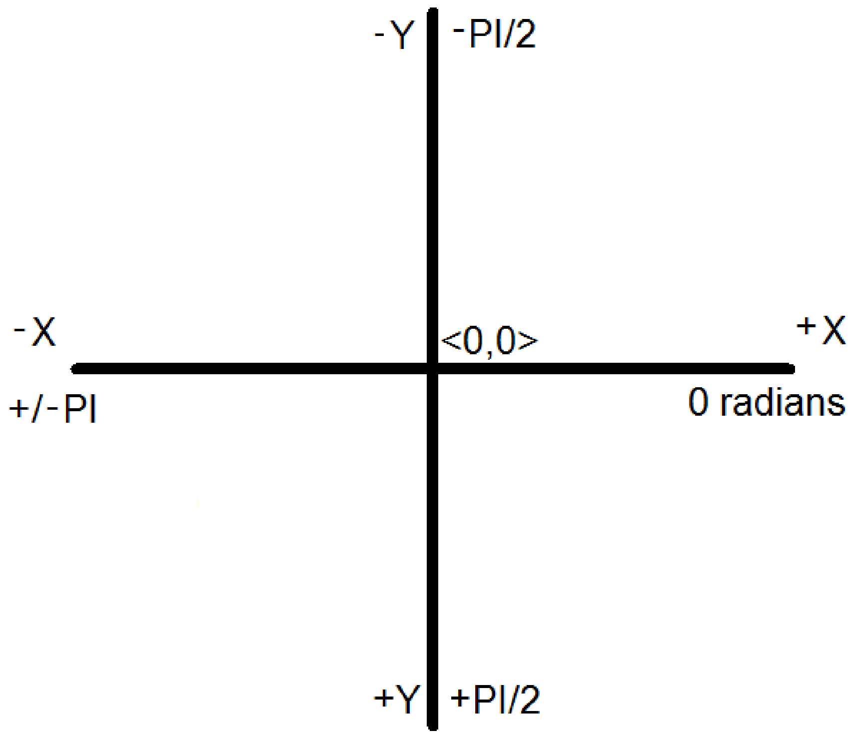
2D Vector Math for Games

Transformation

- We are talking about planar spaces
- 3d spaces need to be transformed
  - For instance, <x,y> = <-a,0,b>
  - There are infinite numbers of possible transformations
    - May include rotations
    - May include scaling
    - Probably won’t require translations

Planar Coordinates

- We will use the following planar coordinate system:



Basic 2d Vector Operations

- Vector Addition (and implicitly subtraction)
- Scalar Multiplication (division, negation)
- Magnitude (vector length)
- Unit Vectors (magnitude, division)
- Vector Comparison (FP precision errors)
- Angle Conversion (to/from radians)
- Dot Product

Variables

- Uppercase: Vector Lowercase: Scalar
- <x,y> - A Vector comprised of Scalar x and y
- Vectors- P: Point, V: Velocity
- Scalars- h: Heading, s: Speed

- $D = P_2 - P_1$
  - D is a vector from P_1 to P_2
  - $|D|$=Distance between P_1 and P_2

Angle Conversion

- Basic Trigonometry – RADIANS!
- From Angle to Vector:

  x = cos(h)   y = sin(h)

  <x,y> is a unit vector, say V_U: $V = V_U$ *s for Velocity
- From Vector to Angle

  h = atan2(y, x)

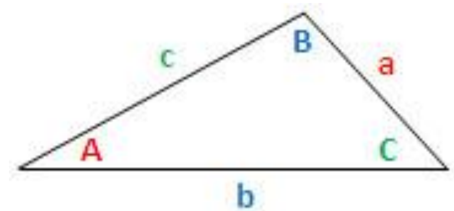
  s = length(<x,y>)

$$\operatorname{atan2}(y,x) = \begin{cases} \arctan(\frac{y}{x}) & x > 0 \\ \pi + \arctan(\frac{y}{x}) & y \geq 0, x < 0 \\ -\pi + \arctan(\frac{y}{x}) & y < 0, x < 0 \\ \frac{\pi}{2} & y > 0, x = 0 \\ -\frac{\pi}{2} & y < 0, x = 0 \\ \text{undefined} & y = 0, x = 0 \end{cases}$$

Dot Product

- Analogous to the Law of Cosines

  $c^2 = a^2 + b^2 - 2ab\,cos(\text{angle})$



$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

- Dot Product

  $A \cdot B = |A||B|\,cos(\text{angle})$

- Rearranged

  $cos(\text{angle}) = (A \cdot B) / (|A||B|)$

  $\text{angle} = cos^{-1}((A \cdot B) / (|A||B|))$

- ***Very useful for Interception of Moving Objects***

Interception of Moving Objects

- Things We Know about Coyote and Roadrunner

  P_C, P_R, V_R, s_C: Positions, Tgt Velocity and My Speed

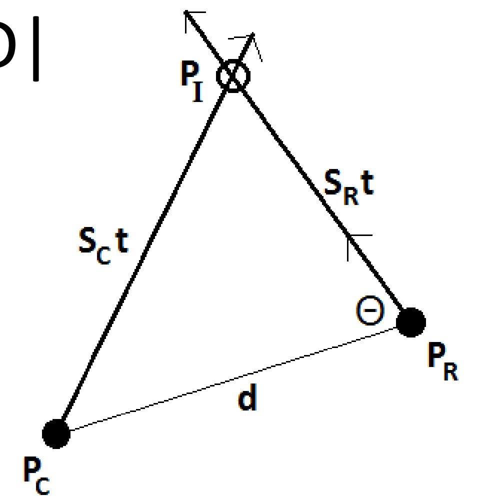
  t = time, $s_R = |V_R|$, $D = P_R - P_C$, $d=|D|$

  P_I = Point of Interception

  $\cos\theta = (V \cdot D) / (ds_R)$

- Law of Cosines tells us:

  $(s_C t)^2 = (s_R t)^2 + d^2 - 2 s_R t d \cos\theta$



This reduces to a Quadratic Equation in 't'