# Establishing Evolutionary Game Models for CYBer security information EXchange (CYBEX)

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**Abstract**

The initiative to protect critical resources against cyber attacks requires security investments complemented with a collaborative sharing effort from every organization. A CYBersecurity information EXchange (CYBEX) framework is required to facilitate cyber-threat intelligence (CTI) sharing among the organizations to abate the impact of cyber attacks. In this research, we present an evolutionary game theoretic framework to investigate the economic benefits of cybersecurity information sharing and analyze the impacts and consequences of not participating in the game. By using micro-economic theory as substrate, we model this framework as human-society inspired evolutionary game among the organizations and investigate the implications of information sharing. Using our proposed dynamic cost adaptation scheme and distributed learning heuristic, organizations are induced toward adopting the evolutionary stable strategy of participating in the sharing framework. We also extend the evolutionary analysis to understand sharing nature of participants in a heterogeneous information exchange environment.

*Keywords:* Cybersecurity, Information exchange, CYBEX, Evolutionary Game Theory, Cyber-threat intelligence, Replicator dynamics

## 1. Introduction

The horizon of cyberwarfare has been evolving rapidly from the past decade, which is not only severely impacting national critical resources but also disrupting the privacy of intellectual properties. The sophistication of cyberattacks is also emerging at an increasing pace, where high quality automated malwares are being developed by cyber criminals and targeted to particular kinds of organizations to

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† Approved for Public Release; Distribution Unlimited: 88ABW-2016-3432, Dated 11 July 2016.

‡ The preliminary version of this manuscript has appeared in IEEE International Conference on Communications (ICC-2015) [1].

steal valuable resources or information [2]. Well organized attacks smoothly sneak into clients' systems using technical exploits & social engineering techniques and persist until detection, but by that time attackers get enough advantages of the vulnerability. In recent times, a string of cyber attacks has been targeted against (1) retail stores such as Target, Home Depot, Neiman Marcus, KMart, Staples, Sony etc. [3], (2) financial institutions like J.P Morgan & Chase Co. [4], HSBC [5], (3) health care insurance companies like Anthem, Primera Blue, (4) government sectors like Department of Energy (DoE) [6], and many more. Since cyber notoriety is at very high flame, significant efforts from government as well as other industries are being made to abate the cyber-criminal activities.

Considering the impact and severity of present day cyber threats, real-time preventive actions are required to stop major disruptions, for which the organizations must posses the knowledge of latest information on various exploits, attack vectors, vulnerabilities, etc. Therefore, cyber-threat information exchanges [7][8] are envisioned to help firms in staying updated and informed with recent incidents, vulnerabilities, malware signatures etc. to develop proactive defenses. Without support from inter-sector organizations for such sharing, establishment of functional information exchanges is nearly impossible. Promoting such information sharing, the Department of Homeland Security (DHS) has introduced several information sharing programs [9]. The recent passed bill, "Cybersecurity Information Sharing Act (CISA) [10] from U.S Senate promotes such information exchange among private/non-profit organizations, and government institutions, because cybersecurity has become a national issue.

Irrespective of all these efforts, firms still hesitate to share their cyber-threat information with other organizations for various reasons: (1) open sharing of threat intelligence might not give competitive advantages in the market; (2) skeptical about the incident reporting process because it might create a channel to the competitors or malicious agents to violate trust and exploit the reporting firm using its own information; (3) shared information might reveal violations of federally controlled regulations, where the firms do not want to get involved. Traditional cybersecurity handling practices can now be complemented with an initiative of collaborative threat intelligence sharing, which can help the organizations to better understand the threat landscape. Such exchange can facilitate knowledge sharing, re-usability of patches/fixes, and security cost reduction for a firm. Additionally, the prior-incidence and post-incidence information can altogether give better knowledge about the adversaries, incident and the targeted assets etc. Many community-driven technical specifications, such as STIX, CYBOX, and TAXII [11], have been introduced to represent threat information in structured manner, and provision transport services too. To provision a globally common format and framework for assured cybersecurity information exchange, ITU-T (International Telecommunication Union-Telecommunication) has taken the initiative to adopt a framework called CYBersecurity information EXchange (CYBEX) [12], which is briefly discussed in the following subsection.

*1.1. CYBersecurity information EXchange (CYBEX) Framework*

ITU-T had attempted to build an emerging standard, CYBEX X.1500 [12], which is aimed to provide a common global format and assured automated platform for exchanging cyber-threat intelligence. The CYBEX framework is mainly built upon five important functional blocks: Information Description, Information Discovery, Information Query, Information Assurance and Information Transport. Figure 1 depicts the functional blocks along with the their supported standards.

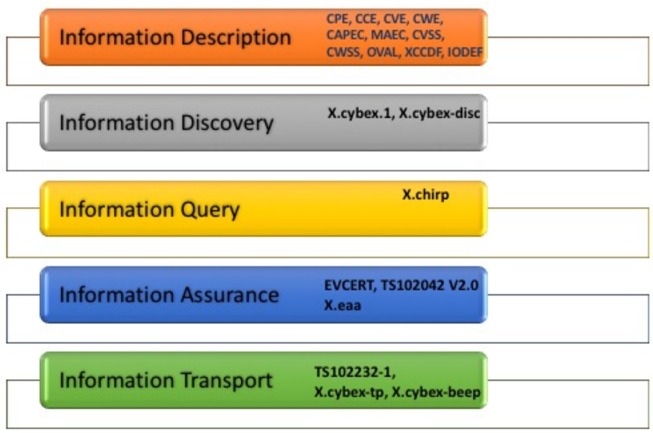



Figure 1: CYBEX X.1500 Functional Blocks [12]

- The `description block` takes care of structuring the cyber information for the purpose of sharing with others. Based on various operation domains, such as knowledge accumulation, incident handling, IT asset management etc., specific ontologies are defined by several institutions and CYBEX framework supports most of them. For example, (1) describing information in knowledge base, CYBEX introduced Common Attack Pattern Enumeration and Classification (CAPEC)[13][14], Common Vulnerabilities and Exposures (CVE) [15], Common Weakness Enumeration (CWE) [16], Malware Attribute Enumeration and Characterization (MAEC) [17]; (2) for accumulating countermeasure knowledge for cyber-risks, CYBEX adopts Common Vulnerability Scoring System (CVSS) [18], Open Vulnerability and Assessment Language (OVAL) [19], and eXtensible Configuration Checklist Description Format (XCCDF) [20]; (3) presenting and storing cyber-incidents in incident/warning database, CYBEX allowed Incident Object Description Exchange Format (IODEF) [21] X.pfoc, and Common Event Expression (CEE) [22].

- The `discovery block` is meant to discover cyber-threat information and entities, which uses two paradigms: centralized and decentralized discovery. In centralized case, information providers maintain hierarchical registries to store cyber-information using object identifiers (OID) [23]. Users can access the central registries and find what they are looking for. However, the main disadvantage is that the users first need to know if a particular registry exists or not. Discovering information in distributed manner can be achieved using resource description framework (RDF) [24].

- `Query block` mainly used to request and respond cyber related information from the framework. CYBEX introduced X.chirp for this purpose, which is an extended SQL form to provide reliable query transactions.
- `Assurance block` is designed to check the validity of the information transacted in the framework. This module is important from the perspective of providing assurance to CYBEX users that the cyber-information is genuine and authentic. To enable such service, CYBEX introduced X.evcert, X.eaa and ETSI TS 102042 V2.0.
- The `transportation block` mainly serves the purpose of exchanging cyber-threat intelligence with other users over networks. X.cybex-tp describes the details transport protocols which is based on Blocks Extensible Exchange Protocol (BEEP) [25].

Noting that, the goal of CYBEX is to provide a framework for users, not limited to businesses, corporations, end-users, government etc., where they can look for cybersecurity solutions and access them over Internet in an automated way. However, the basic assumption of CYBEX is that the firms must be always cooperative and truthful with each other, but in reality the firms compete with each other for better competitive advantages, revenue, market share, and stakeholders etc. Additionally, the market competition is very distributive in nature where rational firms never cooperate without any profit. In such scenario, motivating the firms to participate in the sharing framework and share their proprietary cyber-threat information truthfully with others is a challenging task to achieve. Therefore, self-enforcement schemes for the firms to drive them towards participation are of high requirement to succeed in CYBEX framework. This will necessarily maximize the social welfare of both the participating firms and CYBEX. Since CYBEX is also considered as another rational player in the market, it also intends to maximize its profit by enforcing as many firms in the market to the sharing framework, so that the net sum of the participation costs can be maximum. Thus, from CYBEX's point-of-view, it is critical to study how the participation cost can be smartly used to bring more participants to the sharing framework, eventually evolving to a win-win scenario.

### *1.2. Contributions*

In this paper, we study the effectiveness of dynamic incentives/participation cost on inducing firms to participate and share their threat information truthfully with each other. Since the goal of CYBEX sharing framework is to enable information exchange without intervention of any external means, it is compelling to answer how and when the incentive/participation cost can be enforced such that the sharing system self-sustains. The firms must decide whether to participate in CYBEX or not under such dynamic cost adaptation mechanism adopted by CYBEX. In this work, we formulate and analyze the CYBEX participation game from the evolutionary game perspective to achieve evolutionarily stable strategy (ESS), where, firms are rational, adaptive, dynamically evolving and interact in a non-cooperative manner. In evolutionary games, a strategy is said to be evolutionarily stable [26][27], if the the members taking this strategy cannot be invaded by a mutant strategy through natural selection operation. Once the participation conflict of the firms is resolved, it is crucial to understand how much a firm is willing to share with others since the benefits of sharing are unclear to the firm. Thus, we formalize an information sharing game where firms are required to resolve how much information they want to share in the CYBEX framework or stay out of it. Similar evolutionary analysis is applied to derive the evolutionarily stable strategies along with the conditions under which they can be achieved. Dynamic variation of participation cost and incentives from CYBEX can motivate the firms to participate and truthfully share their threat information with each other.

In summary, the key contributions of the paper are:

- We derive the evolutionary stable strategies for the CYBEX participation game as well as the information sharing game and the underlying conditions which when satisfied the corresponding ESS can be achieved by the firms. The conditional constraints are the major guidelines for CYBEX to self-enforce the firms toward ESS and motivate them to participate and share truthfully in the sharing framework.

- We propose a dynamic cost adaptation heuristic for CYBEX that aims to alter the role of participation cost as incentive for providing profitable environment for the firms to participate in CYBEX and the incentive is gradually reduced as the participation strength grows over time. Eventually the incentive is converted to cost so that CYBEX starts generating revenue without affecting the purpose of CYBEX.

- We design and implement a distributed learning heuristic for the firms to learn about the evolutionary stable strategy in the non-cooperative CYBEX participation game. Rational firms adopt the dynamically update the strategy selection probability at each game stage based on their history information and decide their best responses.

- The differentiated information sharing game enables us to understand the sharing nature of firms and how the incentivization from CYBEX can help the firms to act truthfully and achieve the required ESS. The conditions derived from evolutionary analysis gives better insights to CYBEX to lead the firms to adopt "high sharing" strategy.

### *1.3. Paper Organization*

The remainder of the paper is organized as follows: The components of the CYBEX Participation game are described in Section 3. Section 4 describes the motivations behind adopting evolutionary game approach to analyze our models. In Section 5, the game is formalized and analyzed to find the conditions under which ESS can be achieved. The insights for CYBEX and the proposed distributed learning heuristic is also detailed in this section. Extending the firms' participation towards sharing threat intelligence, we formulate a 3-strategy information sharing game in Section 6. We then analyze this game from evolutionary point-of-view and verified their stability in Section 7. Section 8 presents results achieved via simulation, where players in the population follow the proposed learning heuristic to achieve ESS. Finally, Section 9 concludes the paper.

## 2. Research Background

The information security has been a major concern for the organizations since businesses adopted the cyberspace as their medium for all sorts of business operations. To protect the critical assets from cyber-exploitation, corporations are required to invest resources in terms of both monetary and man power. However, timely information on incidents, vulnerabilities, threat signatures etc., may not be discovered from sole cybersecurity research. The emerging standard of cyber-threat intelligence sharing can complement the security investments made by the firms if they decide to take part in such sharing frameworks. This topic has gained significant attentions and is being investigated by government, policy makers, economists, non-profit organizations, industries, cybersecurity and network professionals with researches in this particular area still emerging [28, 29, 30].

Emphasizing more on cybersecurity information sharing, [29] analyzed the economic (dis)advantages if cybersecurity information sharing and showed that such exchange activity improves the social welfare as well as security level of the firms at a minimum expenditure. Additionally, an incentive mechanism is provided to prohibit free-riding on other firms' expenditure so that no firm can gain more by making underinvestment. As the underinvestment can bring serious cyber issues, [31] constructed a game theoretic model to address economic motivation of security investment and mentions the challenges of regulatory scheme design that mandates specific security standards. As the economic loss of a firm depends on the nature of information that is being shared with others, which has not been addressed in [29], Campbell et.al. and Goel et.al. in [30][32] analyzed the market reaction of firms' in terms of their stock value depending on the involvement of confidential information in the disclosed breaches. Cavusoglu et.al. proposed a similar game theoretic model in [33] like Gordon et.al. to determine the IT security investment levels and compared it with a decision theoretic approach on various dimensions such as vulnerability, payoff from investment etc. [34][35] study the decision of security investment and information sharing with other VMs by modeling the problems as non-cooperative games.

The quest of finding optimal amount of expenditure [36] and information to exchange [37, 38] with others for strengthening a firm's information security gained its importance and many researchers looked into this problem by considering a social planner in the system who can help firms in deciding the mentioned decision parameters so that their social welfare is improved. Using game theory, authors of [39] investigated the competitive implications of information sharing and security investment and showed that benefit of this sharing is contingent on product substitutability. They also analyzed the consequences of establishing information sharing analytic centers (ISAC) that will help in coordinating the information sharing process without allowing free-riding. However, the social outcome of every participating firms is dependent on the extent firms' rely on the social planner's decision. Hausken in [40] also addressed the similar problem by proposing a simultaneous and a two-stage game in the presence of an attacker but considered a new parameter called interdependency between the firms to analyze how this factor affects the social outcome of each firm. He showed that the two-stage game helps the firms to achieve higher outcome compared to simultaneous game, because the social planner cannot impose its decision on the firms when the interaction is simultaneous in nature. As the past works do not consider any specific type of information to share and the range of information sharing amount varies between 0 to 1, authors of [41] adopted a 2-stage Bayesian game considering the information as the number of bugs and using backward induction they derive the optimal investment quantity and number of bugs to share with the other firm.

Though the researches on security information sharing have been there from the past decade, the nation is still facing an increasing amount of cyber criminal activities. Even though there are several ISACs, a significant improvement has not been observed in developing protective measures towards cybersecurity. The problems for insufficient participation in information exchange could be: (1) absence of standardized structured mechanism to exchange the discovered information (2) insecure feeling of firms to participate in the framework due to the fear of reputation loss (3) inefficient incentive model to attract corporations for sharing information. The first point is being addressed by CYBEX framework that will provide a platform of assured information exchange in a global scale. However, the second and third point must be addressed to stay insulated from external malicious activities as mentioned in the recent presidential executive order [8]. This area of research has still a long way to go before reaching the ultimate goal of information sharing in a fully distributed environment without seeking help from any external agent. There are still many unanswered questions that an information exchange framework must resolve before adopting any particular idea. Some of those questions are: (1) how to design and analyze various self-enforcing incentive mechanisms when there are more than two firms can be incentivized to participate in the sharing framework? (2) Can cyber-insurance be used as a potential parameter that motivate and maximize the information sharing activity with in the framework? (3) What conditions on cyber-insurance can lead the system of participants to adopt a stable strategy where maximum sharing is achieved? (4) How can we devise a decentralized learning in the sharing framework that will help the individual agents to learn independently about the sharing attitude of its opponents and accordingly adapt their information sharing nature?

## 3. CYBEX Participation Game

Though the corporations or firms understand the benefits and costs of cyber-threat information sharing, not everyone of them takes the risk of participating in CYBEX framework. Thus, motivating the firms to participate in the exchange framework and guiding them toward a self-sustained framework are some of the critical issues to address from the CYBEX's point-of-view. We aim to present a self-enforcement mechanism that will attract firms to participate in the sharing framework which can maximize the net benefit of participants while increasing the proportion of firms in the framework. From the perspective of CYBEX, charging a cost for participation will maximize its net revenue only if the number of participants is maximized. This might be difficult to achieve without adopting a scheme for dynamic participation cost. To analyze such scenarios, we model the CYBEX participation game and analyze it using evolutionary game dynamics later in the section.

### *3.1. Game Model*

In this work, we consider the generic abstraction of "always rational and profit-seeking" CYBEX and firms. We consider a market of $N$ firms playing independently in this game and trying to decide whether to participate in the CYBEX framework and share with other firms by incurring a participation cost or not. From CYBEX's point-of-view, the decision problem is how much incentive/participation costs should be induced and when, to motivate the firms to participate in the CYBEX framework. If CYBEX charges too high to increase its revenue, the firms may possibly get deterred from participation, eventually reducing CYBEX's revenue. On the other hand, if CYBEX charges too low to attract firms, the revenue generated by CYBEX might be insufficient to sustain in the market. Thus, it is important to investigate, under what conditions and how CYBEX can dynamically decide on incentive/participation cost to attract increasing number of participants to share (which will increasingly strengthen their cyber-defense capability), yet increase CYBEX's revenue. To model the firms' payoff, the following two components are considered.

### *3.2. Sharing and Investment Gain*

In this evolutionary information exchange framework, assuming the firms invest for their own cybersecurity R&D, the firm directly benefits from its own investment. Additionally, an indirect reflected gain is received from the other firms' shared information, which can produce proactive defense, patches and fixes. Therefore, exchange of this valuable information with other firms improves their overall utility. Though participating in CYBEX and sharing information is beneficial for protecting the firms' assets from cyber criminal activities, the participation in the CYBEX architecture and sharing information among the firms are not cost-free.

### *3.3. Modeling Costs in CYBEX*

There exists a cost of participation in the CYBEX architecture, which is defined by the cost that the CYBEX architecture charges the firms for maintenance of the architecture as well as certification (for sharing) and to ensure liability of the firms. Apart from the participation cost, there also exists a cost of information sharing, which has two parts: retrieving the information for relevance, and the potential loss of reputation. Next, we formalize the participation game model using strategic form.

### *3.4. Participation Game in Strategic Form*

As far as a decision strategy in this game model is concerned, every firm has the binary strategy set ($\mathcal{SS}$):

$$\mathcal{SS} = \{\text{Participate and Share in CYBEX}, \text{Not Participate}\} \tag{1}$$

With the strategy set defined, we now define the pairwise strategic form payoffs in Table 1, when any two of the firms engage in a pairwise interaction.

| | Participate & Share | Not Participate |
|---|---|---|
| Participate & Share | $Sa\log(1+I)-x-c$, $Sa\log(1+I)-x-c$ | $a\log(1+I)-x-c$, $a\log(1+I)$ |
| Not Participate | $a\log(1+I)$, $a\log(1+I)-x-c$ | $a\log(1+I)$, $a\log(1+I)$ |

Table 1: Payoffs in Strategic-Form for Participation Game

When firms are not involved in the CYBEX framework (i.e., they neither participate nor share), the utility reward to the firms is dependent only on their own investment, which can be presented as the following variant of logarithmic function, $a\log(1+I)$, where $I$ is the amount of investment made by the firms and $a$ is a simple scaling parameter that maps user satisfaction/benefit to a dimension equitable to the price/monitory value [42][43]. For the rationality constraint, $a\log(1+I) > 0$ must be held, otherwise, the firms would prefer to not make any investment. The logarithmic gain function motivates the players by rewarding for increasing steps towards security investment. However, the reward eventually saturates with gradually increasing investment. This is because increasing the investment further even beyond a certain threshold does not necessarily increase the overall utility with a high rate of increment, rather limiting and saturating the reward obtained [42][43]. In this symmetric game work, we assume a fixed information sharing by every participant, which is also considered as maximum available information to a firm. To analyze the sharing scenario more realistically, we later extend this work to formulate an information sharing game, where firms have choices to share different amount of cybersecurity information.

We also assume, when the engaged firms participate in mutual sharing, the resulting benefit for them would then stem, not just from their own investment, but also from their sharing. Thus, we consider this utility (when both the firms sharing mutually) as $Sa\log(1+I)$, which can be considered as return on both investment and sharing. Again for the rationality constraint, the sharing benefit ($S$) is more than 1, otherwise the player does not have any incentive of sharing. $c$ is the cost of participation in the CYBEX architecture, i.e., the amount charged by CYBEX system of governance for participating and $x$ reflects the cost of information sharing as explained earlier in Subsection 3.3.

However, when a pair of firms are mutually interacting, while one of them is part of CYBEX and the other is not, then the utility to the firms are given in the top right corner and bottom left corner cells. This scenario depicts the risk of participating, where the participating firm incurs the cost due to participation in CYBEX without any additional sharing gain and the other non-participating firm incurring no cost but also not gaining anything due to not sharing. Note that, we could always use any other complex values or functions for depicting the utilities and cost, however, our aim here is to analyze the game from the players' evolution point-of-view regardless of the exact utility or cost values as long as the nature of utility and the costs follow the rationality constraints required in a real market.

## 4. Motivations of using Evolutionary Game Approach

The motivations of opting evolutionary games to model firms' participation and information sharing decision comes from the nature/quality of strategies (solutions) obtainable from the evolution process of the players in such games. Especially, the rational organizations would continuously evolve in real time until every player adopts to a stable steady-state strategy. In the process, the fitter strategies get prevalent and the unfit ones become extinct over time. To model such situations, evolutionary games come handy that are useful to understand the stability of strategies over a finite population of players. Hence, any game can be analyzed using evolutionary concepts to find the stable strategies irrespective of number of players in the system [44]. However, stability of strategies may not be understood from non-cooperative game solutions.

As far as non-cooperative game models are concerned, they are widely used to model problems where, players of the game make their decisions independently without any sort of help from the competitors. The outcomes of these games are the equilibrium strategies with respect to other players' actions. However, evolutionary games extend the idea of non-cooperative games by introducing a population of agents (a group of players), who are interacting with each other dynamically and evolving to figure out the fittest strategies that could help them to survive in the game while the unfit players' strategies get invaded over time. Such games are highly applicable when the individuals exhibit different behaviors at different times and posses the ability to evolve over time for their betterment. The interaction of a player with a group of players exhibiting different behaviors may give him/her a scope for evolution, which can eventually lead to adopt a stable strategy. Thus, it is important to understand the dynamics behind such group interactions, which cannot be measured effectively in isolation. Rather it has to be evaluated in the context of entire population where the player lives.

Additionally, the following characteristics of evolutionary games [45][46][47] motivated us to model our problem accordingly in this research.

1. **Equilibrium Solution Refinement:** The evolutionary games always provide a refined solution that ensures stability of a strategy adopted by a population, where **no small subgroup of deviants could successfully invade the whole population**. Such strategy is known as evolutionary stable strategy (ESS). However, in case of non-cooperative games, Nash equilibrium (NE) is considered as the traditional solution concept, which by definition ensures that no player can gain more by deviating unilaterally to a different strategy [27]. But if a group of players collaboratively change their strategies simultaneously instead of adopting NE, then they may increase their net payoffs [48] and this importantly differentiates ESS from NE solutions. If both of the solutions are compared, outcomes of evolutionary games are stronger and efficient than the outcomes of a non-cooperative games. In general, ESS are nothing but the refined subset of NE strategies, that provides no lesser payoff than a NE strategy [44][49], if opted.

2. **Bounded Rationality:** In traditional game theory, the individuals are assumed as rational and the players believe that their opponents act rationally throughout the game. Based on which the common utility function is maximized to derive the optimal strategy. However, the underlying rationality assumption is often unrealistic. This situation is avoided in evolutionary games by introducing the concept of bounded rationality, where players adopt dynamic strategies that lead them to sustain in the population without caring about instant payoff maximization. This dynamic strategy alteration process eventually leads the individuals to achieve the equilibrium solution.

3. **Game Dynamics:** Since players in the evolutionary games interact with each other in the population for multiple rounds by adopting different strategies, the state of the interaction game varies over time according to the replicator dynamics. Thus, the evolutionary game provides a natural way to introduce dynamics in the system, where successful strategies are imitated by other individuals and propagate over interaction rounds. In particular, the state of the evolutionary game at any point can be captured using the replicator dynamics, which becomes very handy to understand the evolution trajectory of players' behaviors/strategies over time.

## 5. Analyzing CYBEX Participation Game

Once the problems are identified and the game is formalized, we need to solve the game for the firms. Solving a game means predicting the steady state strategy of each player considering the information the game offers and assuming that the players are rational. One can see that if the strategies from the players are mutual best responses to each other, no small group of players would have a reason to deviate from their best response strategies and the game would reach an evolutionary stable state.

In this section, we now analyze the CYBEX participation game in-depth and investigate if the game has any ESS and if exists under what conditions they can be achieved. We are particularly interested in modeling participation cost, which can be used as an initial incentive to attract the firms to share in the CYBEX framework. The system is aimed to be independent and self-enforced, so that the participation nature of the firms is enhanced even without any external stimulant. This will help the system also lead to an evolutionary stable state in self-enforced manner.

### *5.1. Evolutionary Analysis of the Game:*

To analyze the evolutionary stability of the game, we assume $\alpha \in (0, 1]$ is the proportion of population participating and sharing in CYBEX. Then, according to replicator dynamics [26, 27], the transformation speed can be given by

$$g(\alpha) = \alpha\big(\mathbb{E}[U_{sh}] - \mathbb{E}[U_{avg}]\big) \tag{2}$$

where, $\mathbb{E}[U_{sh}]$ is the expected payoff of a player for participating and sharing, and $\mathbb{E}[U_{avg}]$ is the average payoff of the population. The expected utility of "participate & share" strategy can be given as

$$\mathbb{E}[U_{sh}] = \alpha\big[Sa\log(1+I) - x - c\big] + (1-\alpha)\big[a\log(1+I) - x - c\big] \tag{3}$$

Similarly, $\mathbb{E}[U_{not}]$ is the expected payoff of a player for not sharing, where $\mathbb{E}[U_{avg}] = a\log(1+I)$. Hence,

$$\mathbb{E}[U_{avg}] = \alpha\mathbb{E}[U_{sh}] + (1-\alpha)\mathbb{E}[U_{not}]$$

The replicator equation given in Eqn. (2) can be rewritten as:

$$g(\alpha) = \alpha\big[\alpha\big(Sa\log(1+I) - x - c\big) + (1-\alpha)\big(a\log(1+I) - x - c\big) - \alpha\mathbb{E}[U_{sh}] - (1-\alpha)a\log(1+I)\big] \tag{4}$$

After simplifications,

$$g(\alpha) = \alpha(1-\alpha)\big[\alpha(S-1)a\log(1+I) - x - c\big] \tag{5}$$

For ESS to be achieved, the following two conditions must be satisfied [26, 27]: (1) the transformation rate should be zero, i.e., $g(\alpha) = 0$, and (2) the neighborhood of the equilibrium states (found through condition (1)) must also be stable. To prove a strategy to be evolutionarily stable, it is necessary to verify that the population playing with ESS cannot be invaded by any small group of individuals playing with strategy other than ESS. If condition (2) is not met, then there is a chance that any small subgroup of player playing with a random strategy other than ESS can invade the total population of players playing ESS.

For the transformation rate to be zero, i.e., $g(\alpha) = 0$, there exists three distinct solutions of $\alpha$, (i.e., three potential equilibrium states):

$$\alpha_{sol_1} = 0 \tag{6}$$

$$\alpha_{sol_2} = 1 \tag{7}$$

$$\alpha_{sol_3} = \frac{x + c}{(S - 1)a\log(1 + I)} \tag{8}$$

With these three potential equilibrium states, we now need to check the stability of their neighborhood and then only the equilibrium states can be recognized as ESS. For the neighborhood strategies to be stable, the condition of $g'(\alpha) < 0$ must hold true at each of the equilibrium states. With the three solutions of $\alpha$, the first order differential of the replicator expressions can be:

$$g'(\alpha^*_{sol_1} = 0) = -x - c \tag{9}$$

$$g'(\alpha^*_{sol_2} = 1) = -(S - 1)a\log(1 + I) + x + c \tag{10}$$

$$g'\left(\alpha^*_{sol_3} = \frac{x + c}{(S - 1)a\log(1 + I)}\right) = (x + c) - \frac{(x + c)^2}{(S - 1)a\log(1 + I)} \tag{11}$$

Therefore it is clear that ESS is conditioned upon the wise choice of incentives and participation costs ($c$) and this cost can be used to motivate the socially optimal behavior and deter non-cooperative behaviors. Next, we analyze each of the conditional constraints for ESS and derive under what bounds the population will evolve toward participation and under what bounds they would not.

### *5.2. Analyzing conditional constraints for ESS*

As can be seen in the following, we analyze all possible conditional constraints for ESS, depending on the participation cost ($c$), governed by the CYBEX system for governance. Note that, the cost of information exchange, $x > 0$ as this is an inherent cost by the firms for information sharing.

**Case (i):** Let us first assume, $c > 0$ & $c \geq (S - 1)a\log(1 + I)$. Therefore, $g'(\alpha^*_{sol_1} = 0) < 0$ and $g'(\alpha^*_{sol_2} = 1) > 0$.

It can be seen that $g'(\alpha^*_{sol_3})$ itself does not hold as $\alpha^*_{sol_3} > 1$ however, it must lie between 0 and 1. Hence $\alpha^*_{sol_1} = 0$ is the only ESS under this condition, which implies that evolutionary stable strategy for the population would be to "Not Participate" in the CYBEX architecture due to high cost for such activity. Though it is intuitive that the population will never participate in the sharing framework because of high participation cost ($c$), this cost has an important role in motivating the players to participate, which is discussed in the later case. For numerical analysis, we show a simple scenario following the above conditions even when the evolutionary game initiates from a high "participate & share" population proportion $\alpha^* = 0.8$, it is found from Fig. 2 that the individuals taking "Not Participate" strategy could successfully invade the individuals that are participating and sharing because of no cost for taking "Not Participate" strategy. For all the results found from numerical analysis, we assumed the rationality constant $S = 2$; scaling constant $a = 3$; and investment ($I$) as 5 units. The values of participation cost ($c$) and cost of information sharing ($x$) are suitably varied for different cases based on each condition. For this case, we assumed $c = 7.4$, and $x = 3$ units.

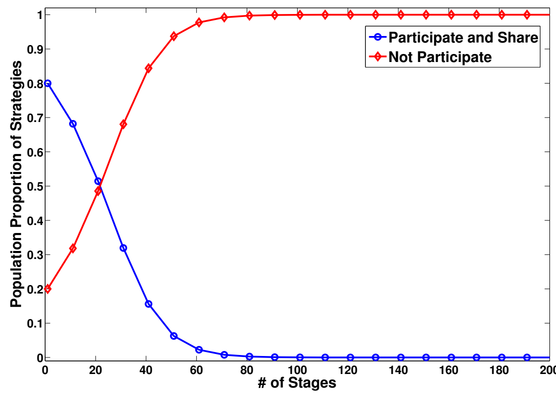



Figure 2: Population proportion variation under constraint (i)

**Case (ii):** When $c > 0$ & $c < (S-1)a\log(1+I)$ such that $(c+x) \geq (S-1)a\log(1+I)$. Therefore, $g'(\alpha^*_{sol_1} = 0) < 0$ and $g'(\alpha^*_{sol_2} = 1) > 0$

It can be seen that $g'(\alpha^*_{sol_3})$ itself does not hold true, as $\alpha^*_{sol_3}$ does not lie between 0 and 1. Hence, under this condition, again, $\alpha^*_{sol_1} = 0$ is the only ESS implying that evolutionary stable strategy for the population would still be not to participate in the CYBEX architecture regardless of the initial participating strategy population. As the total cost component exceeds the sharing gain in this case, the initial population taking the "Participate and Share" strategy can easily be invaded by a small group of individuals taking the "Not Participate" strategy. The result from numerical analysis is presented in Fig. 3 by assuming $c = 3.4$ and $x = 3$, which demonstrates that irrespective of any initial $\alpha$ value, the ESS is always found to be "Not Participate" strategy and always gets invaded by the population of "Participate and Share" strategy.

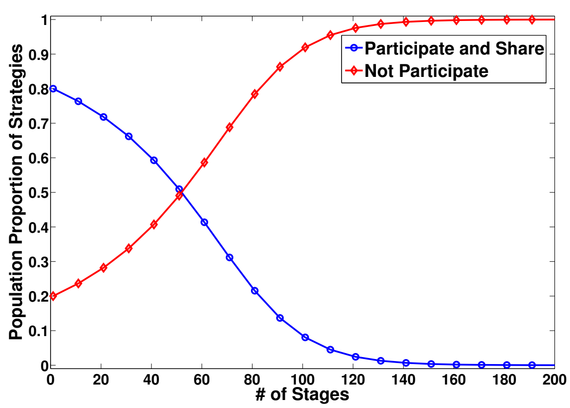



Figure 3: Population proportion variation under constraint (ii)

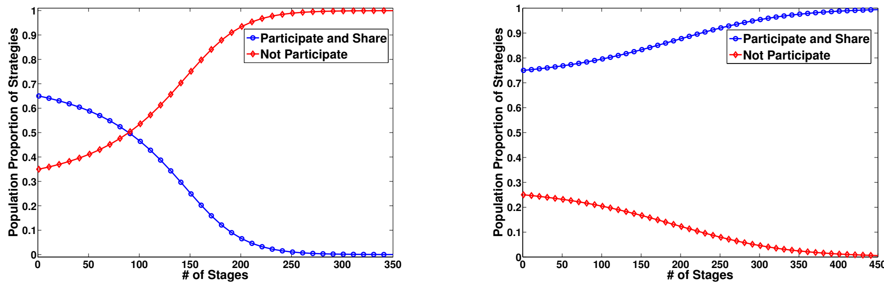
**Case (iii):** When $c > 0$ & $c < (S-1)a\log(1+I)$ such that $(c+x) < (S-1)a\log(1+I)$. Therefore,

$$g'(\alpha^*_{sol_1} = 0) < 0$$
$$g'(\alpha^*_{sol_2} = 1) < 0$$
$$g'(\alpha^*_{sol_3}) = (c+x)\left[1 - \frac{c+x}{(S-1)a\log(1+I)}\right]$$
$$g'(\alpha^*_{sol_3}) > 0$$

Hence, two possible ESS ($\alpha^*_{sol_1} = 0$ and $\alpha^*_{sol_2} = 1$) exist in this case, however, achieving a particular ESS depends on the initial "participate and share" population distribution. ESS tends to "Not Participate" if $0 < \alpha^* < \frac{c+x}{(S-1)a\log(1+I)}$ and ESS tends to "Participate and Share" if $\frac{c+x}{(S-1)a\log(1+I)} < \alpha^* < 1$, where $\alpha^*$ is the initial population fraction playing with participate and share strategy. This clearly implies that if the initial "Participate & Share" population fraction is more than a certain threshold/tipping value, $\alpha_{thres} = \frac{c+x}{(S-1)a\log(1+I)}$, then the rest of the population fraction (which are not sharing) will evolve over time and will participate in CYBEX, thus, ESS tends toward "Participate and Share" strategy. Alternatively, if the initial "Participate & Share" population fraction is less than $\alpha_{thres}$, then the gain from the system would not be sufficient enough to enforce the entire population toward sharing rather ESS will tend towards "Not Participate" strategy, thus, showing the significance of the initial "Participate and Share" population strength as well as the significance of participation cost ($c$). Fig. 4 presents two sample numerical results where the initial population proportion of "Participate and Share" strategy $\alpha^* = 0.65$ and 0.9 respectively, assuming $c = 2.4, x = 1.5$. The simulation results validate the deflecting nature of ESS based on the theoretical threshold/tipping value that can be computed numerically by using the $\alpha_{thres}$ expression, and found to be 0.72. From Fig. 4(a), it is shown that most individuals lean towards the "Not Participate" strategy, when the initial participating population proportion $\alpha^*$ is below the threshold value. However, when the initial "Participate and Share" population is above the threshold value, the population evolves towards more participation as shown in Fig. 4(b). The expected individual utility is the reason for this kind of deflection in ESS because the average utility to a firm playing "Not Participate" strategy is more, when less players are playing the same strategy, compared to the "Participate and Share" strategy and vice-versa.

**Case (iv):** When $c < 0$ such that $(c+x) \leq 0$, i.e., the cost of participation is negative implying the fact that it is no longer a cost but rather a positive incentive given to the firms for enrolling in CYBEX architecture. Therefore, $g'(\alpha^*_{sol_1} = 0) > 0$ and $g'(\alpha^*_{sol_2} = 1) < 0$

It is clear that $g'(\alpha^*_{sol_3})$ itself does not hold true. Hence $\alpha^*_{sol_1} = 0$ is the only ESS under this condition, which implies that ESS for the population would be to participate and share in the CYBEX architecture regardless of initial $\alpha^*$ value. According to this case, the total cost $(c+x)$, appears to be an incentive for firms to participate, hence the population will eventually be inclined towards the




(a) $\alpha^*_{sol_3} < \alpha_{thres}$ (b) $\alpha^*_{sol_3} > \alpha_{thres}$

Figure 4: Population proportion variation under constraint (iii)

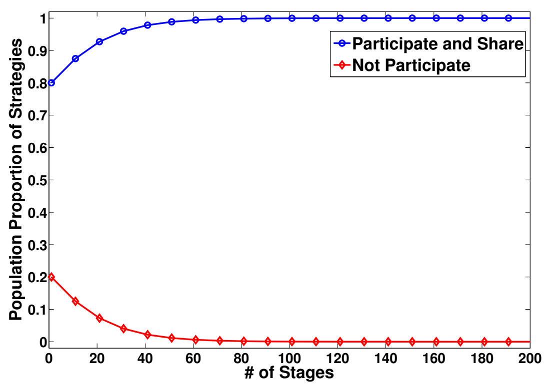



Figure 5: Population proportion variation under constraint (iv)

"Participate & Share" strategy irrespective of any $\alpha^*$ value as shown in Fig. 5, where $c + x$ is assumed to be -1. The result shows that the individuals with "Participate and Share" strategy could successfully invade the "Not Participate" strategy individuals.

**Remark:** Hence, ESS is not unique for the CYBEX participation game. Both strategies "Participate & Share" and "No Participate" have potential to be evolutionarily stable depending on which of the above presented cases are satisfied.

To give a clear picture that the CYBEX participation game has two potential evolutionary equilibrium strategies that are dependent on certain conditions as given above, Figure 6 encapsulates the summary of above discussions. We can see that when the case (i) or (ii) is satisfied, the population preferably chooses "No Participate" strategy to be evolutionarily stable, whereas Participate strategy happens to be the ESS, when case (iv) is satisfied. The most interesting scenario of our model is the condition (iii), where both "Participate" and "No Participate" strategy could possibly be ESS but dependent on the initial proportion of participants in CYBEX. These ESSs can be triggered based on the appropriate incentive/cost from CYBEX.

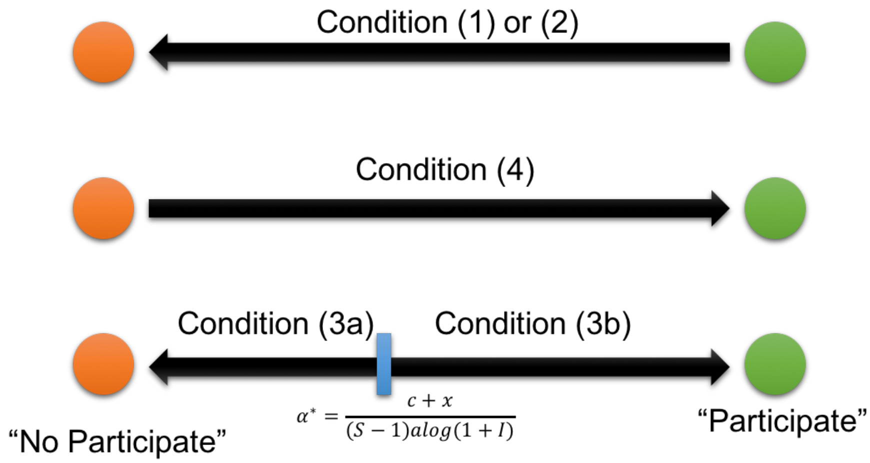



Figure 6: Population preferred stable strategies based on conditional constraints

*5.3. Understanding the impact of conditional constraints on firms' participation*

**Guidance for CYBEX:** The above discussion illustrates how the evolutionary stability structure of CYBEX is directly dependent on the incentive/participation cost along with initial Participating population strategies. Thus, it is of utmost importance to model the cost of participation according to the conditional constraints presented in above model to establish and maintain an effective CYBEX system. These conditions not only show that ESS can be achieved, but also demonstrate how the participation cost is a factor for improving participation in CYBEX and the utility obtained through sharing.

Since CYBEX is also a rational player in the participation game, its aim is to charge a cost to the firms for their participation. To do so, if CYBEX introduces a flat participation cost, then it is possible that firms will not be interested to take part in the framework as positive cost may dominate over sharing gain. Hence, the goal of CYBEX, i.e. cyber-threat information sharing, may not be fulfilled. However, if CYBEX provides incentives for firms' participation, then CYBEX cannot gain anything in return as it goes against its rationality. Hence, a dynamic cost of participation mechanism is necessary to let both CYBEX and firms coexist in a sharing market such that firms can take the advantage of information sharing and CYBEX can profitably manage the participation as well as CTI sharing, thus leading us to present Algorithm 1.

Our proposed algorithm works in the following manner. At the start of the game, when the initial strength of participating population is very small in CYBEX, our analysis shows that using case (iv), incentives can be given to help and evolve the system toward more participation rather than charging with a cost for participation. Once the system goes beyond the threshold (in terms of number of players enrolled in CYBEX), then moving into case (iii), would still ensure that the system will self-sustain in terms of participation without any external positive incentive. Now CYBEX can impose a particular cost for participation knowing how many participants are in the system. The same process is repeated to impose the set of participation costs iteratively. Thus, the dynamic cost adaptation algorithm can provision benefits to CYBEX by maximizing the participation in the system, thus providing benefits to both the participants and CYBEX.

It should be noted that the algorithm does not require lot of information and exchange prior to making decisions. However, in our algorithm a small set of parameters are required to be exchanged with CYBEX for only once, which are the fixed values for each firm. Since our algorithm considers that all the firms are homogeneous in nature, the parameters remain same across all other players too. The necessary information that need to be exchanged with CYBEX are the sharing cost ($x$), investment ($I$) amount of a firm, and rationality constant for sharing ($S$). CYBEX can use all these parameters to calculate what proportion of participants in the system is needed to impose a particular participation cost. Therefore, these information are not required to be exchanged multiple times with CYBEX but only once. As the game progresses, CYBEX counts the number of enrollments and uses it to know which cost ($c$) from the set of all participation costs it can charge to the firms.

In the algorithm 1, the set of $k$ ordered participation costs are stored in the array $CYBEX\_Cost\_INC[1 \cdots k]$. In line 2, we initialize the incentive ($c$) from CYBEX. The initial proportion of participants ($\alpha$) is assumed to be very small (say $\epsilon$). At stage $t = 0$, the individuals initialize their probability of choosing "Participate" strategy ($p_1^i(0)$) to $\epsilon$ and probability of choosing "Not Participate" to $1 - \epsilon$. At stage $t = 1$, CYBEX plans to enforce a minimum participation cost ($c$) of $CYBEX\_Cost\_INC[1]$, but to do that it has to wait until proportion of participating population reaches to at least $\alpha_{thres} = \frac{CYBEX\_Cost\_INC[1]+x}{(S-1)a\log(1+I)} \times |N|$. After this goal is achieved, the players participating in CYBEX need to pay $c = CYBEX\_Cost\_INC[1]$ amount. In line 12-21, each player takes its action based on its mixed strategy probability vector and CYBEX imposes a participation cost if number of participants exceeds the threshold. Based on the cost, the players calculate an updated payoff which is also used to update their mixed strategy probability vector. This process is repeated for all players until each of the $k$ participation costs is successfully charged or maximum number of game stages is reached.

**Algorithm 1:** Dynamic Cost Adaptation algorithm

**Assumptions**: CYBEX has $k$ of different costs to ask the firms.
**Data**: $S, I, x, \kappa$ and $CYBEX\_Cost\_INC[1...k]$
**Result**: Participant population proportion ($\alpha$) to reach 100%
Initialize $c$ with a value s.t. $c < 0$ and $c + x < 0$;
$\alpha \leftarrow \epsilon$, where $0 < \epsilon << 1$ ;
$Payoff_matrix \leftarrow$calculate_payoff($c$);
Initialize Prob. of choosing strategy "Participate" $p_1^i(0) \leftarrow \epsilon \ \forall i \in N$;
Initialize Prob. of choosing strategy "No Participate" $p_2^i(0) \leftarrow 1 - \epsilon \ \forall i \in N$;
$U_i(0) \leftarrow$Pairwise_Interaction($Payoff\_matrix$);
$k_i \leftarrow 1$;
**for** $t \leftarrow 1$ *to maxT* **do**
    $\alpha_{thres} \leftarrow \frac{CYBEX\_Cost\_INC[k_i]+x}{(S-1)a\log(1+I)}$;
    **for** *each* $i \in N$, **do**
        Take a uniform random strategy decision based on $p^i = [p_1^i(t-1), p_2^i(t-1)]$;
        $\alpha_{new} \leftarrow \frac{1}{|N|} \times$ count_participants();
        **if** $\alpha_{new} > \alpha_{thres}$ *and* $k_i \leq k$ **then**
            $c \leftarrow CYBEX\_Cost\_INC[k_i]$;
            $k_i \leftarrow k_i + 1$;
            $Payoff_matrix \leftarrow$calculate_payoff($c$);
        **end**
        $U_i(t) \leftarrow$Pairwise_Interaction($Payoff\_matrix$);
        Update the strategy selection probability $p_1^i$ according to $p_1^i(t) = p_1^i(t-1) + \kappa(U_i(t) - U_{i,1}^{avg}(t))$
        $p_2^i(t) = 1 - p_1^i(t)$
        *//* $U_{i,1}^{avg}(t)$ *: Avg. payoff from "Participate" strategy until stage t*
    **end**
**end**

### *5.4. Learning heuristic for evolutionary stable strategy*

In the previous section, we presented the detailed theoretical analysis and impact of conditional constraints for ESS, which clearly demonstrates how CYBEX architecture can dynamically induce participation cost/incentive to attract and self-enforce firms toward sharing and achieve stability. However, in a simultaneous distributed non-cooperative CYBEX participation game, it is also necessary to design a distributed learning heuristic for the firms to decide which strategy to play at each stage, and how to update their "strategy selection probability" based on the utility feedback obtained from the past game stages. As the game unfolds, the firms would then learn about their best responses and eventually converge to ESS.

In the following, we detail the description of the distributed learning algorithm for the firms to obtain ESS by following the natural evolution similar to replicator dynamics. For choosing a strategy based on the firms' past experience, each firm $i \in \mathcal{N}$, maintains a probability vector, $p^{(i)}(t) = \{(p_1^{(i)}(t), p_2^{(i)}(t)) : p_1^{(i)}(t) + p_2^{(i)}(t) = 1\}$, which defines the probability of choosing "Participate & Share" and "Not Participate" strategy by firm $i$ at game stage $t$ respectively. In each stage, all possible pairwise simultaneous interactions are conducted from each firm's perspective, where, each firm $i \in \mathcal{N}$ sticks to a single strategy throughout the stage and observes the average pairwise utility $\bar{U}_{pair}^{(i)}(t)$ for stage $t$, which is given by:

$$\bar{U}_{pair}^{(i)}(t) = \frac{\sum_{j \neq i} U_i^{(t)}(s_i, s_j)}{|\mathcal{N}| - 1} \tag{12}$$

where, $U_i^{(t)}(s_i, s_j)$ is the payoff to player $i$ from the simultaneous pairwise game between firm $i$ and $j$ by playing with strategy $s_i$ and $s_j$ respectively at game stage $t$.

After each game stage, the player $i$ update its probability of selecting strategy $s_i$ by utilizing two different average utility vectors: (1) $\bar{U}_{avg}^{(i)}(t)$: average received utility, and (2) $\bar{U}_{s_i \in S}^{(i)}(t)$: average utility obtained by playing "strategy $s_i$ only" until stage $t$, which are defined as follows:

$$\bar{U}_{avg}^{(i)}(T) = \frac{\sum_{t=1}^{T} \bar{U}_{pair}^{(i)}(t)}{T} \tag{13}$$

$$\bar{U}_{s_i \in S}^{(i)}(T) = \frac{\sum_{t=0}^{T} \{\bar{U}_{pair}^{(i)}(t) | a_i(t) = s_i\}}{T'} \tag{14}$$

where, $a_i(t)$ is the action of player $i$ at game stage $t$ and player $i$ played strategy $s_i$ for $T'$ number of stages until stage $T$, such that $T' \leq T$.

To learn a stable strategy from the strategy set $S$, the probability of choosing a particular strategy must be reflected from the average utility it receives by playing that strategy. Hence the difference between player $i$'s average utility obtained by playing a particular strategy $s_i$ and average utility out of all game stages will help to decide the probability of choosing $s_i$ in future. Assuming player $i$ played strategy $s_i \in S$ at $(t-1)^{th}$ stage, the probability of playing the same strategy ($p_{s_i}^{(i)}(t)$) at $t^{th}$ stage can be computed using the update rule given in Eqn. (15) and the probability of playing with complementary strategy ($s_i'$) can be given as: $p_{s_i'}^{(i)}(t) = 1 - p_{s_i}^{(i)}(t)$.

$$p_{s_i}^{(i)}(t) = p_{s_i}^{(i)}(t-1) + \kappa(\bar{U}_{s_i}^{(i)}(t) - \bar{U}_{avg}^{(i)}(t)) \tag{15}$$

Where, $\kappa \in (0, 1)$ represents the learning constant that determines how fast or slow the players will move towards the optimal probability of choosing a particular strategy. It is an input parameter for the learning algorithm and must be chosen wisely for faster convergence. If $\kappa$ is too small, then they require more

**Algorithm 2:** Learning Heuristic for ESS Convergence

Initialize the initial "participating" population proportion $\alpha(0)$ for "Participate and Share" strategy, and utility matrix $U$;
Initialize random strategy profile, $p^{(i)}(0) = (p_1^{(i)}(0), 1 - p_1^{(i)}(0))\ \forall i \in \mathcal{N}$ ;
**while** *stage* $t = 1$ *to* $MaxT$ **do**
    **for** *each firm* $i \in N$ **do**
        Select a strategy $s_i \in S$ based on its mixed strategy profile $p^{(i)}(t)$;
        Observe the average utility reward $\bar{U}^{(i)}_{pair}(t)$ from all simultaneous pairwise interactions;
        Update the probability of selecting strategy $s_i$ ($p^{(i)}_{s_i}(t+1)$) for player $i$ according to equation 15;
        Update the probability of playing with complementary strategy $s'_i$ as $(1 - p^{(i)}_{s_i}(t+1))$;
        $t \leftarrow t + 1$;
    **end**
**end**

iterations to converge, however, if $\kappa$ is very large, the players might skip the optimal solution. The game is played repeatedly until the probability of choosing certain action becomes stable and does not change more than a small value $\epsilon > 0$ over the stages. The Algorithm 1 summarizes the distributed learning heuristic which is employed by the players to learn and play with ESS eventually.

The following lemma helps to prove the convergence of the proposed learning heuristic to an ESS.

**Lemma 5.1.** *The sequence of processes $\{P(t)\}$, where $P(t) = (P_1(t), P_2(t), \cdots, P_N(t))$ be the state of the population at stage $t$, converges as the learning constant $\kappa \to 0$.*

**Proof:** Without loss of generality, a player $i$ in the game updates its probability of choosing a strategy $s_i$ using the following update rule.

$$p^{(i)}_{s_i}(t) = p^{(i)}_{s_i}(t-1) + \kappa(\bar{U}^{(i)}_{s_i}(t) - \bar{U}^{(i)}_{avg}(t)) \tag{16}$$

$$\text{where, } \bar{U}^{(i)}_{avg}(T) = \frac{\sum_{t=1}^{T} \bar{U}^{(i)}_{pair}(t)}{T} \tag{17}$$

$$\bar{U}^{(i)}_{s_i \in S}(T) = \frac{\sum_{t=0}^{T} \{\bar{U}^{(i)}_{pair}(t) | a_i(t) = s_i\}}{T'} \tag{18}$$

We consider that $\bar{U}^{(i)}_{pair}(t)$ is the average utility observed by the player $i$ at stage $t$ for all possible pairwise interactions in the population.

Now we can consider that $P(t) = (P_1(t), P_2(t), \cdots, P_N(t))$ be the state of the population at stage $t$, where $P_i(t) = \{p^{(i)}_{s_i}(t), p^{(i)}_{s'_i}(t)\}$ such that $p^{(i)}_{s_i}(t) + p^{(i)}_{s'_i}(t) = 1$. From the learning rule given in Eqn. 16, we can see that probability of choosing a strategy is dependent on the previous stage probability and the accumulated utility until current stage. Hence it is clear that $\{P(t) : t \geq 0\}$ is a Markov process. To prove that the learning process eventually converges to an equilibrium strategy, we basically need to analyze the asymptotic behavior of $\{P(t)\}$ when the learning constant $\kappa \to 0$. We can apply the Theorem 3.1 from [50] (presented below) and state that when $\kappa \to 0$, the Markovian sequence $\{P(t)\}$ converges to the solution of following ordinary differential equation (ODE).

$$\frac{dP}{dt} = f(P), P(0) = f(0) \tag{19}$$

where, $P(0)$ is the initial strategy selection probability of the population and $f(P)$ is the conditional expected function defined as:

$$f(P) = \mathbb{E}[G(P(t), \bar{U}^{(i)}_{s_i \in S}(T), \bar{U}^{(i)}_{avg}(T), a(t))|P(t)] \tag{20}$$

where, $G(\cdots)$ can be correlated with Eqn.16 by considering the learning equation for the population as $P(t) = P(t-1) + \kappa G(P(t), \bar{U}^{(i)}_{s_i \in S}(T), \bar{U}^{(i)}_{avg}(T), a_i(t))$.
*Theorem 3.1* [50]: Consider the sequence of interpolated processes $\{P_b(.)\}$. Let $X_0 = P_b(0) = P(0)$. Then the sequence converges weakly, as $b \to 0$ to $X(.)$ which is a solution of the ODE, $\frac{dX}{dt} = f(X), X(0) = X_0$.

The above theorem is a particular case of generalized result presented in Theorem 3.2 of Kushner [51], which can be stated as follows.
*Theorem 3.2* [51]: Assuming the conditions of Canonical algorithm [51], each subsequence of $\{\theta^\epsilon(q_\epsilon \epsilon + \boldsymbol{.}), \epsilon > 0\}$ has a further subsequence which converges weakly to a bounded solution $\theta(\boldsymbol{.})$ of $\dot{\theta} = g(\theta)$ on $[0, \infty)$ if $q_\epsilon = 0$ and on $(-\infty, \infty)$ if $q_\epsilon \epsilon \to \infty$, where $g(\boldsymbol{.})$ is a continuous function of $\theta$.

Using the above theorem, we can note the following points about our learning algorithm.

1. It is true that $\{P(t), (a(t-1), U_{diff}(t-1)), t > 0\}$ is a Markov process, where $U_{diff}(t) = \bar{U}^{(i)}_{s_i}(t) - \bar{U}^{(i)}_{avg}(t)$. Additionally, the tuple $(a(t), U_{diff}(t))$ takes value from the compact metric space.

2. The function $G(\cdots)$ is a continuous and bounded function.

3. If $P(t) = P$ is a constant at stage $t$, then the process $\{(a(t), U_{diff}(t)), t > 0\}$ is i.i.d. sequence.

4. The differential equation given in Eq. 19 has a unique solution for every initial condition.

Using Theorem 3.2 from [51], it can be stated that the sequence $\{P(t)\}$ converges to the solution of the differential equation presented in Eqn. 19 when the learning constant $\kappa \to 0$. Hence the lemma follows.

It is evident that the individuals preferably take an action that could lead them to survive in the population. According to the learning rule provided in Eqn.16, we can state that if the pairwise interactions provide higher utility for taking a certain strategy then the individuals can invade the rest of the unsuccessful strategies in the population, hence leading to a stable equilibrium strategy. As in our model, we have two pure strategies available for the players, and both can potentially be evolutionarily stable, our algorithm always leads to one of the pure strategy solutions.

## 6. Information Sharing Game

The above proposed game model best presents a scheme to motivate firms toward participating in CYBEX like sharing framework. Using evolutionary analysis, we have derived conditional constraints on participation cost ($c$), a driving parameter for attracting firms to join and transact cyber-threat intelligence (CTI) with other firms. For the sake of simplistic analysis, we also assumed that every participating firm in CYBEX shares a constant amount of CTI. However, realistically some rational firms may share less whereas some share high based on their best interest. Hence constant sharing may not successfully capture the true sharing nature of firms, which is why we no longer restrain this assumption in our extended model. As an extension, we address how the information exchange can be enhanced when firms differentiate their threat knowledge sharing nature after they participate in the framework. This is a very crucial problem because it can lead us to understand whether the firms truthfully intend to share all of their information or only exchange minimally to free-ride on others' threat intelligence. In such cases, it is important to have different benefit components for differentiated sharing with additional exogenous support in terms of incentive from CYBEX to enhance and motivate the firms to truthfully share their complete information. To analyze this differentiated sharing scenario, we extend the evolutionary analysis for a strategic game called "information sharing game", where players are mixture of firms participating in CYBEX and non-participants. The strategic form game of information sharing is discussed extensively in the following subsections.

### *6.1. Formulating Information Sharing Game*

To measure advantages of information sharing from the perspective of participants compared to the ones who are not willing to participate in CYBEX, we considered mix of $N$ rational firms (both participants and non-participants) in the strategic game. For the sake of simplicity, we assumed that firms have three choices in the strategic game irrespective of their participation. If the firm is participating, it can either choose all of its discovered CTIs or choose to share less CTI than its potential or if it is not satisfied with the sharing benefits it can leave the framework and be a non-participant. Thus, from a firm's perspective the objective is to find a stable equilibrium strategy among the three above discussed ones that will maximize the best interest of the firm, which is the CTI sharing benefits. On the other hand, CYBEX is aiming to bring more firms to the sharing framework and lead them to share all of their CTIs truthfully rather than allowing the firms to free-ride. To do so, CYBEX introduces two different incentive parameters for two different sharing levels which we call as high sharing (HS) strategy and low sharing (LS) strategy. Now it is vital to investigate, under what conditions these incentives can bring the group of firms taking LS strategy to HS strategy so that every firm can truthfully share all of its threat knowledge in the community. To model the payoff of the firms, we have extended the components used in CYBEX participation game to the followings.

*6.1.1. Differentiated Sharing Gain*

When a firm is not participating in the sharing framework, then we can infer that the firm is not interested in sharing its CTIs with others and decides to tackle cybersecurity issues solely. Therefore, these non-participants do not expect anything from CYBEX. However, some firms out of curiosity might want to get involved in the exchange framework to check whether they can resolve any security issues by exchanging a few information. This acute distinction between two actions made us to formulate two of the three strategies, i.e. no participation (NP) and low sharing (LS), for the information sharing game. However, there can be some firms who actually realized about the advantages of information sharing and are willing to share all of their information truthfully. This action of firms is denoted as high sharing (HS) strategy. LS strategy is only favorable in two scenarios, (1) when the firms do not get the worth of their truthfully shared cyber-threat information, (2) when firms decide not to share all of their information and free-ride on others' CTIs, so that the cost of information sharing is minimized. These cases motivated us to have only three strategies for the information sharing game which will also help in simplifying the evolutionary analysis of the game.

Since the firms have two choices of action for sharing their CTIs, the reward for both actions must be different. When the firms choose to share few information, i.e. take low share (LS) strategy, the reward for such action is directly dependent on strategy taken by the other interacting firm. Therefore, if the interacting firm is sharing less or fully sharing, then the gain for LS strategy will be low or high respectively. However, when a firm which shares information, interacts with a non-participating firm, then we assume that the sharing firm do not receive any benefit for its sharing rather incurs cost for such action. The rewards are nothing but the quantitative form of incidence responses, patches/fixes for vulnerabilities etc. In addition to the gain out of distinct sharing nature, the firms get benefited from their direct investment towards cybersecurity too.

*6.1.2. Incentive Integrated Cost Component*

It is clear that when a firm does not participate in the sharing framework, it does not share any of its threat intelligence, which is a risk averse and cost reduction strategy that might not be helpful in long-term. Whereas, the firms who risk to participate and share information incur a cost for both participation as well as CTI exchange. The cost of CTI exchange might be due to risk of reputation loss or chances of getting flooded with attacks based on the shared vulnerabilities. Since the firms are participating in CYBEX, they can help out each other to recover from hazardous situations by sharing necessary threat related responses. So we integrate an incentive parameter from CYBEX that can waive some of the cost incurred with firms' participation and CTI sharing. The incentives are not necessarily same for HS and LS strategies rather have a strict difference. Incentive of minimal sharing cannot exceed the incentive for fully share strategy. However, these incentives can be wisely chosen by CYBEX to motivate firms to truthfully share all their CTIs.

*6.2. Strategic Form of Information Sharing Game*

In this subsection, we formulate the strategic form of the information sharing game. As per our above discussion, the players in this game have three strategies namely (i) High Sharing (HS) (ii) Low Sharing (LS) (iii) No Participation (NP). We consider that when the firms do not participate, they do not incur the cost of participation and also do not gain anything from information sharing since they don't have to share. However, they only gain from their own investment. When the firms share low, which means they participate in the information sharing game, they incur a participation cost and the sharing cost at the same time while getting benefited not only from their sharing activity and investment but also externally from the CYBEX as incentives. The similar case happens when the firms share fully too, but the firm taking this strategy is assumed to gain more compared to the ones who choose low sharing strategy. As a matter of fact, if the incentives are not worthy enough, the firms always choose not to participate at all.

Once the game is formalized, it is our best interest to solve it from an evolutionary game point-of-view to define what strategies are evolutionarily stable and under what conditions they can be achieved. By saying equilibrium stable strategy (ESS) we are required to verify that no other population deviating from the ESS can completely invade the population playing with ESS strategy. Through such analysis, we can come up with scenarios, where incentives from CYBEX can be controlled tactfully to avoid free-riding and at the same time firms can truthfully share all of their CTIs with each other. Assuming the strategy set $S_i$ for firm $i$ can be the following, Table 2 represents the payoff in strategic form for the information sharing game, when a pairwise interaction among two firms occurs.

$$S_i = \{\textbf{High Sharing (HS)}, \textbf{Low Sharing (LS)}, \textbf{No Participation (NP)}\}$$

| | HS | LS | NP |
|---|---|---|---|
| HS | $S_h a\log(1+I) - x_h - c + \delta_1$, $S_h a\log(1+I) - x_h - c + \delta_1$ | $S_l a\log(1+I) - x_h - c + \delta_1$, $S_h a\log(1+I) - x_l - c + \delta_2$ | $a\log(1+I) - x_h - c + \delta_1$, $a\log(1+I)$ |
| LS | $S_h a\log(1+I) - x_l - c + \delta_2$, $S_l a\log(1+I) - x_h - c + \delta_1$ | $S_l a\log(1+I) - x_l - c + \delta_2$, $S_l a\log(1+I) - x_l - c + \delta_2$ | $a\log(1+I) - x_l - c + \delta_2$, $a\log(1+I)$ |
| NP | $a\log(1+I)$, $a\log(1+I) - x_h - c + \delta_1$ | $a\log(1+I)$, $a\log(1+I) - x_l - c + \delta_2$ | $a\log(1+I)$, $a\log(1+I)$ |

Table 2: Strategic–form of Information Sharing Game

We have extended the similar game structure as we proposed for CYBEX participation game previously by integrating differentiated sharing gain/cost component and CYBEX controlled incentives. From Table 2, it can be seen that when two firms have decided to participate, their CTI sharing strategies can be distinguished into two categories: share minimally or fully share. If both firms choose to take the same strategies (either LS or HS), then their payoffs received are same because the gain and cost components are assumed to be same for same strategies.

However, when one firm takes LS strategy and other takes HS strategy, this situation can be classified as free-riding of the former firm. Since the firm playing LS strategy receives complete set of CTIs exchanged by the firm taking HS strategy, its indirect sharing gain is $S_h > 1$, whereas the truthful firm does not receive enough support from the other firm and ends up having a sharing gain of $S_l > 1$, where $S_h > S_l$. Thus, to prevent such free-riding situations, external incentives $\delta_1, \delta_2$ from CYBEX are introduced. When CYBEX observes the firm's truthful nature towards CTI sharing, it rewards a higher incentive $\delta_1$, otherwise a reduced incentive of $\delta_2$ is awarded. These incentives also keep the firms motivated toward information sharing, however can be used to punish them if free-riding is detected. The cost component for a participating firm is composed of participation cost ($c$) and the sharing cost based on chosen strategy ($x_h$ for HS and $x_l$ for LS strategy). Since there is a clear distinction between high share and low share strategy in terms of quantity of information exchanged, the cost of taking such strategies is also ordered according to same metric. Thus, we assume that $x_h > x_l$. When a participating firm interacts with a non-participant, it is clear that the non-participating firm cannot get advantages of CTI shared by the participating firm. Whereas if the participating firm shares less or high CTIs, a sharing cost of $x_l$ or $x_h$ will be incurred respectively but the non-participant neither incurs any cost, nor gains anything in terms of threat intelligence.

## 7. Evolutionary Analysis of Information Sharing Game

To find the evolutionary stable strategy of the above formulated three-strategy game, we assume $p_1, p_2, p_3$ are the proportions of total $N$ population taking strategy HS, LS and NP respectively, where $p_1 + p_2 + p_3 = 1$. To understand the dynamics of firm population taking LS strategy and HS strategy, we derive the replicator equations for both population groups.

$$g_1(p_1) = p_1(\mathbb{E}[U_{HS}] - \mathbb{E}[U_{net}]) \tag{21}$$

$$g_2(p_2) = p_2(\mathbb{E}[U_{LS}] - \mathbb{E}[U_{net}]) \tag{22}$$

where, $\mathbb{E}[U_{HS}]$ and $\mathbb{E}[U_{LS}]$ are the expected payoffs of a firm when it chooses "High Sharing" and "Low Sharing" strategy respectively, and $\mathbb{E}[U_{net}]$ is the average utility of the population of firms.

$$\mathbb{E}[U_{HS}] = a\log(1+I)(p_1 S_h + p_2 S_l + p_3) - x_h - c + \delta_1 \tag{23}$$

$$\mathbb{E}[U_{LS}] = a\log(1+I)(p_1 S_h + p_2 S_l + p_3) - x_l - c + \delta_2 \tag{24}$$

$$\mathbb{E}[U_{NP}] = a\log(1+I) \tag{25}$$

$$\mathbb{E}[U_{net}] = p_1\mathbb{E}[U_{HS}] + p_2\mathbb{E}[U_{LS}] + p_3\mathbb{E}[U_{NP}] \tag{26}$$

Replacing the expected utility expressions from equation 23, 24, 26 in replicator dynamics equations 21 and 22, we find,

$$g_1(p_1) = p_1\{a\log(1+I)(1-p_1-p_2)[p_1(S_h-1)+p_2(S_l-1)] \\ -(x_h+c-\delta_1)(1-p_1)+(x_l+c-\delta_2)p_2\} \quad (27)$$

$$g_2(p_2) = p_2\{a\log(1+I)(1-p_1-p_2)[p_1(S_h-1)+p_2(S_l-1)] \\ +(x_h+c-\delta_1)p_1-(x_l+c-\delta_2)(1-p_2)\} \quad (28)$$

Evolutionary stability for the information sharing game can be achieved when the following conditions are achieved: (1) the rate of change of different population proportions are zero, (2) neighborhood of the fixed points or possible stable strategies found must be also stable. Therefore, the replicator equation $g_1(p_1)$ and $g_2(p_2)$, a.k.a transformation speed of high sharing and low sharing population group must be equated to zero and solved to obtain the possible fixed points. Hence by solving $g_1(p_1) = 0, g_2(p_2) = 0$, and taking all possible combinations of following expressions, we can find the set of possible solutions of $p_1$ and $p_2$.

$$p_1 = 0 \text{ or, } a\log(1+I)(1-p_1-p_2)(p_1S'_h+p_2S'_l) = \tilde{x_h}(1-p_1)-\tilde{x_l}p_2 \quad (29)$$

$$p_2 = 0 \text{ or, } a\log(1+I)(1-p_1-p_2)(p_1S'_h+p_2S'_l) = -\tilde{x_h}p_1+\tilde{x_l}(1-p_2) \quad (30)$$

where, $\tilde{x_h} = x_h+c-\delta_1$, $\tilde{x_l} = x_l+c-\delta_2$, $S'_h = S_h-1$, and $S'_l = S_l-1$

Thus, the set of solutions for triple $(p_1, p_2, p_3)$ can be listed as the followings:

$$\alpha_{sol_1} = (0,0,1) \quad (31)$$

$$\alpha_{sol_2} = (0,1,0) \quad (32)$$

$$\alpha_{sol_3} = (0, \frac{\tilde{x_l}}{AS'_l}, 1-\frac{\tilde{x_l}}{AS'_l}) \quad (33)$$

$$\alpha_{sol_4} = (1,0,0) \quad (34)$$

$$\alpha_{sol_5} = (\frac{\tilde{x_h}}{AS'_h}, 0, 1-\frac{\tilde{x_h}}{AS'_h}) \quad (35)$$

where, $A = a\log(1+I)$. Two more solutions can be found by solving the following two expressions from Eqn. 29 and Eqn. 30.

$$a\log(1+I)(1-p_1-p_2)(p_1S'_h+p_2S'_l) = \tilde{x_h}(1-p_1)-\tilde{x_l}p_2$$
$$a\log(1+I)(1-p_1-p_2)(p_1S'_h+p_2S'_l) = -\tilde{x_h}p_1+\tilde{x_l}(1-p_2)$$

Since the l.h.s of both equations are same, we can equate the r.h.s expressions and by solving we can find that $\tilde{x_l} = \tilde{x_h} = x$. This condition will only be satisfied if $p_1 = p_2 = p$, hence the sharing gains must be same, i.e. $S'_h = S'_l = s$. Now solving for one of the above equations including all the assumptions, we find

$$(1-2p)(2Asp-x) = 0$$
$$p = 0.5 \text{ or } p = \frac{x}{2As}$$

Thus, the new possible solutions triples can be

$$\alpha_{sol_6} = (0.5, 0.5, 0) \tag{36}$$

$$\alpha_{sol_7} = (\frac{x}{2As}, \frac{x}{2As}, 1 - \frac{x}{As}) \tag{37}$$

We found 7 possible solution triples as fixed points of the evolutionary game. However, we must check which of these solutions are in fact stable by verifying the neighborhood stability criteria.

*7.1. Stability Verification of Fixed Points*

According to Taylor and Jonker [52], a fixed point $p = (p_1, p_2)$ is said to be strictly stable equilibrium strategy, if the eigen values [53] of $\mathbf{J}(g_1, g_2)$ matrix have negative real part, where $\mathbf{J}(.)$ is the Jacobian matrix of replicator dynamics equations and represented by the following.

$$\mathbf{J}(g_1, g_2) = \begin{bmatrix} \frac{\partial g_1}{\partial p_1} & \frac{\partial g_1}{\partial p_2} \\ \frac{\partial g_2}{\partial p_1} & \frac{\partial g_2}{\partial p_2} \end{bmatrix}$$

where, first order differentials are the followings:

$$\frac{\partial g_1}{\partial p_1} = A(1 - 2p_1 - p_2)(p_1 S_h' + p_2 S_l') + Ap_1(1 - p_1 - p_2)S_h' - (1 - 2p_1)\tilde{x_h} + p_2\tilde{x_l}$$

$$\frac{\partial g_1}{\partial p_2} = -Ap_1(p_1 S_h' + p_2 S_l') - Ap_1(1 - p_1 - p_2)S_l' + p_1\tilde{x_l}$$

$$\frac{\partial g_2}{\partial p_2} = A(1 - p_1 - 2p_2)(p_1 S_h' + p_2 S_l') + Ap_2(1 - p_1 - p_2)S_l' + p_1\tilde{x_h} - (1 - 2p_2)\tilde{x_l}$$

$$\frac{\partial g_2}{\partial p_1} = -Ap_2(p_1 S_h' + p_2 S_l') - Ap_2(1 - p_1 - p_2)S_h' + p_2\tilde{x_h}$$

Now, we are required to check which of the fixed points from $\alpha_{sol_1}$ to $\alpha_{sol_7}$, when imposed on the Jacobian matrix, can produce an eigen vector that has negative real part. This mandatory check can take care of the neighborhood stability criteria. Now, we derive the Jacobian matrix at each fixed point found previously (Eqn. 31 – 37) in the following:

$$\mathbf{J}|_{\alpha_{sol_1}=(0,0,1)} = \begin{bmatrix} -\tilde{x_h} & 0 \\ 0 & -\tilde{x_l} \end{bmatrix} \tag{38}$$

$$\mathbf{J}|_{\alpha_{sol_2}=(0,1,0)} = \begin{bmatrix} -\tilde{x_h} - \tilde{x_l} & 0 \\ -AS_l' - \tilde{x_h} & -AS_l' + \tilde{x_l} \end{bmatrix} \tag{39}$$

$$\mathbf{J}|_{\alpha_{sol_3}=(0, \frac{\tilde{x_l}}{AS_l'}, 1 - \frac{\tilde{x_l}}{AS_l'})} = \begin{bmatrix} \tilde{x_l} - \tilde{x_h} & 0 \\ J_{11} & \tilde{x_l}(1 - \frac{\tilde{x_l}}{AS_l'}) \end{bmatrix} \tag{40}$$

$$\mathbf{J}|_{\alpha_{sol_4}=(1,0,0)} = \begin{bmatrix} -AS_h' + \tilde{x_h} & -AS_h' - \tilde{x_l} \\ 0 & -\tilde{x_h} - \tilde{x_l} \end{bmatrix} \tag{41}$$

$$\mathbf{J}|_{\alpha_{sol_5}=(\frac{\tilde{x_h}}{AS'_h},0,1-\frac{\tilde{x_h}}{AS'_h})} = \begin{bmatrix} \tilde{x_h}(1-\frac{\tilde{x_h}}{AS'_h}) & J_{12} \\ 0 & \tilde{x_h}-\tilde{x_l} \end{bmatrix} \tag{42}$$

$$\mathbf{J}|_{\alpha_{sol_6}=(0.5,0.5,0)} = \begin{bmatrix} \frac{-A}{4}(S'_h+S'_l)+\frac{\tilde{x_l}}{2} & \frac{-A}{4}(S'_h+S'_l)+\frac{\tilde{x_l}}{2} \\ \frac{-A}{4}(S'_h+S'_l)+\frac{\tilde{x_h}}{2} & \frac{-A}{4}(S'_h+S'_l)+\frac{\tilde{x_h}}{2} \end{bmatrix} \tag{43}$$

$$\mathbf{J}|_{\alpha_{sol_7}=(\frac{x}{2As},\frac{x}{2As},1-\frac{x}{As})} = \begin{bmatrix} 0.5x-\frac{x^2}{2As} & -0.5x+\frac{x^2}{2As} \\ -0.5x+\frac{x^2}{2As} & 0.5x-\frac{x^2}{2As} \end{bmatrix} \tag{44}$$

We now need to verify which of the fixed points are stable according to the neighborhood stability criteria. This will help us in finding the conditional constraints on the incentive parameters that will lead the population towards adopting ESS. To demonstrate stability of a fixed point mathematically, we must show that the eigen values of the corresponding Jacobian matrix have negative real part [52]. To find the eigen values ($\lambda$) of a matrix $A$, we solve the equation $\det(A-\lambda I)=0$, where $\lambda$ denotes the eigen values of matrix $A$ and $I$ represents the identity matrix of same dimension as $A$. In the following, we find the eigen values of the Jacobian matrix at each fixed point and derive the conditions at which the stability can be achieved.

1. Eigen values of $\mathbf{J}|_{\alpha_{sol_1}}$ are $\lambda_1=-\tilde{x_h}$ and $\lambda_2=-\tilde{x_l}$, which have negative real part if the following conditions are satisfied.

$$\delta_1 < x_h + c \text{ and } \delta_2 < x_l + c \tag{45}$$

2. Eigen values of $\mathbf{J}|_{\alpha_{sol_2}}$ are $\lambda_1=-\tilde{x_h}-\tilde{x_l}$ and $\lambda_2=-AS'_l+\tilde{x_l}$. Since the eigen values are real by themselves and they can be negative if the following conditions are held.

$$\delta_1+\delta_2 < x_h+x_l+2c \text{ and } \delta_2 > x_l+c-A(S_l-1)$$
$$\implies x_l+c-A(S_l-1) < \delta_2 < (x_h+x_l+2c)-\delta_1 \tag{46}$$
$$\text{and } \delta_1 < x_h + A(S_l-1) \tag{47}$$

3. Eigen values of $\mathbf{J}|_{\alpha_{sol_3}}$ are $\lambda_1=\tilde{x_l}(1-\frac{\tilde{x_l}}{AS'_l})$ and $\lambda_2=-\tilde{x_h}-\tilde{x_l}$. From the fixed point $\alpha_{sol_3}=(0,\frac{\tilde{x_l}}{AS'_l},1-\frac{\tilde{x_l}}{AS'_l})$, it is clear that $\frac{\tilde{x_l}}{AS'_l}\leq 1$ and $\tilde{x_l}>0$. Hence the eigen value $\lambda_1$ cannot be negative. Therefore, the solution $\alpha_{sol_3}$ is not a stable equilibrium. By similar reasoning, we can also prove that the fixed point $\alpha_{sol_5}$ is not a stable strategy.

4. Eigen values of $\mathbf{J}|_{\alpha_{sol_4}}$ are $\lambda_1=-\tilde{x_h}-\tilde{x_l}$ and $\lambda_2=-AS'_h+\tilde{x_h}$. The eigen values are real and they can be negative if the following conditions are satisfied.

$$\delta_1+\delta_2 < x_h+x_l+2c \text{ and, } \delta_1 > x_h+c-A(S_h-1)$$
$$\implies x_h+c-A(S_h-1) < \delta_1 < (x_h+x_l+2c)-\delta_2 \tag{48}$$
$$\text{and, } \delta_2 < x_l + A(S_h-1) \tag{49}$$

5. Finding the eigen values of $\mathbf{J}|_{\alpha_{sol_6}}$ leads to solve the following equation

$$\lambda\left(\lambda+\frac{-A}{2}(S_h'+S_l')-\frac{\tilde{x_h}+\tilde{x_l}}{2}\right)=0$$

   It is clear that one of the eigen values is zero, which by itself invalidates the stability criteria. Therefore the fixed point $\alpha_{sol_6}=(0.5,0.5,0)$ is not a stable equilibrium. Using the similar analysis on $\mathbf{J}|_{\alpha_{sol_7}}$, we can find that one of the eigen values is zero. Since it cannot be negative, we simply discard this unstable solution.

After analyzing the above scenarios, we conclude that each of the three strategies has potential to be an evolutionarily stable strategy for the firms. However, the decision of a particular ESS can be controlled by the incentive parameters that is guided by CYBEX. In other words, we can state that the CYBEX controlled incentive parameters $\delta_1, \delta_2$ can be suitably altered to achieve one of the desired equilibrium scenario. The incentive parameters can essentially be used to motivate the firms taking LS strategy towards sharing all of their information truthfully, i.e. adopting HS strategy.

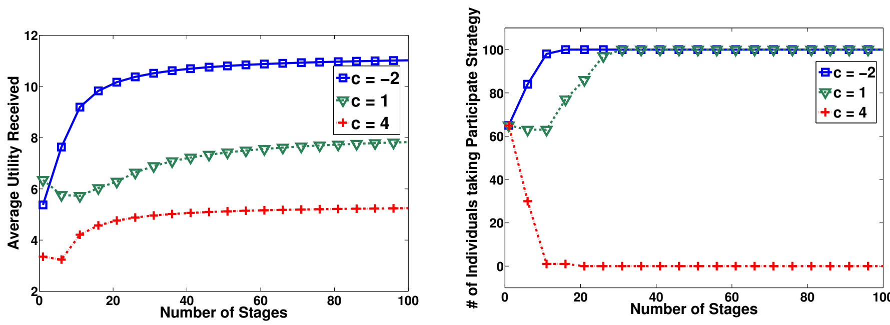
## 8. Results and Discussion

To demonstrate the validity of our proposed model, we have simulated the CYBEX participation and information sharing nature of organizations, when different participation costs ($c$) and sharing costs ($x$) are imposed. Due to lack of standard valuation of parameters related to CTI sharing, we have chosen to validate our model by assigning appropriate scaled values for both cost components. Although we have chosen the values of parameters arbitrarily in our experimental scenarios, it has been made sure that the parameters follow the conditional constraints, derived in the paper.

### *8.1. Simulation for CYBEX Participation Game*

In this subsection, we present the simulation results that have been obtained from the proposed distributed learning heuristic to attain ESS under different conditions. The population size is assumed as 100 throughout the simulation. The rationality constant $a$ and investment $I$ are assumed to be 2, and 5 units respectively, which are kept same for all the experiments. The value of sharing cost ($x$) and participation cost ($c$) is varied dynamically to maintain different conditions described in Section 5. The learning constant ($\kappa$) is assumed to be 0.07. Each stage represents all possible simultaneous pairwise interactions between the players, and the game is played for 500 stages at max in each experiment.

In Fig. 7a, we plot the evolution of average utility over the number of stages for different participation cost ($c$) values. It is observed that when $c$ is negative, the individuals find an incentive to participate and share. However, when $c > 0$, the individuals choose to take part and share in the framework opportunistically




(a) Average utility growth (b) Evolution of participating population

Figure 7: Evolution dynamics and average utility of participating population under different participation costs

depending on how many other players participate and share in the framework. Therefore, the average utility converges at high value when the participation cost is minimum, where the population unanimously play the "Participate & Share" strategy. As $c$ increases above certain threshold, the individuals find that participate strategy is costly and switch to "Not Participate" strategy, which is why the saturated average utility is less for $c = 4$ than 1. It is shown that the proposed heuristic helps the individuals reach the evolutionary stable state within fewer game stages by making them learn about the expected utilities of different strategies. We have experimented to understand how quickly the population adapts to ESS, we plot the growth of "Participate and Share" strategy population in Fig. 7b. It is clear that a population type either invades another type or gets invaded by the other type, depending on the cost constraints. If the participation cost ($c$) is negative, then it is intuitive that everybody will be willingly participate and share because the participation cost is nothing but an incentive. However, when the cost is positive, then the stable strategy depends on how many other members adopt that particular strategy. In our experimental setup, the population converges to "Participate and Share" when the initial sharing strategy population is 65% or more and cost ($c$) is 1, but they get invaded by the rest of "Not Participate" strategy individuals if $c$ increases to 4 because the population threshold requirement is now well above .65. The important point to notice here is the convergence speed of the proposed learning heuristic, which enables the firms to obtain their ESSs within a few number of game stages.

In Fig. 8, we present the evolution of average utility with respect to different values of learning rate $\kappa$. The reason of having this experiment is to understand which learning rate would preferably help the system to quickly reach ESS. It is found that low as well as high $\kappa$ value takes relatively longer time to stabilize the average utility because low $\kappa$ takes longer duration to reach the optimal selection probability and high $\kappa$ oscillates around the optimal solution. Hence, the step size $\kappa$ must be chosen carefully to achieve quick convergence to the equilibrium strategy. From the result reported, we observed that the value of $\kappa$ in the range of 0.05 to 0.09 works better for our simulation and helps the players to quickly adapt to the ESS.

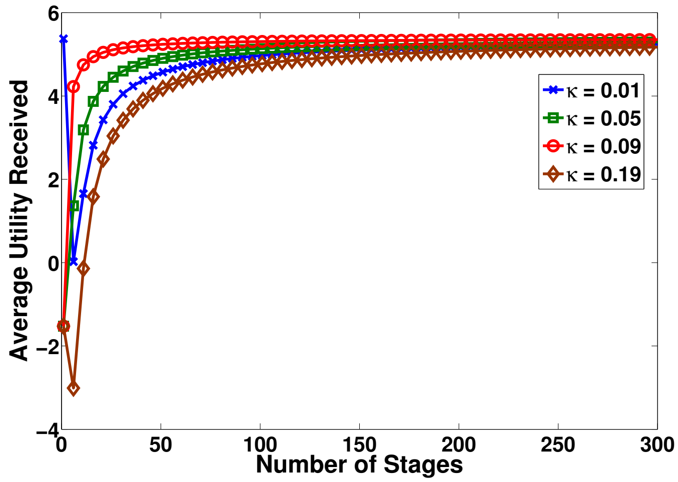
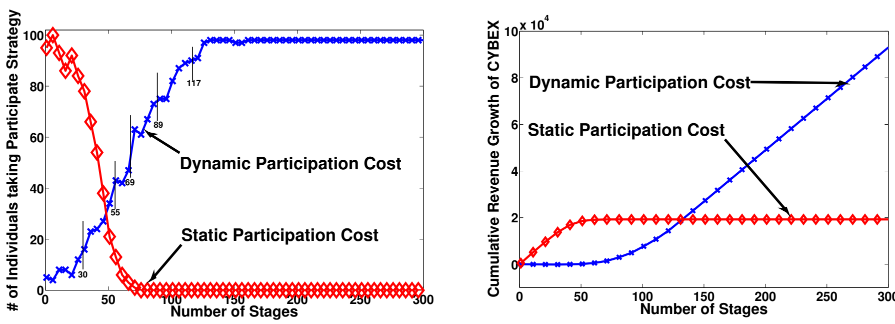


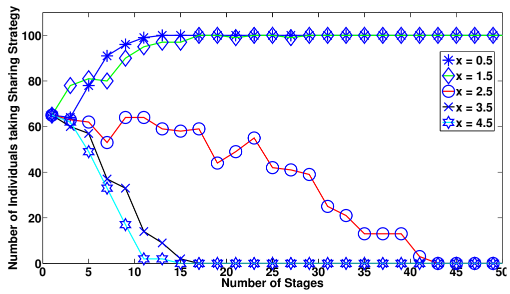

Figure 8: Evolution of average utility as the game progresses under different learning constant



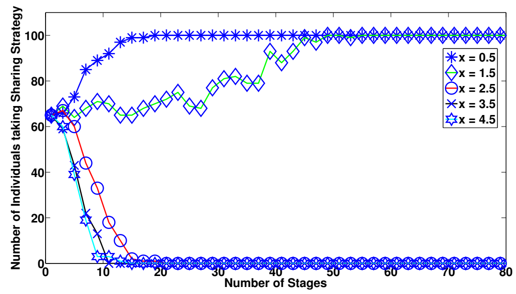

(a) Dynamics of Participating Population (b) CYBEX's Gross Revenue comparison

Figure 9: Evolution of participating population and revenue of CYBEX under static and dynamic participation cost

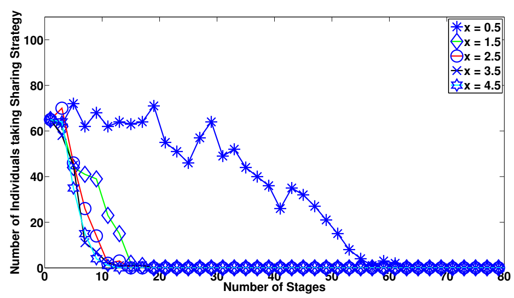
To understand how our proposed dynamic participation cost/incentive heuristic can help CYBEX to increase its revenue, we simulate two scenarios presented in Fig. 9a: where (1) 95% individuals initiated with "participate and sharing" strategy in the beginning but CYBEX charges a static amount ($c = 5$) towards participation all along, and (2) CYBEX uses our proposed dynamic participation cost/incentive mechanism (based on conditions of case (iii & iv) presented in Section 5), even when only 5% of total population were participating in the beginning. It is observed that in the scenario (1), the participating population percentage decreases over stages due to the high cost charged by CYBEX (as seen in Fig. 9a, in red color plot). It is also seen that the cumulative revenue of CYBEX over time does not increase any more as firms leave the framework gradually (as seen in Fig. 9b, in red color plot). However in scenario (2), CYBEX could manage to attract more firms to participate by rewarding ($-c = 0.5$) them in the beginning. As the number of participants started growing (going beyond the population threshold/tipping point given in case (iii), Section 5), CYBEX dynamically updates its participation cost within a certain limit (based on the cost conditions presented in case (iii), Section 5) to generate revenue. But it ensures that the cost raise does not lead the participating population to leave the framework rather it can still attract more participants to join so that eventually every firm will be inside the sharing framework. Thus, CYBEX's incremental cost raise can lead to a win-win situation, where every firm participates and shares to strengthen their security infrastructure, and CYBEX also generates an increasing revenue as depicted in Fig. 9a, and 9b respectively (blue color plots).



(a) Cost of participation ($c = 1$)



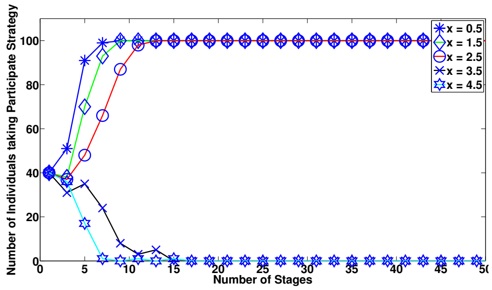
(b) Cost of participation ($c = 2$)



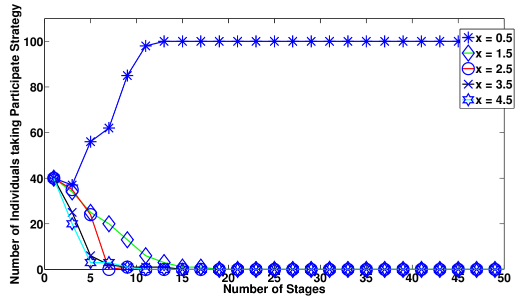
(c) Cost of participation ($c = 3$)

Figure 10: Evolution of participating population at different participation cost, when initial participant proportion is 0.65

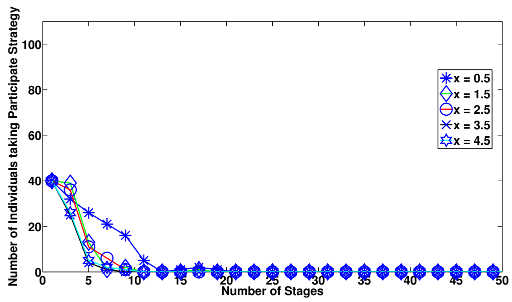
In our next set of simulation results presented in Fig.10 and Fig.11, we demonstrate the evolution of the population, when cost of participation ($c$) is varied and initial participant population is below/above than 50%. At a participation cost of 1, it is observed that case (3) gets satisfied until $x$ reaches 3.5. But we know that as x increases, the value of required participation threshold ($\alpha_{thres}$) increases too. We can find that, for $x = 0.5, 1.5, 2.5, 3.5$, the required $\alpha_{thres} = 0.28, 0.47, 0.651, 0.84$ respectively. Hence, the population can converge to "Participate" strategy when $x = 0.5$ and 1.5, provided $\alpha_{init} = 0.65$, which is true according to Fig10a. But, when $\alpha_{init} = 0.4$, the population can converge to "Participate" strategy when $x = 0.5$ only as seen in Fig.11b. However, when




(a) Cost of participation ($c = -2$)




(b) Cost of participation ($c = 1$)



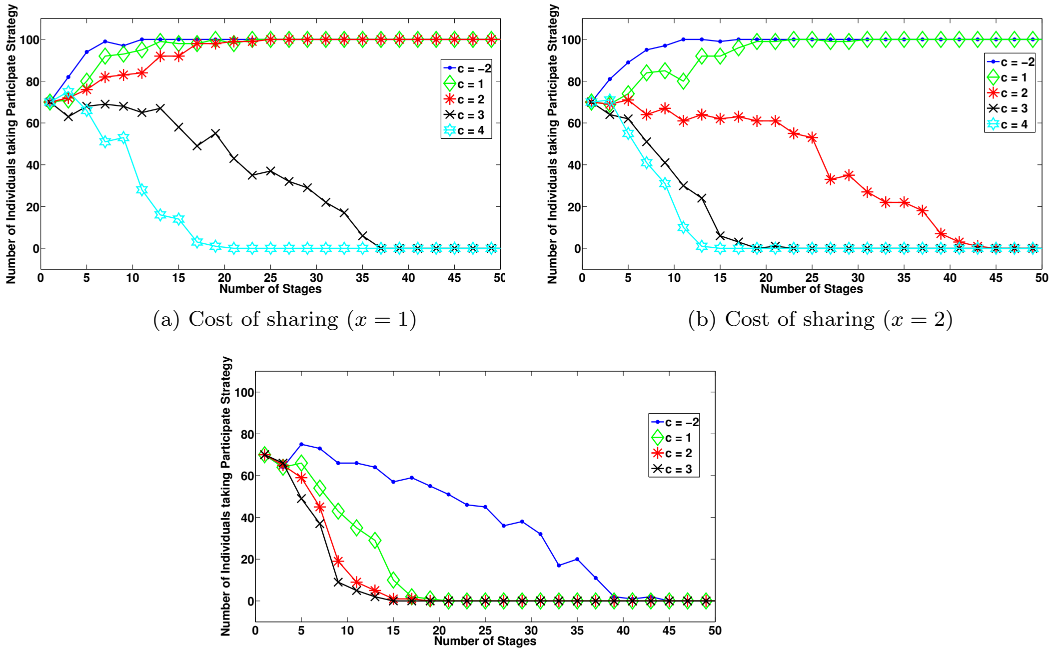

(c) Cost of participation ($c = 2$)

Figure 11: Evolution of participating population at different sharing cost, when initial participant proportion is 0.4

sharing cost ($x$) increases beyond 3.5, the total cost dominates over the sharing gain and hence case (1) or (2) is triggered, which is why "No Participate" strategy turns out to be evolutionarily stable.

If the participation cost is raised to 2, and cost of sharing is high ($x > 1.5$), then the cost component exceeds over the total gain, hence case (1) or (2) becomes applicable. We can see in Fig.10b and Fig:11c that high $x$ always disappoints the population, leading to zero participation eventually. Since the low value of $x$ can enforce the case (3), it requires at least 27% and 47% participants for $x = 0.5$ and 1.5 respectively, which cannot be satisfied when $\alpha_{init} = 0.4$ as shown in Fig.11c. However, the population can converge to full participation in case of $\alpha_{init} = 0.65$ as shown in Fig.10b. When the participation cost increases to 3 or more, then high sharing cost will always lead to no participation in the framework as we have seen in above cases. Fig.10c verifies that when $x$ is high, the case (1) or (2) is induced that leads the system to no participation altogether. When sharing cost is small, the strength of participating population is not enough to enforce all firms toward participation. However, when cost of participation is negative, i.e. CYBEX is providing incentives for participation, then firms would prefer to participate if the sharing cost is not dominating over the sharing benefits. We can observe from Fig.11a that when $c = -2$ and $x < 2$, the firms always get profit out of participation, hence it is wise for each firm to participate. However, as the cost of sharing increases gradually, firms' behavior changes because of higher cost, which is why the evolutionary stable strategy for the population turns out to be "No Participate".

**Remarks:** Since research in the CYBEX domain is still in its inception, there does not exist any standardized sharing platform yet to provide us actual field-data. Due to the lack of such field-data, our selection for parameter values not only adheres to the real-world rationality constraints but also conveys how these values would originate in the real-world. The values of the participation cost in Figure 10 are $1, 2$, and 3. While one can quibble that maximum costs may not be shown, as opposed to appropriate costs, these value assignments lay the stage for a conventional participant's expectations. First, a compulsory up-front unit cost (i.e., $c = 1$) is accrued when a participant elects to join in the sharing framework. Once having joined, a jump can ensue: a participant may witness a doubling (i.e., $c = 2$) or even a tripling (i.e., $c = 3$) of their costs in order to sustain the framework as it goes through its perturbations of evolution. A participant's acceptance of these cost-jumps reflect reluctance on the part of a participant to divest of the framework and thus lose their initial investment. Recognizing that participants would only embrace the framework because they might ultimately gain a benefit, the value assignment of $c = -2$ in Figure 11 demonstrates the participant earning back its initial investment (of $c = 1$) together with an additional payback of the same amount. The aforementioned cost values depict a mix of favorable and unfavorable situations. Both types must be represented because professionals would recognize that inevitably both would be encountered in practice. Likewise, our values for the initial participating population ($\alpha_{init}$) are placed in the realm of reality. We assert that an outsider (that is a non-participant) cannot observe the sharing framework's outcomes; instead they have to be in-situ to make meaningful determinations about the framework's performance. Therefore, values of $\alpha_{init}$ are in the vicinity of 50% to indicate that about half of the initial population takes the optimistic approach (i.e., anticipates success) whereas the remaining part of the population tends toward the pessimistic (i.e., fears failure). These values are based on experience: computer science is populated with protocols that can be either optimistic or pessimistic. For example, one can optimistically assume that deadlocks seldom occur and thus (hopefully) only have to rarely kill a deadlocked process; whereas, pessimistically, waits-for graphs are painstakingly constructed when a deadlock is fearfully anticipated - and subsequently - avoided. The choice to adopt an optimistic or pessimistic approach is based on finding a balance between performance expectations and risk, and in the evolution of computer science both types of approaches have consistently co-existed because that can be equally valuable. We therefore assigned $\alpha_{init}$ values that are near the pivotal balance between one of two choices. With a similar parameter selection approach, the nature of firms' participation in CYBEX is derived with respect to different sharing costs ($x = 1, 2,$ and 3), which are depicted in Figure 12 and 13. Since exchanging CTI comes at a non-zero cost, a participating organization may incur a unit cost ($x = 1$) or multiples of it, depending on the nature of the information involved and the consequences of sharing such CTI. The investment and incentive




(a) Cost of sharing ($x = 1$)

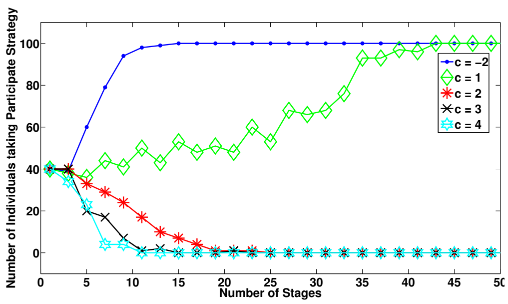
(b) Cost of sharing ($x = 2$)

(c) Cost of sharing ($x = 3$)

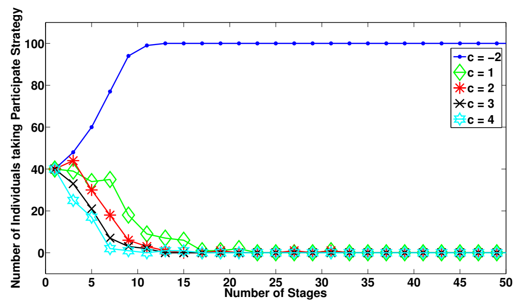
Figure 12: Evolution of participating population w.r.t. participation and sharing cost, when initial participant proportion is 0.65

parameters are represented in the form of monetary quantities, hence their values can be straightforwardly chosen by the organization and CYBEX respectively (with appropriate normalization). As CYBEX evolves, we believe that real data will become available and that such data can be readily used to further verify our proposed model.

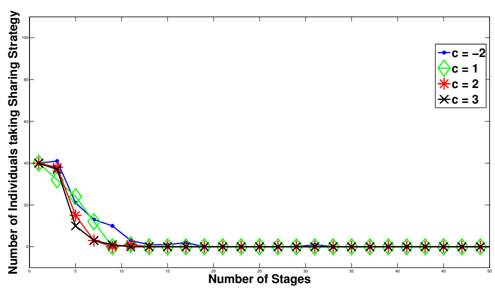
To verify our model for various values of $\alpha_{init}$, and sharing cost ($x$), an extensive set of simulation results are presented in Fig.12 and Fig.13. For the sake of generalization, we generate the results for two different values $\alpha_{init}$; (1) above 50%, where $\alpha_{init} = 0.65$ (2) below 50%, where $\alpha_{init} = 0.4$. The result set given in Fig.12 represents the evolution of individuals taking participate strategy over stages, when the initial proportion of participants ($\alpha_{init}$) is 65% and CYBEX imposes different participation costs ranging from $c = -2$ to 4. A similar simulation is conducted but with lesser initial participation, $\alpha_{init} = 0.4$, and the results are presented in Fig.13. It is observed that when cost of sharing is low ($x = 1, 2$) and firms are given incentives (i.e. $c < 0$), the population prefers the "Participate" strategy that eventually becomes ESS as seen in Fig.12a, Fig.12b, Fig.13a and Fig.13b irrespective of the initial population proportion. This is because, the condition (4) of our evolutionary analysis gets enforced here, where we know that "participate" strategy is evolutionarily stable. As $c$ increases to 1, the case (3) gets activated, where the required population threshold ($\alpha_{thres}$) is 0.37 when $x = 1$, 0.56 when $x = 2$, and 0.74 when $x = 3$. If we observe Fig.12a, where $\alpha_{init} = 0.65 > \alpha_{thres} = 0.37$, the population converges to complete participation and same happens in Fig.12b too $\alpha_{init} > \alpha_{thres} = 0.56$. However, in Fig.12c the population prefers not to participate because it did not have enough participants initially compared to $\alpha_{thres}$. We can now imagine that what situation will arise, when $\alpha_{init} = 0.4$. It is observed that the population will prefer to participate only when $x = 1$ because $\alpha_{init} = 0.4 > \alpha_{thres} = 0.37$ (Fig.13a) but not in any of the other two scenarios.




(a) Cost of sharing ($x = 1$)




(b) Cost of sharing ($x = 2$)




(c) Cost of sharing ($x = 3$)

Figure 13: Evolution of participating population at different sharing cost, when initial participant proportion is 0.4

Now, when $c$ increases to 2, the case (3) is still valid and the required population threshold ($\alpha_{thres}$) is increased for $x = 1, 2, 3$ to 0.56, 0.74, and 0.93 respectively. Now for $\alpha_{init} = 0.65$, we can say that the only scenario when $x = 1$ will converge to "participate" stable strategy as shown in Fig.12a. However, when $x = 2, 3$, 'No Participate" becomes ESS since $\alpha_{init} = 0.65 < \alpha_{thres}$. We can see that minimum participation threshold required for $x = 1, 2, 3$ is 0.56 which is more that $\alpha_{init} = 0.4$, none of the scenarios given in Fig.13 will converge to full participation.

When $c$ increases to 3, the case (3) will be valid only if $x \leq 2$, otherwise the cost component will dominate over the sharing gain and hence case (1) gets activated; which is why the population converges to "No Participate" strategy as shown in Fig.12c,and Fig.13c. At $x = 1, 2$, required $\alpha_{thres} = 0.74$, and 0.93 respectively, which is more that $\alpha_{init}$. Hence the population converges to "No

Participate" in Fig.12a, Fig.12b, Fig.13a, and Fig.13b for $c = 3$. When $c > 3$, the case (1) is applicable where ESS is "No Participate" strategy, as seen in Fig.12 and Fig.13 for $c = 4$.

*8.2. Simulation Results for Information Sharing Game*

To demonstrate that different stable equilibrium strategies can be achieved for the information sharing game, we conduct simulations by considering a population of size 100. Throughout the experiments, we fix a unity rationality constant ($a$) and investment ($I$) of 15. The participation cost is assumed to be 1 unit. The sharing costs for HS and LS are considered as 8 and 3 units respectively. From our evolutionary analysis, we understand that if the value of incentives $\delta_1$ and $\delta_2$ are taken in such a way that the $\tilde{x_h}, \tilde{x_l}$ are positive, the stable equilibrium strategy of the population will be not to participate provided conditions for other ESSs are not satisfied. An instance is shown in Figure 14, where $\delta_1 = 3$ and $\delta_2 = 2$ and it can be observed that even if the population proportion of all three different strategies are same in the beginning, the participating firms are invaded by the group of firms taking "No Participate" strategy. Eventually, all firms decide not to participate as the average utility reward out of sharing is dominated by utility obtained from not participating.

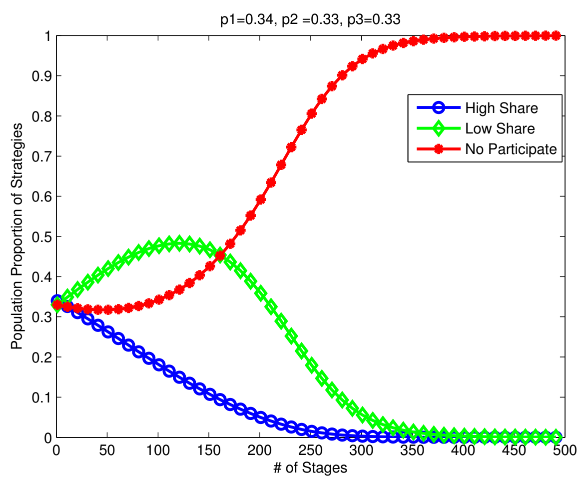



Figure 14: Evolution of population demonstrating "No Participation" strategy as ESS

In some situations, CYBEX may look forward in motivating firms to start sharing some of their CTIs, which may not be maximal. Thus, the incentive for LS strategy $\delta_2$ can be suitably increased to grow the population towards LS strategy such that the corresponding ESS condition is satisfied. In the plot given in Figure 15, we can see that when incentive $\delta_2$ is increased to 10 and $\delta_2$ set to 12, the population from no-participant group and high sharing group got invaded easily, leaving the LS strategy as evolutionarily stable. On the other hand, if the incentive for high sharing is appropriately set by CYBEX so that the respective

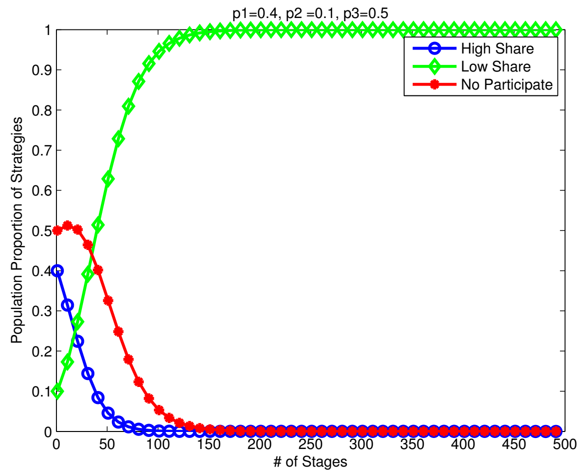



Figure 15: Evolution of population demonstrating “Low Sharing (LS)” strategy as ESS

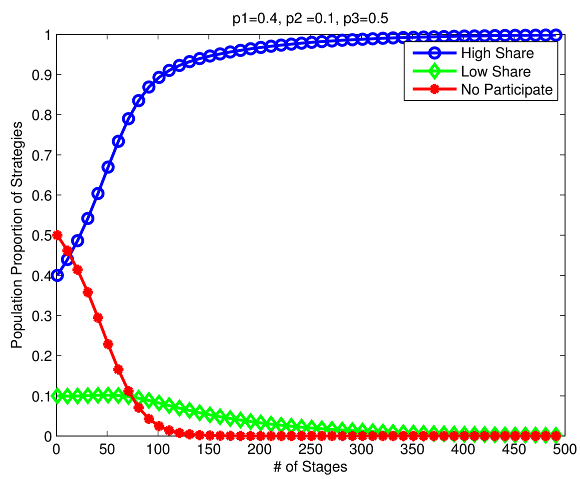



Figure 16: Evolution of population demonstrating “High Sharing (HS)” strategy as ESS

ESS criteria is satisfied, then every firm prefers to take HS strategy due to high payoff. However, when sufficient firms have switched their strategy to share completely and truthfully, the incentive of HS strategy can be reduced to make the sharing system self-sustained. A sample simulation scenario is presented in Figure.16, where we set the incentives $\delta_1 = 8.5$ and $\delta_2 = 3$. It can be observed that the population of HS strategy group could easily invade other populations when the external incentive is increased to 8.5. Thus, we can conclude from these three instances that the incentive parameters are crucial guiding parameter that can suitably modified by CYBEX according to conditions derived in Eqn.45–49 to decide which strategy can be evolutionarily stable. This will also help to guide the firms to share their complete cyber-threat information in CYBEX.

## 9. Conclusions and Future Study

In this research, we study the two crucial problems related to cybersecurity: (1) CYBEX participation game, where firms can be motivated to participate in the CYBEX framework, (2) Information sharing game to understand how the firms can be evolved to truthfully share all of their cyber-threat information in a differentiated sharing environment. We use evolutionary dynamics to analyze both participation and information sharing game and understand the outcomes of the game, i.e. evolutionary stable strategies (ESS). Various conditional constraints derived from the analysis have helped to devise a dynamic cost adaptation algorithm that exploits the participation cost to act like an incentive in the beginning so as to motivate as many firms to participate. However, as the participation strength grows, the incentive gradually turns to be a cost and let the sharing system to self-sustain, leading to a win-win situation. We also propose a distributed learning heuristic for the participating firms to let them attain ESS by learning from the history information. After the participation decision is resolved, we formulate an information sharing game, and analyze using similar evolutionary dynamics. We find that external incentivization from CYBEX can motivate the firms to share more information truthfully instead of staying out of the sharing framework or sharing minimally. In future, we plan to study the participation and information sharing game under differentiated investment scenarios, which will give a better vision of how security investment affects the cybersecurity information sharing decisions of the firms.


## Acknowledgement

This research is supported by the National Science Foundation (NSF), USA, Award #1528167.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