r_i} \), where \( r_i \) is the rate at which the \( i \)th transmitter transmits. If the BER experienced by the \( i \)th receiver is \( BER_i(x_i) \), the effective rate of overlay transmission of the \( i \)th transmitter, \( u_i \), is

\[
u_i = r_i (1 - 2BER_i(x_i)) = r_i f(x_i),
\]

where the factor 2 appears in order to avoid degenerate solutions and infinite utility [16]. The effective goodput of the \( i \)th timing channel, \( r_e^{(i)} \), is then given by (11) by replacing \( t_0 \) by \( t_0^{(i)} \), \( \sigma_0 \) by \( \sigma_0^{(i)} \), \( \sigma_1 \) by \( \sigma_1^{(i)} \) and \( t_p \) by \( t_p^{(i)} = L_p/u_i \).

It is observed that the effective goodput of the \( i \)th timing channel depends on the SINR, \( x_i \), which, in turn, depends on the transmit power \( P_i \) of the \( i \)th transmitter. Therefore, it is essential to determine the optimal transmit power for each transmitter that maximizes \( r_e^{(i)} \) for each \( i \). It is also observed from (11) that the effective goodput, \( r_e \), increases when \( t_p \) decreases, i.e., \( u_i \) increases and is maximum when \( u_i \) is maximum. Therefore, in order to maximize the effective goodput of the \( i \)th timing channel, it is essential to maximize \( u_i \) and hence, determine the optimal transmit power for each transmitter that maximizes \( u_i \) \( \forall i \). Thus, it is observed that the optimal transmit powers assist in maximizing the effective goodput of all the timing channels. Hence, the network is a “power assisted multi-terminal covert timing network”.

Determining \( P_i \) for each \( i \) so as to maximize \( u_i \) for each \( i \) can be formulated as the following optimization problem

\[
\max_{P_i} u_i \quad \forall i
\]

subject to \( 0 \leq P_i \leq P_{max} \) \( \forall i \),

where \( P = [P_1 \ P_2 \ \cdots \ P_M] \). Since \( u_i \) depends on \( x_i \), which, in turn, depends on the powers transmitted by all

\footnote{It is noted that the power control is only to maximize the effective rate of overlay transmission and hence, does not affect the covertness aspect as well as the packet count distribution.}
the transmitters, the optimization problem in (17) subject to the constraints (18) can be modeled as an $M$-player non-cooperative game. It was shown in [17] that the Nash equilibrium for the game described by the optimization problem in (17) subject to the constraints in (18) occurs when all transmitters transmit at maximum powers. A pricing function was proposed in [17], which depends on the SINR, $x_i$, to improve the energy efficiency. We apply a similar pricing function, which is a function of the SINR, $x_i$ given by [17]

$$u_p(x_i) = \lambda \frac{x_i}{x_i + G_i},$$  

(19)

where $\lambda$ is the pricing parameter. Larger values of $\lambda$ represent heavy pricing and a smaller value indicates light pricing.

Maximizing the goodput of each timing channel in the power assisted multi-terminal covert timing network incorporating the pricing function in (19) can be modeled as the following optimization problem.

$$\max_{p} u_{net} = \max_{p} \left[ u_i - u_p(i) \right] \quad \forall i$$  

(20)

subject to the constraints in (18).

The optimization problem described by (20) can also be modeled as an $M$-player non-co-operative game. Since $x^{(i)}_{net}$ depends only on $x_i$ and not other $x_j$'s, $j \neq i$, one can obtain $x^{*}_i$ which maximizes $u^{(i)}_{net} \quad \forall i$. Let $x^{*} = \begin{bmatrix} x^*_1 & x^*_2 & \cdots & x^*_M \end{bmatrix}$ be the vector of optimal $x_i$'s which solve (20). From $x^{*}$, $p^{*}$ can be obtained by solving the matrix equation [17]

$$p^{*} = N_0 W (I_M - D_1^{-1} A)^{-1} D_1^{-1} D_2^{-1} 1,$$  

(21)

where $I_M$ is the $M \times M$ identity matrix, $1$ is the column vector of all 1's, $D_1 = \text{diag} \left( \frac{G_i}{h_{ij}} \right)_{1 \leq i \leq M}$, $D_2 = \text{diag} \left( h_{ij} \right)_{1 \leq i \leq M}$ and $A = [a_{ij}]_{1 \leq i \leq M}$. It is observed from (21) that the Nash equilibrium for the game described by the optimization problem in (20) is then given by the solution of (21).

It is essential to choose $f(x_i)$ which is a non-negative $(f(x_i) \geq 0)$, increasing $(f'(x_i) > 0)$, concave function $(f''(x_i) < 0)$, $\forall x_i$, with $f(0) = 0$ [18]. The optimum value of $x_i$'s that solve (20) can be obtained from the first order necessary condition (i.e., by solving $\frac{du^{(i)}_p}{dx_i} = 0$) as the value of $x_i$ that solves

$$g(x_i) \triangleq \left( 1 + \frac{x_i}{G_i} \right)^2 f'(x_i) = \frac{\lambda}{W}.$$  

(22)

It is observed from (22) that

$$g'(x_i) = \frac{x_i + G_i}{G_i^2} \left[ 2f'(x_i) + (x_i + G_i)f''(x_i) \right].$$

If $f(x_i)$ is chosen such that

$$2f'(x_i) + (x_i + G_i)f''(x_i) < 0,$$  

(23)

the $g(x_i)$ is a decreasing function of $x_i$. Although (23) appears restrictive, choices of the BER function $BER_i(x_i)$ for a differential phase shift keying (DPSK) and $M$-ary phase shift keying [19] results in $f(x_i)$ that satisfy (23). The following theorems proved in [17] then apply.

**Theorem 4.1:** [17] Let $\lambda_{max} = W f'(0)$. Then, the necessary condition for the optimization problem in (20) to have a feasible solution is $\lambda < \lambda_{max}$.

**Theorem 4.2:** [17] An SINR vector $x^*$ with all positive entries results in a power vector $p^*$ with all positive entries if and only if the $Z$-matrix $(I_M - D_1^{-1} A)$ is an $M$-matrix.

**Theorem 4.3:** [17] $\exists \lambda_{min}$ such that $\forall \lambda \in (\lambda_{min}, \lambda_{max})$, with $\lambda_{max}$ as specified in Theorem 4.1, the $Z$-matrix $(I_M - D_1^{-1} A)$ is an $M$-matrix and hence, the game described by the optimization problem in (20) subject to constraints (18) has a unique feasible Nash equilibrium.

By choosing $\lambda \in (\lambda_{min}, \lambda_{max})$, the Nash equilibrium can be obtained by solving the matrix equation (21). Transmitter $i$ then transmits covert information whose characteristics are specified by the 3-tuple, $(P_i, t_0^{(i)}, \ell_1^{(i)})$. The value of $t_p^{(i)}$ computed as $t_p^{(i)} = L_p/u_i^*$, where $u_i^*$ is the value of $u_i$ corresponding to the value of $x_i$ that solves (22). The maximum value of the goodput of the $i$th timing channel is obtained from (14).

As an illustration, consider an $M = 10$ terminal covert timing network with bandwidth $W = 20$ MHz, $N_0 = 0.01$ pico Watts/Hz with the $H$ matrix generated using the Jake's channel model [20]. Consider a single eavesdropper in the system, with $h_e = [h_{1e} \ h_{2e} \cdots h_{Me}]$, where $h_{ie}$ denotes the channel gain from the $i$th transmitter to the eavesdropper, generated using Jake's model. Let $BER_i(x_i)$ be the bit error rate for a system with DQPSK and hence, given by $BER_i = \frac{1}{2} e^{-x_i}$. [19]. Therefore, $f(x_i)$ is given by

$$f(x_i) = 1 - e^{-x_i}.$$  

(24)

It is observed that $f(x_i)$ in (24) satisfies (23) for $G_i > 2$.

We first study one of the timing channels. Fig. 5 presents the behavior of the timing channel goodput vs the rate of transmission, $r_i$, for one transmit-receive pair in a 10-terminal network. It is observed from Fig. 5 that the effective timing channel goodput increases with increasing rate of transmission. This agrees with the behavior observed in Section IV-A.

![Fig. 5. Effective goodput of one transmit-receive pair.](image)

Table I presents the performance of each timing channel of a sample path of the $H$ matrix and $h_e$ vector generated by the Jake's simulator. The $i$th transmitter transmits at a rate $r_i$ (in Mbps) uniformly distributed in the in $[2, 10]$. It is noted

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6Definitions and properties of $Z$ and $M$-matrices can be found in [17].
that Table I presents only a snapshot of the system, i.e., a particular sample path of the simulation performed using the Jake’s simulation model. One can obtain a different set of values for a different sample path. More precisely, from the results provided in Table I, one must not infer that timing channel 5 always obtains maximum effective timing channel goodput. However, at a particular instant one can obtain a relative behavior of all the timing channels as represented by Table I. Table I indicates that the optimum SINR is almost the same for all the users. This behavior was observed for several sample paths. It then raises the following questions, to answer which, we perform an asymptotic analysis of multi-terminal covert timing networks in the following subsection.

1) Can this behavior be generalized when the number of terminals, $M$, increases?

2) If the generalization is true, then can it be exploited to provide insights on the overlay and covert timing network characteristics?

C. Asymptotic Spectral Efficiency

When the number of transmit-receive pairs, $M$ and the available bandwidth, $W$, become large\(^9\), then solving (22) $M$ times and then solving (21) becomes computationally very complex. We now present the analysis for a system in which the number of transmit-receive pairs, $M$ and the system bandwidth, $W$ become very large such that $M/W = \rho$. The objective is to obtain results that can generalize the optimum $x_i^*$, $\forall i$, when $M$ and $W$ are large. This is used to evaluate the maximum asymptotic spectral efficiency and then applied to obtain the optimal rates of overlay packet transmission.

The spectral efficiency, $S$, is defined as

$$S \triangleq \frac{1}{W} \sum_{i=1}^{M} u_i^*, \quad (25)$$

where $u_i^*$ is the value of $u_i$ for $x_i = x_i^*$. Before evaluating the asymptotic spectral efficiency, we re-state some Definitions and Lemmas from [17], which we will be using in our analysis to obtain the maximum asymptotic spectral efficiency. For any $k$ such that $1 \leq k \leq M$, let $D_1^{(k)}$ and $A^{(k)}$ denote the $k \times k$ leading principal sub-matrices (i.e., the sub-matrices specified by the first $k$ rows and columns) of $D_1$ and $A$, respectively. It is observed that the $Z$-matrix $Z^{(k)} \triangleq \left( I_k - (D_1^{(k)})^{-1} A^{(k)} \right)$ can be written as

$$Z^{(k)} \triangleq \left( I_k - (D_1^{(k)})^{-1} A^{(k)} \right) = \begin{bmatrix} Z^{(k-1)} & f_k \\ g_k^T & 1 \end{bmatrix}, \quad (26)$$

for $k \geq 2$, where $Z^{(1)} = [1]$.

The matrix $(I_M - D_1^{-1}A)$ can be formed by a sequence of matrices $Z^{(1)}, Z^{(2)}, \ldots, Z^{(M)}$, where $Z^{(k)}$ is as defined in (26), $\forall k$ such that $2 \leq k \leq M$.

**Lemma 4.1:** [17] Let $Z^{(k)}$ be as defined in (26) with $f_k$ and $g_k$ as defined in (27) and (28), respectively. Then, the power control game with pricing has a unique Nash equilibrium, i.e., $Z^{(k)}$ is an $M$-matrix if and only if $g_k^T (Z^{(k-1)})^{-1} f_k < 1$.

**Lemma 4.2:** [17] Let $Z^{(k)}$ be as defined in (26) with $f_k$ and $g_k$ as defined in (27) and (28), respectively. Then, the function $g_k^T (Z^{(k-1)})^{-1} f_k$ is an increasing function of $\chi$, $\forall k$ such that $2 \leq k \leq M$, i.e., $g_k^T (Z^{(k-1)})^{-1} f_k$ increases when every term in $\chi$ increases.

Lemmas 4.1 and 4.2 yield the following Theorems.

**Theorem 4.4:** For large $M$ and $W$ such that $M/W = \rho$, all transmit-receive pairs obtain equal optimum SINR, $x_i^*$.

**Proof:** Let $G = \begin{bmatrix} G_1 & G_2 & \cdots & G_M \end{bmatrix}$. Let $G_{\text{max}}$ be the maximum entry in $G$. Let $G_{\text{max}}$ denote the vector $G$ with all entries as $G_{\text{max}}$. Let $\theta(x, G) \triangleq g_{\text{max}}^T (Z^{(M-1)})^{-1} f_{\text{max}}$. Therefore, $\theta(x, G) < 1$ for a Nash equilibrium to exist. Let $\Gamma$ be the values of $x_i$ such that $f'(\Gamma) = \frac{1}{W}$. Since $f(x_i)$ is a concave function, $f'(x_i)$ is a non-increasing function of $x_i$. Therefore, from (22), $x_i \geq \Gamma \forall i$. From Lemma 4.2, and the definition of $\theta(x, G)$, $\theta(x, G) > \theta(\Gamma, G)$, where $\Gamma$ is the vector with all entries as $\Gamma$. Similarly, $\theta(x, G)$ is a decreasing function of $G$ and hence $\theta(x, G) > \theta(x, G_{\text{max}})$. Therefore, $\theta(x, G) > \theta(\Gamma, G_{\text{max}})$. The Perron-Frobenius eigen value of $A$, $\chi(A) < 1$. Therefore, $\theta(\Gamma, G_{\text{max}})$ can be upper bounded by replacing all the off-diagonal elements of $Z^{(M-1)}$ by $-1$. $(Z^{(M-1)})^{-1}$ can then be evaluated using the Sherman-Morrison formula [22] to yield

$$\theta(\Gamma, G_{\text{max}}) \leq \frac{\Gamma^2 (M-1)^2}{G_{\text{max}}^2}. \quad (29)$$

The above bound becomes equal to 1 when

$$\Gamma = \frac{G_{\text{max}}}{M-1} = \frac{1}{r_{\text{min}} (\rho - \frac{1}{W})}, \quad (30)$$

where $r_{\text{min}} = \min_i r_i$. $\Gamma < \frac{1}{r_{\text{min}} (\rho - \frac{1}{W})}$, which is finite for large $M$, $W$. Similarly, using the fact that $\theta(x, G) < \theta(x, G_{\text{min}})$, where $G_{\text{min}}$ is the vector with all entries $G_{\text{min}}$ where $G_{\text{min}}$ is the minimum of all the entries in $G$, $x$ is also finite for large $M$, $W$. Since $G_i = W/r_i$, $G_i$ can be arbitrarily large for large $W$. Therefore from (22), for large $M$, $W$, $f'(\Gamma) = \frac{1}{W}$. Thus all transmit-receive pairs obtain equal optimum SINR.
The maximum asymptotic spectral efficiency of the power assisted multi-terminal covert timing network, \( \eta \), is given by

\[
\eta = \min \left( f'(0), \min_{\rho} \right),
\]

(32)

where \( r_{\text{max}} = \max_i r_i \).

**Proof:** Let \( \Gamma = \sup \lambda \Gamma \). From Theorem 4.5, \( \Gamma = \frac{W}{\sum_{i=1}^{M} r_i} \), and using (25) and Theorem 4.4, the asymptotic spectral efficiency is \( S(\Gamma^*) = \frac{f'(\Gamma^*)}{\Gamma^*} \). From the first order necessary conditions for maxima, the optimum value of \( \Gamma \) that maximizes \( S(\Gamma^*), \Gamma^* \), is obtained by equating \( S'(\Gamma) \) to zero, i.e., as the solution to

\[
\Gamma^* f'(\Gamma^*) - f'(\Gamma^*) = 0.
\]

(33)

The second derivative of \( S(\Gamma) \) evaluated at \( \Gamma = \Gamma^* \) is

\[
S''(\Gamma^*) = \frac{f''(\Gamma^*)}{\Gamma^*},
\]

which, from (33), yields \( Y = X f'(\Gamma^*) \), which represents a line passing through \((0, 0)\). Therefore \( T(\Gamma^*) \), intersects the curve at two points, \((0, 0)\) (since \( f'(0) = 0 \)) and \((\Gamma^*, f(\Gamma^*))\). Since \( T(\Gamma^*) \) is a tangent, \( \Gamma^* = 0 \).

Let the maximum value of \( S(\Gamma) \) (evaluated at \( \Gamma = \Gamma^* \)) be \( \eta \). Since \( \Gamma^* = 0, \eta \) can be written as

\[
\eta = \lim_{\Gamma \to 0} S(\Gamma) = \lim_{\Gamma \to 0} \frac{f'(\Gamma^*)}{\Gamma} = f'(0),
\]

(35)

by applying L'Hospital's rule. Note that since \( r_i \leq r_{\text{max}} \ \forall \ i \), \( S(\Gamma) \leq \rho r_{\text{max}} \) with equality if and only if \( r_i = r_{\text{max}}, \ \forall \ i \). Thus, \( \eta \leq \rho r_{\text{max}} \) and (32) follows.

From Theorem 4.6, the maximum asymptotic spectral efficiency is obtained when \( r_{\text{max}} = f'(0) \), i.e., all transmitters transmit at a rate \( r_{\text{max}} = f'(0) \). Thus, for a system with finite bandwidth, \( W \) and finite number of transmit-receive pairs, \( M \) and \( \rho \), the optimum rate of overlay transmission is \( r_{\text{max}} = \frac{f'(0)}{\rho} \) for all transmit-receive pairs. The power control mentioned in Section IV-B is then applied to determine the optimal SINR's at all the receivers and the optimal transmit powers of all the transmitters. From the values of the optimal SINR's, the effective rate of overlay transmission and thus, the optimal timing channel goodputs can be determined by applying the analysis described in Section IV-A.

**Fig. 6.** Covert timing channel goodput when transmitters transmit over-layer packets at asymptotically optimal rates.

**V. CONCLUSION**

We analyzed the factors of the overlay communication that affect the goodput of multi-terminal covert timing networks. The key inferences include

- Covert timing channels are more effective under overlay applications with larger rates of transmission (e.g., video streaming) or with smaller packet sizes (e.g., ping).
- In a multi-terminal network, asymptotically all the receivers receive equal SINR irrespective of the transmission rates and the relative locations from the transmitters.

**REFERENCES**

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