- 8-bit Numbers

- The Negative of N, written as -N, is computed from

$$2^8 - N = 100000000_2 - N$$

- Example:

$$\begin{aligned} N &= 00000011_2 \\ -N &= \quad \begin{array}{r} 100000000_2 \\ -00000011_2 \\ \hline 11111101 \end{array} \end{aligned}$$

$$-N = 11111101$$

- **Definition:** The negative of N is called the **two's complement of N** and is denoted by **2comp(N)**

- $\boxed{\text{Math Tip:}}$
- When calculating 2comp(N) use this "shortcut"
- 8-Bits

$$
\begin{aligned}
2comp(N) &= 2^8 - N_2 \\
&= ((2^8 - 1) - N_2) + 1 \\
&= (11111111 - N_2) + 1
\end{aligned}
$$

- This is a shortcut because $11111111 - N_2$ can be calculated by simply replacing each 0 by 1, and each 1 with 0 in N_2. (You don't have to worry so much about borrows).
- Two Steps in Calculating 2comp(N)

1. Complement N ; also called 1comp(N)
2. Add 1

$\boxed{\text{N-bit Two's Complement Numbers}}$

1. Two's complement n-bit numbers are binary numbers of length n. Bits are indexed from 0 to $n-1$. For each bit d_i, the subscript corresponds to the power of 2 in the binary expansion.
2. **Range:** $-2^{n-1} \ldots (2^{n-1} - 1)$
3. MSB = Sign bit; $1 \rightarrow -ive$, $0 \rightarrow +ive$
4. For an n-bit number M, Negative of M is 2comp(M)
   (a) 2comp(M) $= 2^n - M$
   (b) Shortcut: 2comp(M) = 1Comp(M) + 1
5. $\forall$ values of M, $M + 2comp(M) = 0$
   - Except for -2^{n-1}, $2comp(-2^{n-1}) = -2^{n-1}$!!!!!!

6. **Magnitude:** The magnitude of a negative number in two's complement representation is hidden. You know it is negative because the MSB = 1, but what is its magnitude?

   - Magnitude of a negative number is calculated by finding its two's complement:
     - example: $M = 10001011_2$
     - M is negative, because MSB = 1
     - What is M's magnitude; M is -ive what?
     - $2comp(M) = 01110100_2{+}1 = 01110101_2 = 117_{10}$
     - Therefore : M is -117_{10}

$\boxed{\text{2's complement Hex numbers}}$

- We can compactly write binary numbers in HEX

Size = 8 bits

2's complement	Decimal value	HEX
0111 1111	127	7F
0111 1110	126	7E
0111 1101	125	7D
.....	.....	.....
0000 0011	3	03
0000 0010	2	02
0000 0001	1	01
0000 0000	0	00
1111 1111	-1	FF
1111 1110	-2	FE
1111 1101	-3	FD
.....	.....	.....
1000 0010	-126	82
1000 0001	-127	81
1000 0000	-128	80

2's complement Hex numbers ... cont'd

16-bit numbers

- example 1: 2comp(06FD)

$$\begin{array}{r} 2^{16} - 06FD \\ \hline 10000 \\ -06FD \\ \hline \\ F903 \\ \hline \end{array} \quad \text{OR} \quad \begin{array}{r} \text{SHORTCUT} \\ \hline FFFF \\ -06FD \\ \hline F902 \\ +0001 \\ \hline F903 \\ \hline \end{array}$$

- example 2: 2comp(903B)

$$\begin{array}{r} 2^{16} - 903B \\ \hline 10000 \\ -903B \\ \hline \\ 6FC5 \\ \hline \end{array} \quad \text{OR} \quad \begin{array}{r} \text{SHORTCUT} \\ \hline FFFF \\ -903B \\ \hline 6FC4 \\ +0001 \\ \hline 6FC5 \\ \hline \end{array}$$