Artificial Intelligence

CS482, CS682, MW 1 – 2:15, SEM 201, MS 227

Prerequisites: 302, 365

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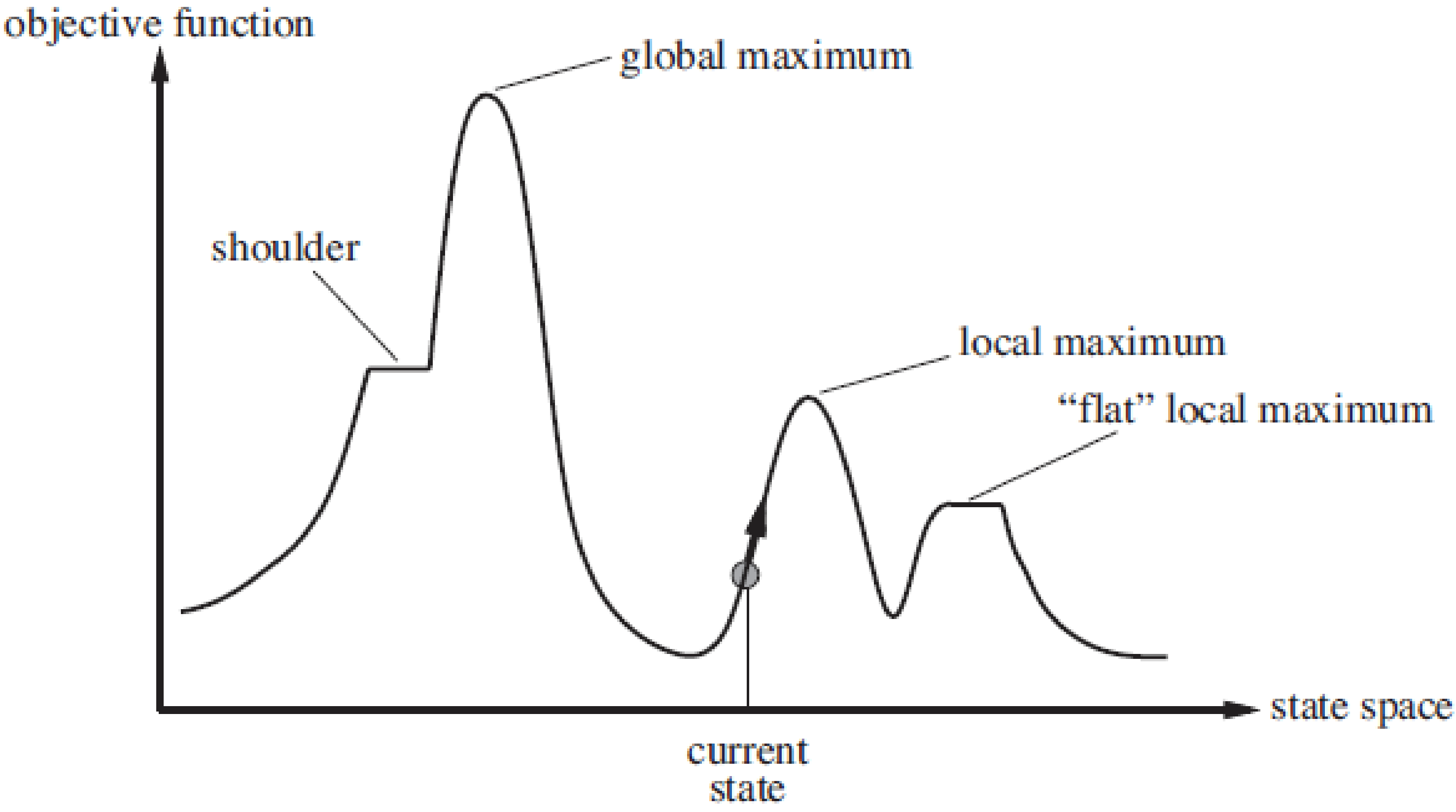
Non-classical search

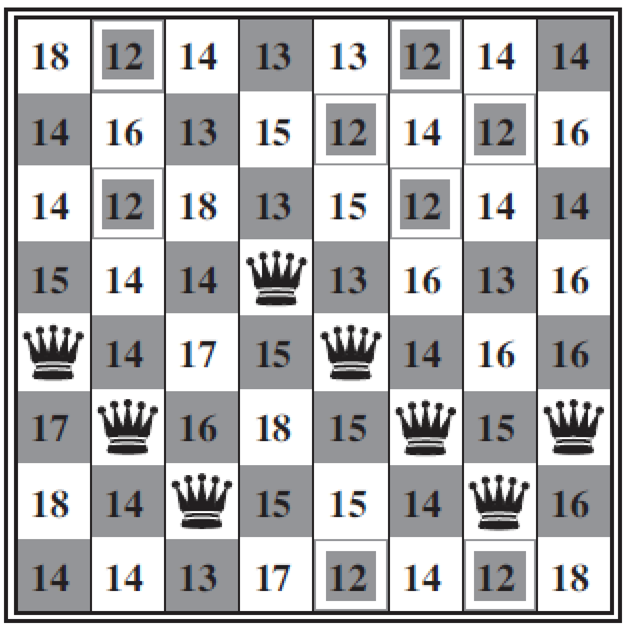


- Path does not matter, just the final state
- Maximize objective function

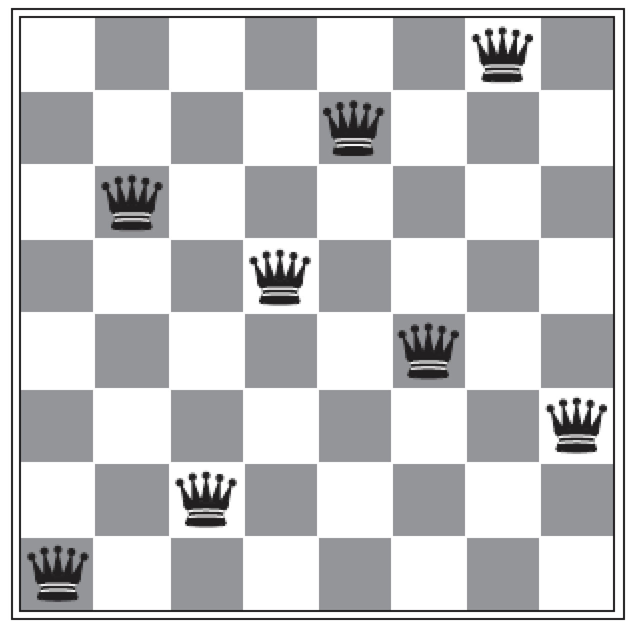
Local optimum

- Heuristic: Number of pairs of queens attacking each other directly
- Movement: only within your column

H = 17

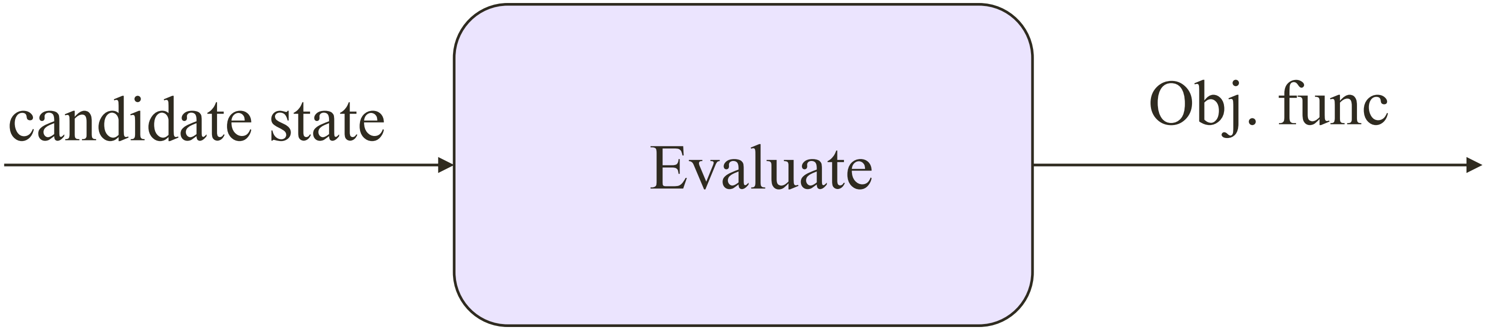


H = 1 but all successors have > 1



Model

- We have a black box "evaluate" function that returns an objective function value



Application dependent fitness function

Local Hill Climbing

function HILL-CLIMBING(*problem*) **returns** a state that is a local maximum

current ← MAKE-NODE(*problem*.INITIAL-STATE)
loop do
neighbor ← a highest-valued successor of *current*
if neighbor.VALUE ≤ current.VALUE **then return** *current*.STATE
current ← *neighbor*

- Move in the direction of increasing value
- Very greedy
- Subject to
  - Local maxima
  - Ridges
  - Plateaux
- 8-queens: 86% failure, but only needs 4 steps to succeed, 3 to fail

Hill climbing

- Keep going on a plateau?
  - Advantage: Might find another hill
  - Disadvantage: infinite loops → limit number of moves on plateau
    - 8 queens: 94% success!!
- Stochastic hill climbing
  - randomly choose from among better successors (proportional to obj?)
- First-choice hill climbing
  - keep generating successors till a better one is generated
- Random-restarts
  - If probability of success is *p*, then we will need 1/p restarts
  - 8-queens: p = 0.14 ~= 1/7 so 7 starts
  - 6 failures (3 steps), 1 success (4 steps) = 22 steps
  - In general: Cost of success + (1-p)/p * cost of failure
  - 8-queens sideways: 0.94 success in 21 steps, 64 steps for failure
    - Under a minute

Simulated annealing

```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
  inputs: problem, a problem
          schedule, a mapping from time to "temperature"

  current ← MAKE-NODE(problem.INITIAL-STATE)
  for t = 1 to ∞ do
     T ← schedule(t)
     if T = 0 then return current
     next ← a randomly selected successor of current
     ΔE ← next.VALUE – current.VALUE
     if ΔE > 0 then current ← next
     else current ← next only with probability e^(ΔE/T)
```

- Gradient descent (not ascent)
- Accept bad moves with probability $e^{dE/T}$
- T decreases every iteration
- If *schedule(t)* is slow enough we approach finding global optimum with probability 1

Beam Search

Idea: keep k states instead of 1; choose top k of all their successors

Not the same as k searches run in parallel!
Searches that find good states recruit other searches to join them

Problem: quite often, all k states end up on same local hill

Idea: choose k successors randomly, biased towards good ones

Observe the close analogy to natural selection!

Genetic Algorithms

- Stochastic hill-climbing with information exchange
- A population of stochastic hill-climbers

```
function GENETIC-ALGORITHM(population, FITNESS-FN) returns an individual
  inputs: population, a set of individuals
          FITNESS-FN, a function that measures the fitness of an individual

  repeat
     new_population ← empty set
     for i = 1 to SIZE(population) do
        x ← -SELECTION(population, FITNESS-FN)
        y ← -SELECTION(population, FITNESS-FN)
        child ← REPRODUCE(x, y)
        if (small random probability) then child ← MUTATE(child)
        add child to new_population
     population ← new_population
  until some individual is fit enough, or enough time has elapsed
  return the best individual in population, according to FITNESS-FN
```

```
function REPRODUCE(x, y) returns an individual
  inputs: x, y, parent individuals

  n ← LENGTH(x); c ← random number from 1 to n
  return APPEND(SUBSTRING(x, 1, c), SUBSTRING(y, c + 1, n))
```

More detailed GA

- Generate pop(0)
- Evaluate pop(0)
- T=0
- While (not converged) do
  - Select pop(T+1) from pop(T)
  - Recombine pop(T+1)
  - Evaluate pop(T+1)
  - T = T + 1
- Done

Generate pop(0)

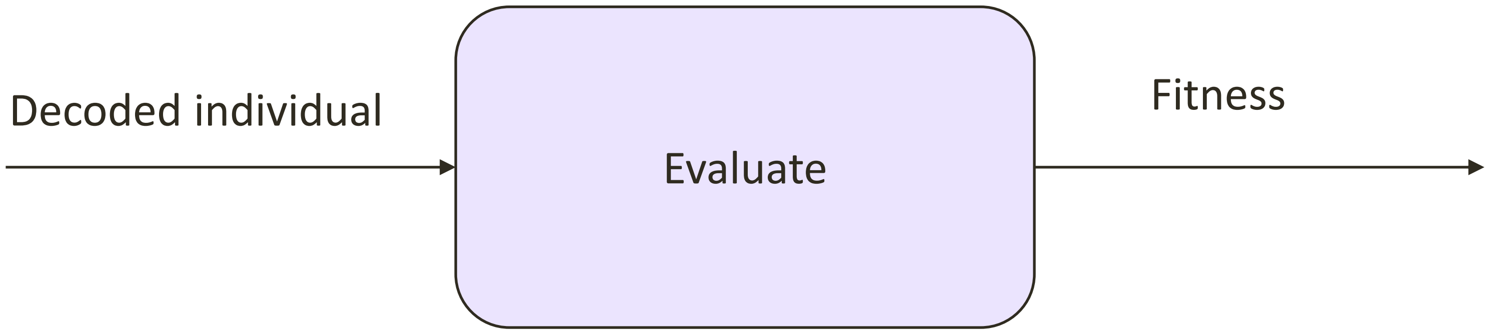
Initialize population with randomly generated strings of 1’s and 0’s

```
for(i = 0 ; i < popSize; i++){
  for(j = 0; j < chromLen; j++){
    Pop[i].chrom[j] = flip(0.5);
  }
}
```

Genetic Algorithm

- Generate pop(0)
- Evaluate pop(0)
- T=0
- While (not converged) do
  - Select pop(T+1) from pop(T)
  - Recombine pop(T+1)
  - Evaluate pop(T+1)
  - T = T + 1
- Done

Evaluate pop(0)



Application dependent fitness function

Genetic Algorithm

- Generate pop(0)
- Evaluate pop(0)
- T=0
- While (T < maxGen) do
  - Select pop(T+1) from pop(T)
  - Recombine pop(T+1)
  - Evaluate pop(T+1)
  - T = T + 1
- Done

Genetic Algorithm

- Generate pop(0)
- Evaluate pop(0)
- T=0
- While (T < maxGen) do
    - Select pop(T+1) from pop(T)
    - Recombine pop(T+1)
    - Evaluate pop(T+1)
    - T = T + 1
- Done

Selection

- Each member of the population gets a share of the pie proportional to fitness relative to other members of the population
- Spin the roulette wheel pie and pick the individual that the ball lands on
- Focuses search in promising areas

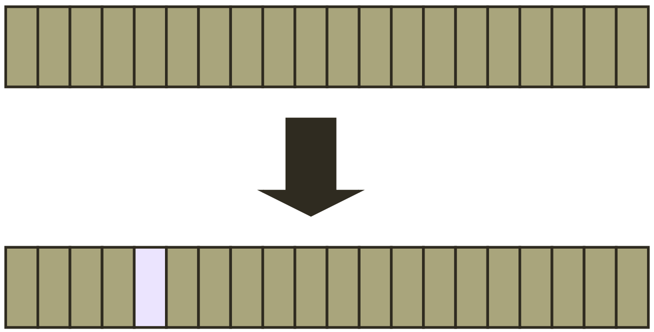
Code

```
int roulette(IPTR pop, double sumFitness, int popsize)
{

  /* select a single individual by roulette wheel selection */

  double rand,partsum;
  int i;

  partsum = 0.0; i = 0;
  rand = f_random() * sumFitness;

  i = -1;
  do{
    i++;
    partsum += pop[i].fitness;
  } while (partsum < rand && i < popsize - 1) ;

  return i;
}
```

Genetic Algorithm

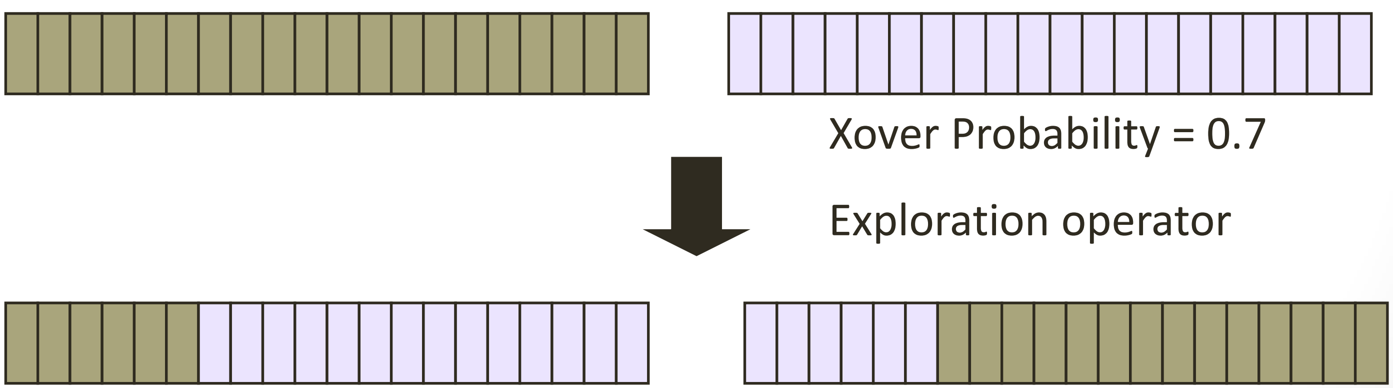
- Generate pop(0)
- Evaluate pop(0)
- T=0
- While (T < maxGen) do
  - Select pop(T+1) from pop(T)
  - Recombine pop(T+1)
  - Evaluate pop(T+1)
  - T = T + 1
- Done

Crossover and mutation



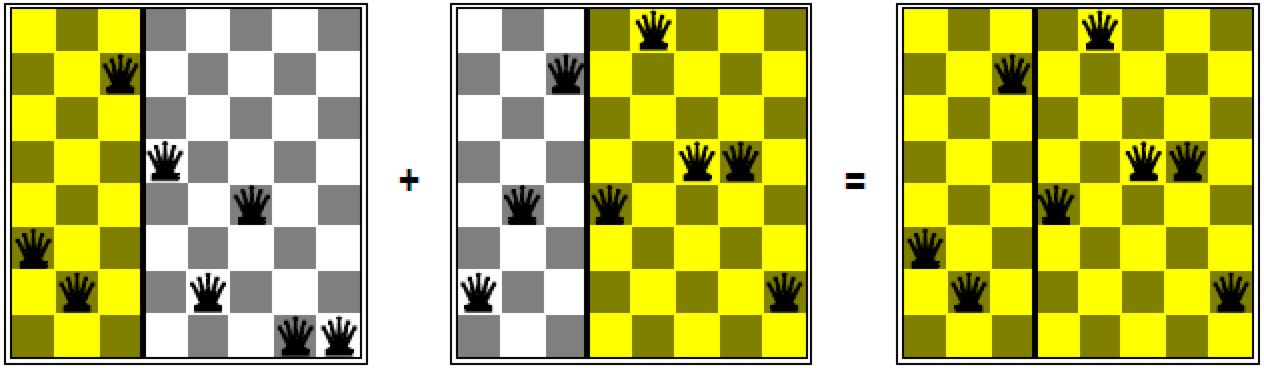
Mutation Probability = 0.001

Insurance



Crossover helps if

Crossover helps iff substrings are meaningful components



Crossover code

```
void crossover(POPULATION *p, IPTR p1, IPTR p2, IPTR c1, IPTR c2)
{
/* p1,p2,c1,c2,m1,m2,mc1,mc2 */
  int *pi1,*pi2,*ci1,*ci2;
  int xp, i;

  pi1 = p1->chrom;
  pi2 = p2->chrom;
  ci1 = c1->chrom;
  ci2 = c2->chrom;

  if(flip(p->pCross)){

    xp = rnd(0, p->lchrom - 1);
    for(i = 0; i < xp; i++){
      ci1[i] = muteX(p, pi1[i]);
      ci2[i] = muteX(p, pi2[i]);
    }
    for(i = xp; i < p->lchrom; i++){
      ci1[i] = muteX(p, pi2[i]);
      ci2[i] = muteX(p, pi1[i]);
    }
  } else {
    for(i = 0; i < p->lchrom; i++){
      ci1[i] = muteX(p, pi1[i]);
      ci2[i] = muteX(p, pi2[i]);
    }
  }
}
```

Mutation code

```
int muteX(POPULATION *p, int pa)
{
  return (flip(p->pMut) ? 1 - pa  : pa);
}
```

How does it work

String	decoded	f(x^2)		fi/Sum(fi)	Expected	Actual
01101	13	169	0.14	0.58	1	
11000	24	576	0.49	1.97	2	
01000	8	64	0.06	0.22	0	
10011	19	361	0.31	1.23	1	
Sum		1170	1.0	4.00	4.00	
Avg		293	.25	1.00	1.00	
Max		576	.49	1.97	2.00	

How does it work cont’d

String mate offspring decoded f(x^2)

0110|1	2	01100	12	144		
1100|0	1	11001	25	625		
11|000	4	11011	27	729		
10|011	3	10000	16	256		
Sum				1754		
Avg				439		
Max				729		

Continuous spaces

Suppose we want to site three airports in Romania:

– 6-D state space defined by (x_1, y_2), (x_2, y_2), (x_3, y_3)

– objective function $f(x_1, y_2, x_2, y_2, x_3, y_3) =$
sum of squared distances from each city to nearest airport

Discretization methods turn continuous space into discrete space, e.g., empirical gradient considers $\pm\delta$ change in each coordinate

Gradient methods compute

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial y_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial y_2}, \frac{\partial f}{\partial x_3}, \frac{\partial f}{\partial y_3}\right)$$

to increase/reduce f, e.g., by $\mathbf{x} \leftarrow \mathbf{x} + \alpha \nabla f(\mathbf{x})$

- What is a good value for α ?
  - Too small, it takes too long
  - Too large, may miss the optimum

Newton Raphson Method

Sometimes can solve for $\nabla f(\mathbf{x}) = 0$ exactly (e.g., with one city).
Newton–Raphson (1664, 1690) iterates $\mathbf{x} \leftarrow \mathbf{x} - \mathbf{H}_f^{-1}(\mathbf{x})\nabla f(\mathbf{x})$
to solve $\nabla f(\mathbf{x}) = 0$, where $\mathbf{H}_{ij} = \partial^2 f / \partial x_i \partial x_j$

Linear and quadratic programming

- Constrained optimization
  - Optimize f(**x**) subject to
    - Linear convex constraints – polynomial time in number of variables
      - Linear programming – scales to thousands of variables
    - Convex non-linear constraints – special cases → polynomial time
      - In special cases non-linear convex optimization can scale to thousands of variables

Games and game trees

- Multi-agent systems + competitive environment → games and adversarial search
- In game theory any multiagent environment is a game as long as each agent has “significant” impact on others
- In AI many games were
  - Game theoretically: Deterministic, Turn taking, Two-player, Zero-sum, Perfect information
  - AI: deterministic, fully observable environments in which two agents act alternately and utility values at the end are equal but opposite. One wins the other loses
- Chess, Checkers
- Not Poker, backgammon,

Game types

	deterministic	chance
perfect information	chess, checkers, go, othello	backgammon monopoly
imperfect information	battleships, blind tictactoe	bridge, poker, scrabble nuclear war

Starcraft? Counterstrike? Halo? WoW?

Search in Games

"Unpredictable" opponent $\Rightarrow$ solution is a strategy specifying a move for every possible opponent reply

Time limits $\Rightarrow$ unlikely to find goal, must approximate

Plan of attack:

- Computer considers possible lines of play (Babbage, 1846)
- Algorithm for perfect play (Zermelo, 1912; Von Neumann, 1944)
- Finite horizon, approximate evaluation (Zuse, 1945; Wiener, 1948; Shannon, 1950)
- First chess program (Turing, 1951)
- Machine learning to improve evaluation accuracy (Samuel, 1952–57)
- Pruning to allow deeper search (McCarthy, 1956)

Tic-Tac-Toe

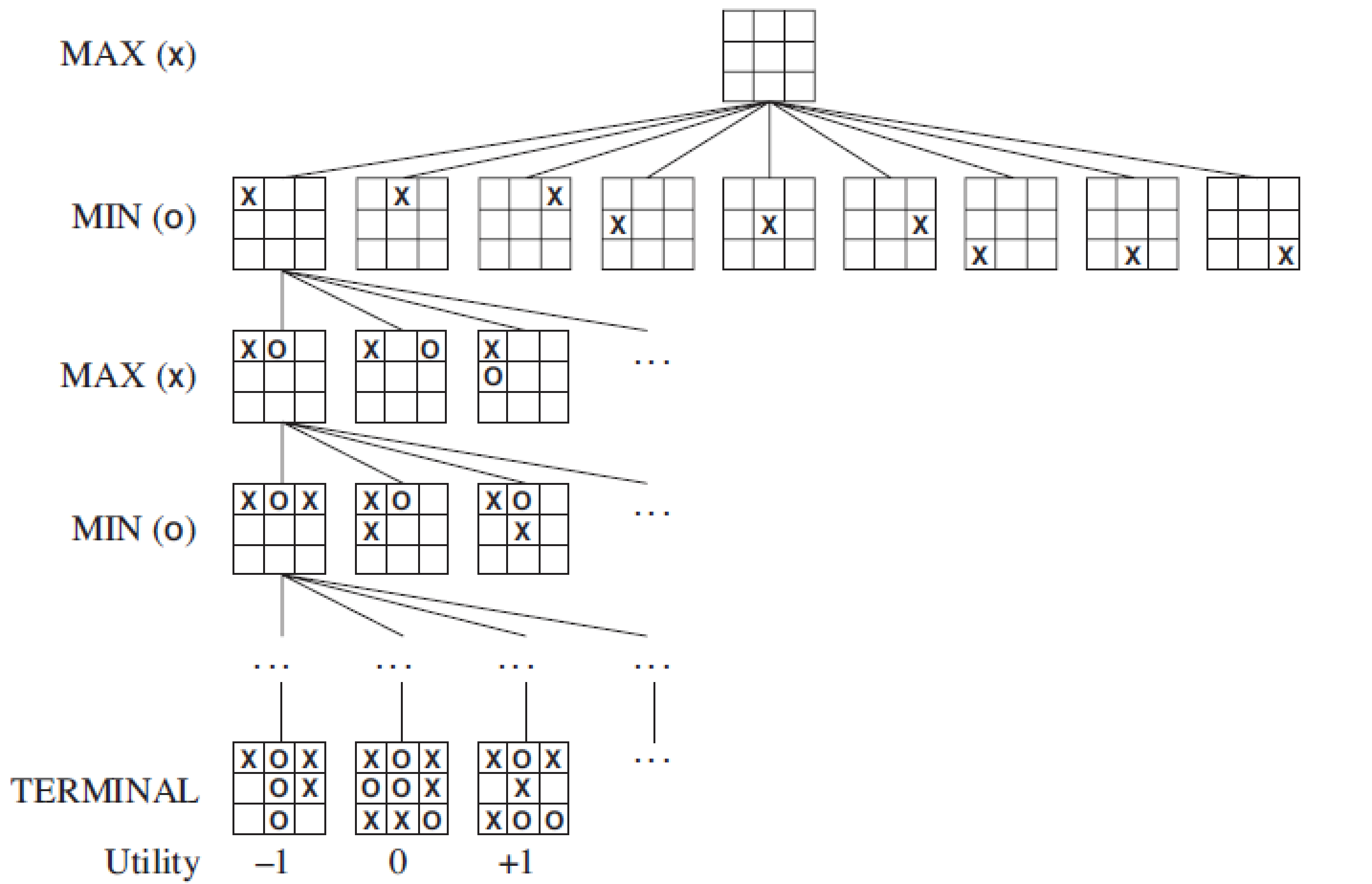


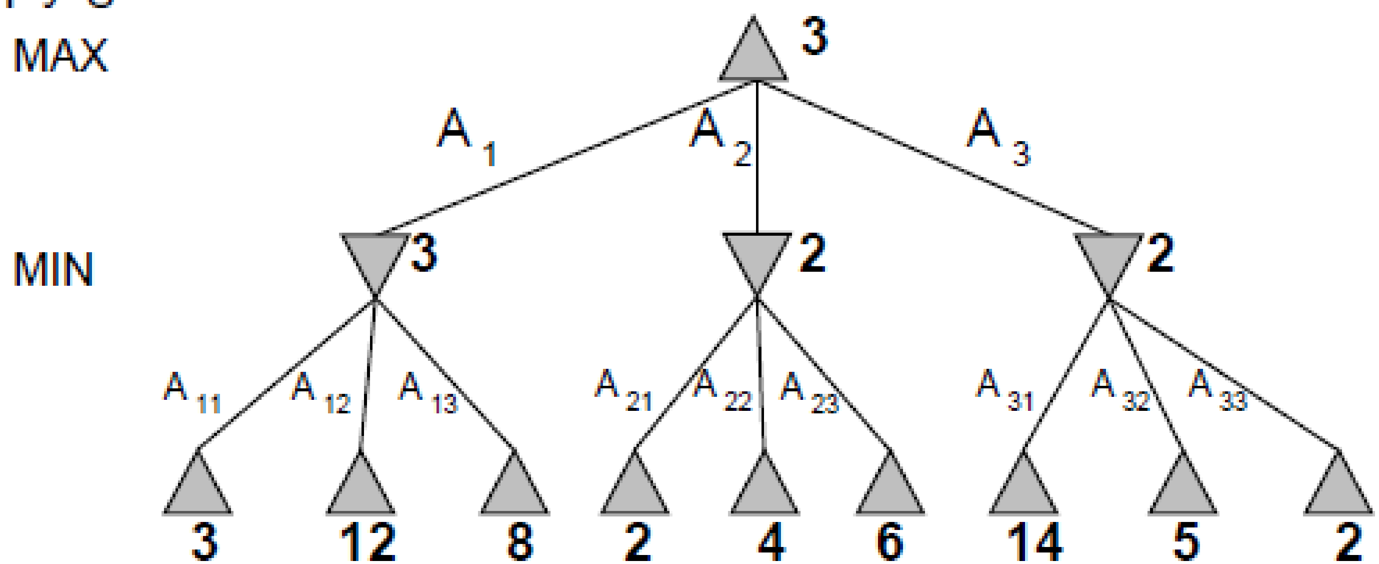
Figure 5.1 **FILES: figures/tictactoe.eps (Tue Nov 3 16:23:55 2009).** A (partial) game tree for the game of tic-tac-toe. The top node is the initial state, and MAX moves first, placing an X in an empty square. We show part of the tree, giving alternating moves by MIN (O) and MAX (X), until we eventually reach terminal states, which can be assigned utilities according to the rules of the game.

Minimax search

Perfect play for deterministic, perfect-information games

Idea: choose move to position with highest minimax value
= best achievable payoff against best play

E.g., 2-ply game:



Minimax algorithm

```
function MINIMAX-DECISION(state) returns an action
   inputs: state, current state in game

   return the a in ACTIONS(state) maximizing MIN-VALUE(RESULT(a, state))
```

```
function MAX-VALUE(state) returns a utility value
   if TERMINAL-TEST(state) then return UTILITY(state)
   v ← −∞
   for a, s in SUCCESSORS(state) do v ← MAX(v, MIN-VALUE(s))
   return v
```

```
function MIN-VALUE(state) returns a utility value
   if TERMINAL-TEST(state) then return UTILITY(state)
   v ← ∞
   for a, s in SUCCESSORS(state) do v ← MIN(v, MAX-VALUE(s))
   return v
```

3 player Minimax

- Two player minimax reduces to one number because utilities are opposite – knowing one is enough
- But there should actually be a vector of two utilities with player choosing to maximize their utility at their turn
- So with three players → you have a 3 vector
- Alliances?

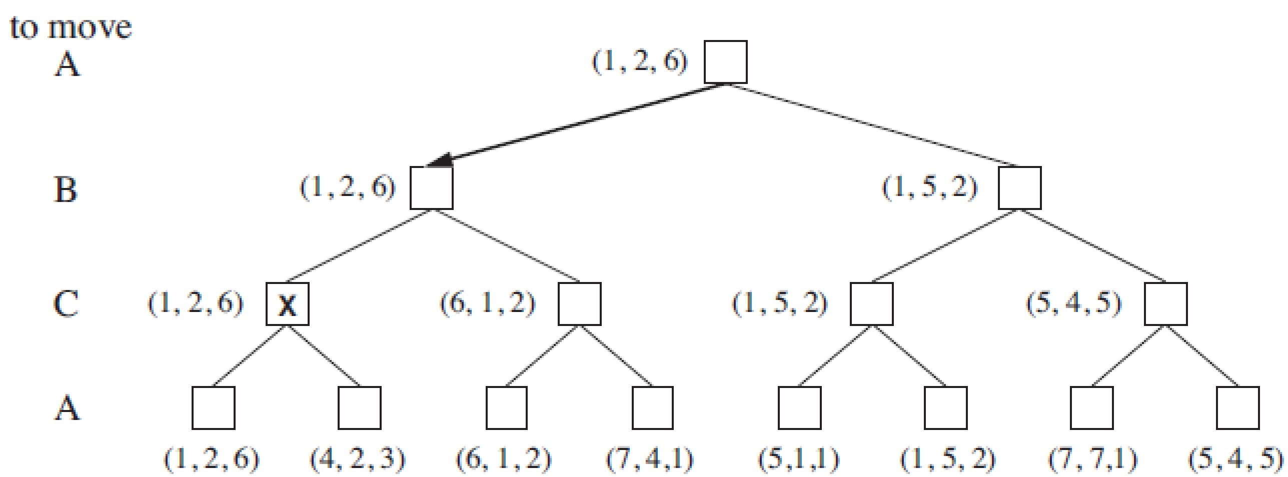
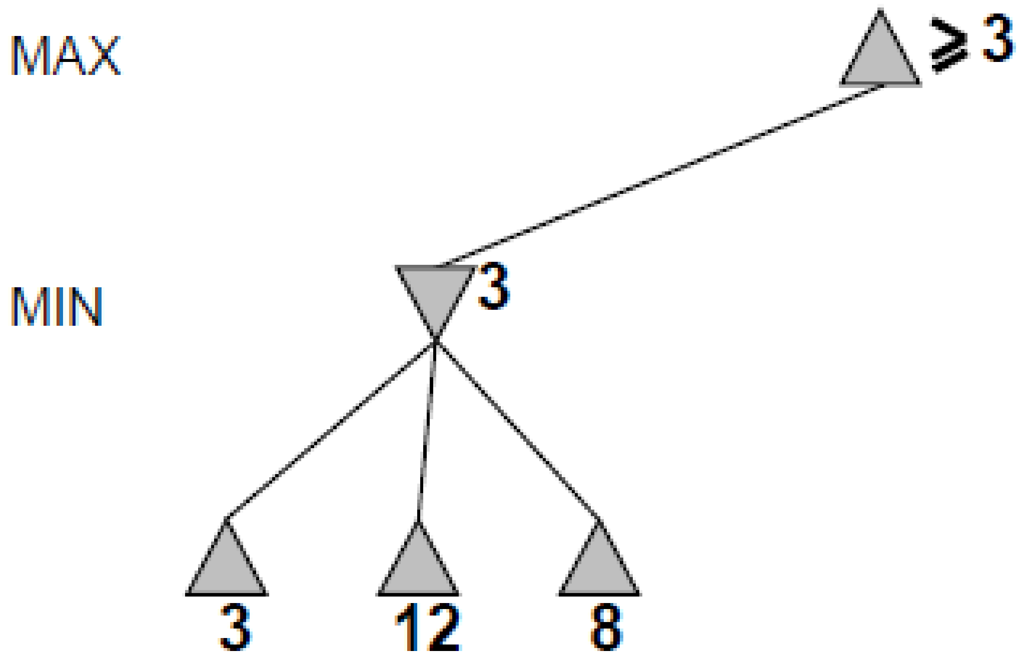


Figure 5.4 **FILES: figures/minimax3.eps (Tue Nov 3 16:23:11 2009).** The first three plies of a game tree with three players (A, B, C). Each node is labeled with values from the viewpoint of each player. The best move is marked at the root.

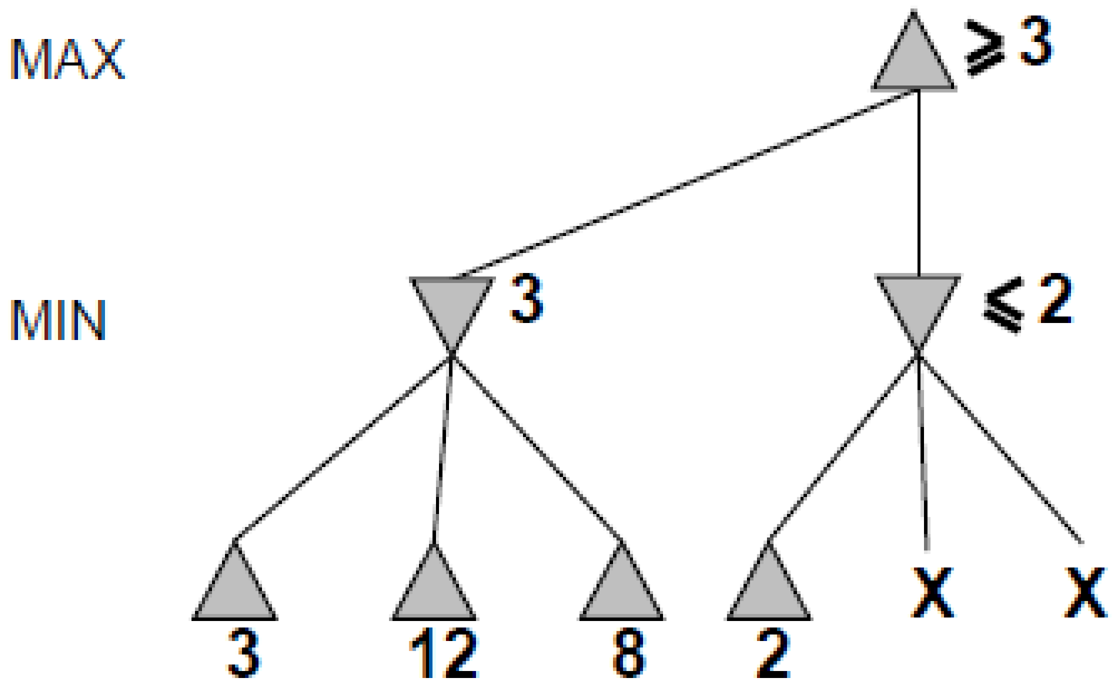
Minimax properties

- Complete?
  - Only if tree is finite
    - Note: A finite strategy can exist for an infinite tree!
- Optimal?
  - Yes, against an optimal opponent! Otherwise, hmmmm
- Time Complexity?
  - O(b^m)
- Space Complexity?
  - O(bm)
- Chess:
  - b ~= 35, m ~= 100 for reasonable games
  - Exact solution still completely infeasible

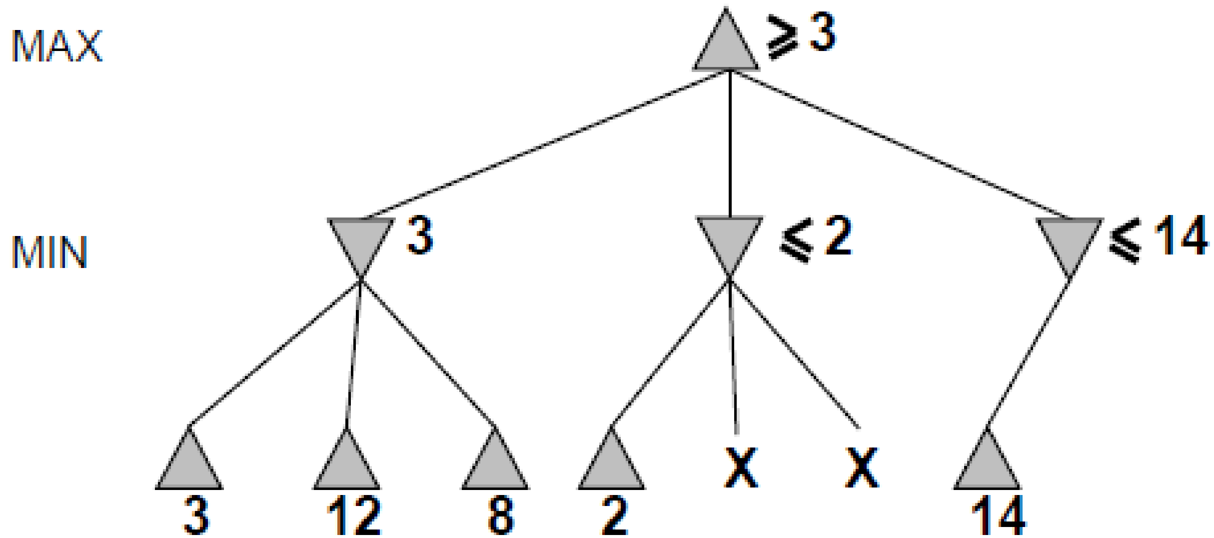
Alpha-beta pruning



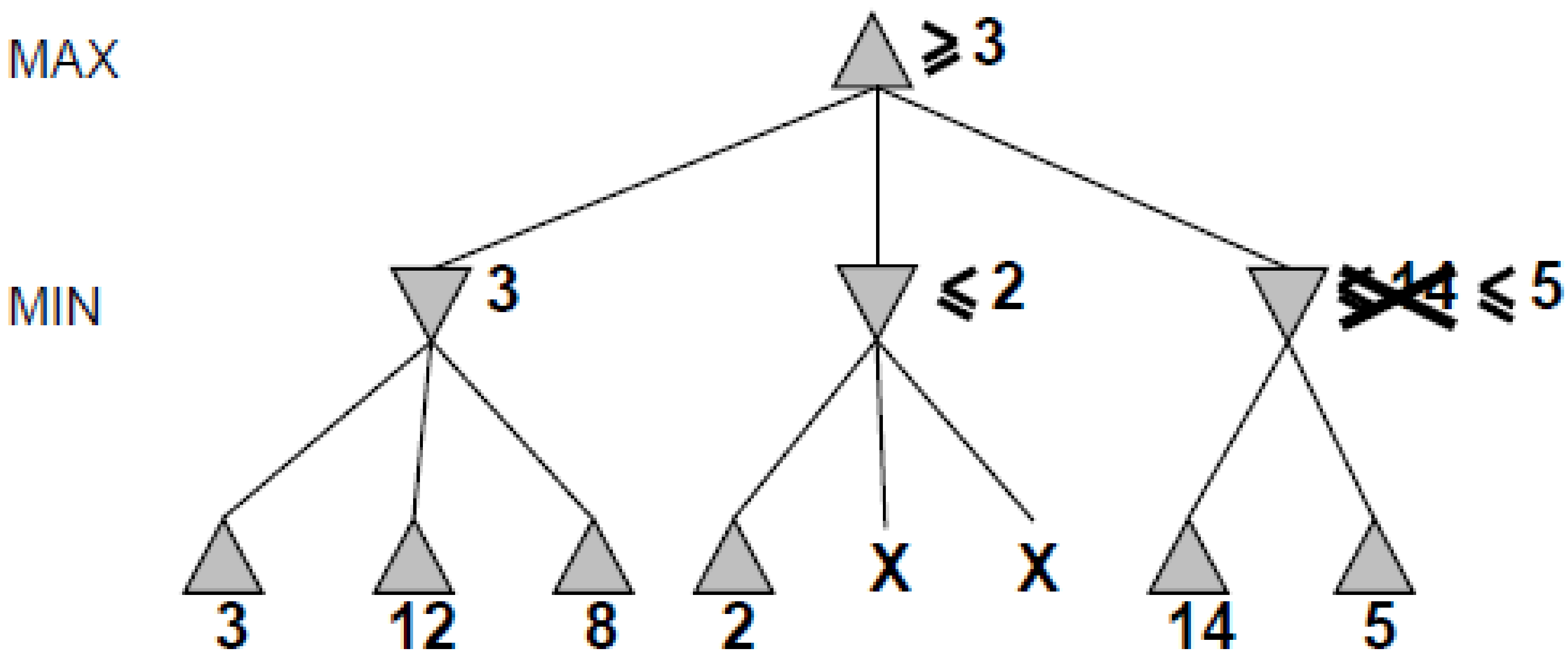
Alpha-beta



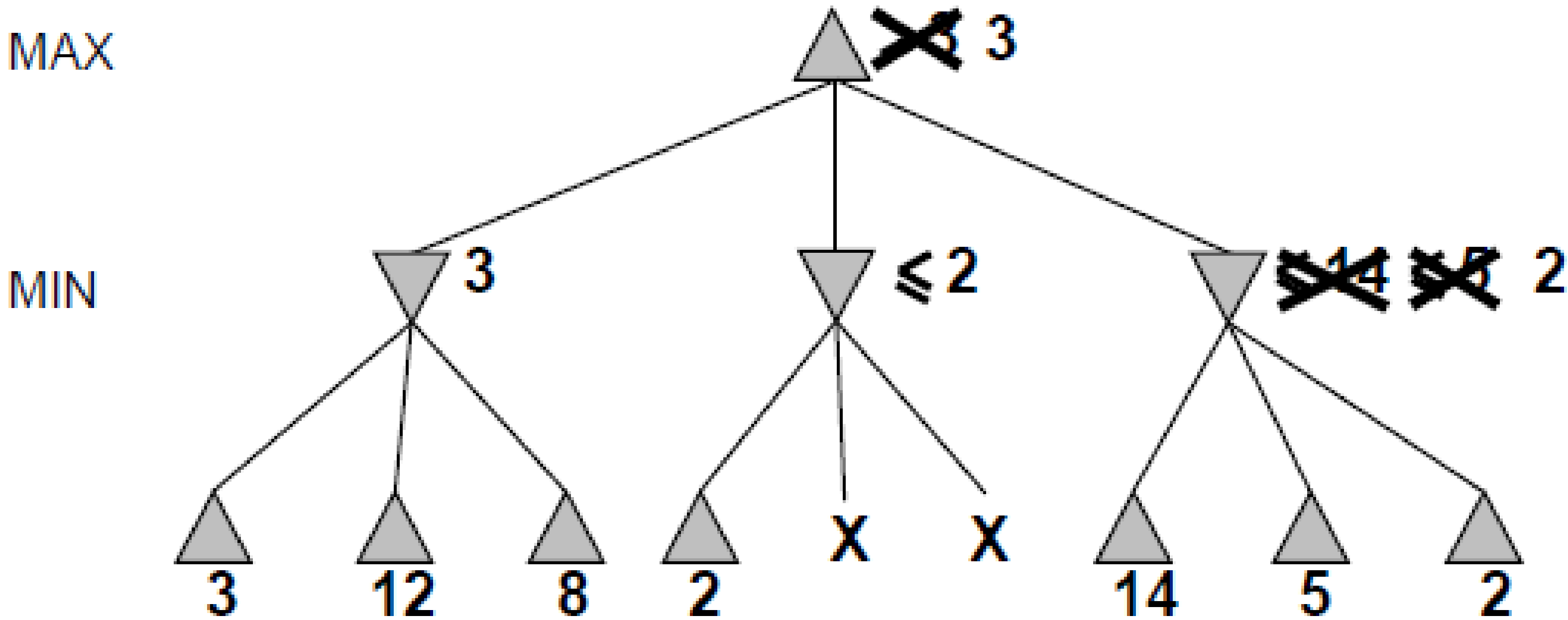
Alpha-beta



Alpha-beta



Alpha-beta



Alpha-beta

- Alpha is the best value (for Max) found so far at any choice point along the path for Max
  - Best means highest
  - If utility v is worse than alpha, max will avoid it
- Beta is the best value (for Min) found so far at any choice point along the path for Min
  - Best means lowest
  - If utility v is larger than beta, min will avoid it

Alpha-beta algorithm

function ALPHA-BETA-DECISION(*state*) **returns** an action
  return the a in ACTIONS(*state*) maximizing MIN-VALUE(RESULT(a, *state*))

---

function MAX-VALUE(*state*, α, β) **returns** *a utility value*
  inputs: *state*, current state in game
    α, the value of the best alternative for MAX along the path to *state*
    β, the value of the best alternative for MIN along the path to *state*

  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for a, s in SUCCESSORS(*state*) **do**
    $v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$
    if $v \geq \beta$ **then return** v
    $\alpha \leftarrow \text{MAX}(\alpha, v)$
  return v

---

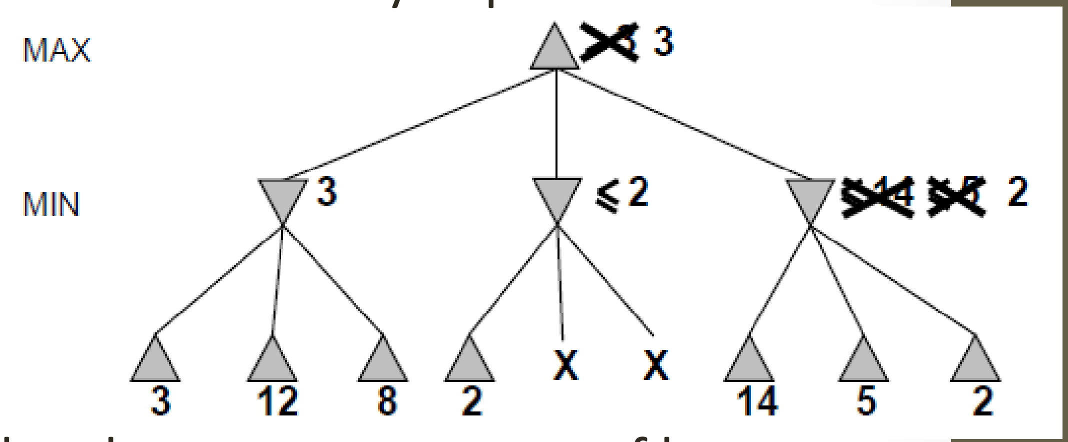
function MIN-VALUE(*state*, α, β) **returns** *a utility value*
  same as MAX-VALUE but with roles of α, β reversed

Alpha beta example

- Minimax(root)
  - = max (min (3, 12, 8), min(2, x, y), min (14, 5, 2))
  - = max(3, min(2, x, y), 2)
  - = max(3, aValue <= 2, 2)
  - = 3

Alpha-beta pruning analysis

- Alpha-beta pruning can reduce the effective branching factor
- Alpha-beta pruning's effectiveness is heavily dependent on **MOVE ORDERING**
- **14, 5, 2 versus 2, 5, 14**
- **If** we can order moves well
- O($b^{\frac{m}{2}}$)
- Which is O(($b^{1/2}$).^{m}
- Effective branching factor then become square root of b
- For chess this is huge → from 35 to 6
- Alpha-beta can solve a tree twice as deep as minimax in the same amount of time!
  - Chess: Try captures first, then threats, then forward moves, then backward moves comes close to b = 12



Imperfect information

- You still cannot reach all leaves of the chess search tree!
- What can we do?
  - Go as deep as you can, then
  - Utility Value = Evaluate(Current Board)
  - Proposed in 1950 by Claude Shannon

Search

- Problem solving by searching for a solution in a space of possible solutions
- Uninformed versus Informed search
- Atomic representation of state
- Solutions are fixed sequences of actions